Improved "magic states" distillation for quantum universality

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[quant-ph/0411036]

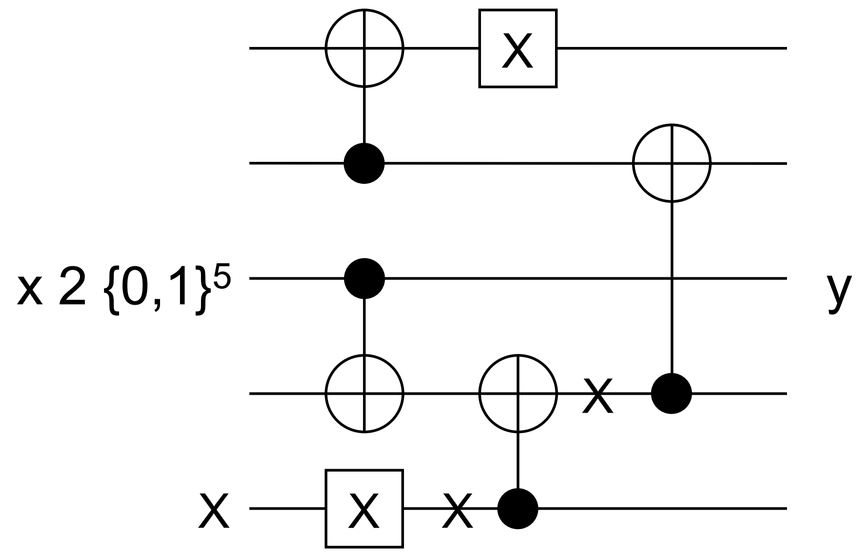
Stabilizer operations

Def: Stabilizer operations are

- Clifford group unitaries

$$\left\langle H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, K = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \mathrm{CNOT} \right\rangle$$

- Preparation of $|0\rangle$
- Measurement in $|0\rangle$, $|1\rangle$



$$y = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + x \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Gottesman-Knill Theorem: Stabilizer operations are efficiently classically simulable.

Main theorem

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- Preparation of $|0\rangle$
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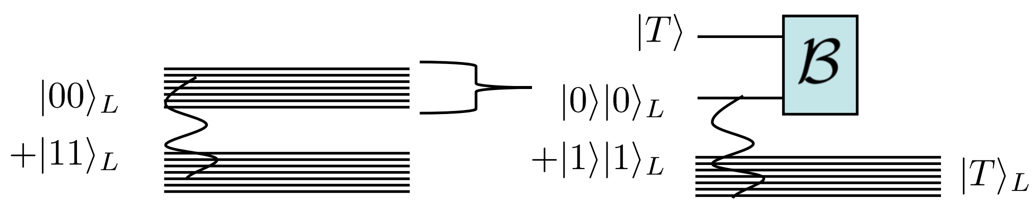
Theorem: [R'05] Stabilizer ops + prepare $|\psi\rangle$ any pure state not a stabilizer state gives quantum universality.

Application: [Knill'04] Estimated threshold of 5-10%.



Stabilizer op. fault-tolerance ⇝ Universal fault-tolerance

Proof of theorem

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Lemma: [R'05] Stabilizer ops + prepare $|\psi\rangle$ any single-qubit pure state not a Pauli eigenstate gives quantum universality.

Fact: Stab ops + prepare $|H\rangle \propto \begin{matrix} (1+\sqrt{2})|0\rangle \\ +|1\rangle \end{matrix}$
! universality.

Fact: Stab ops + $Z^{1/4} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
! universality.

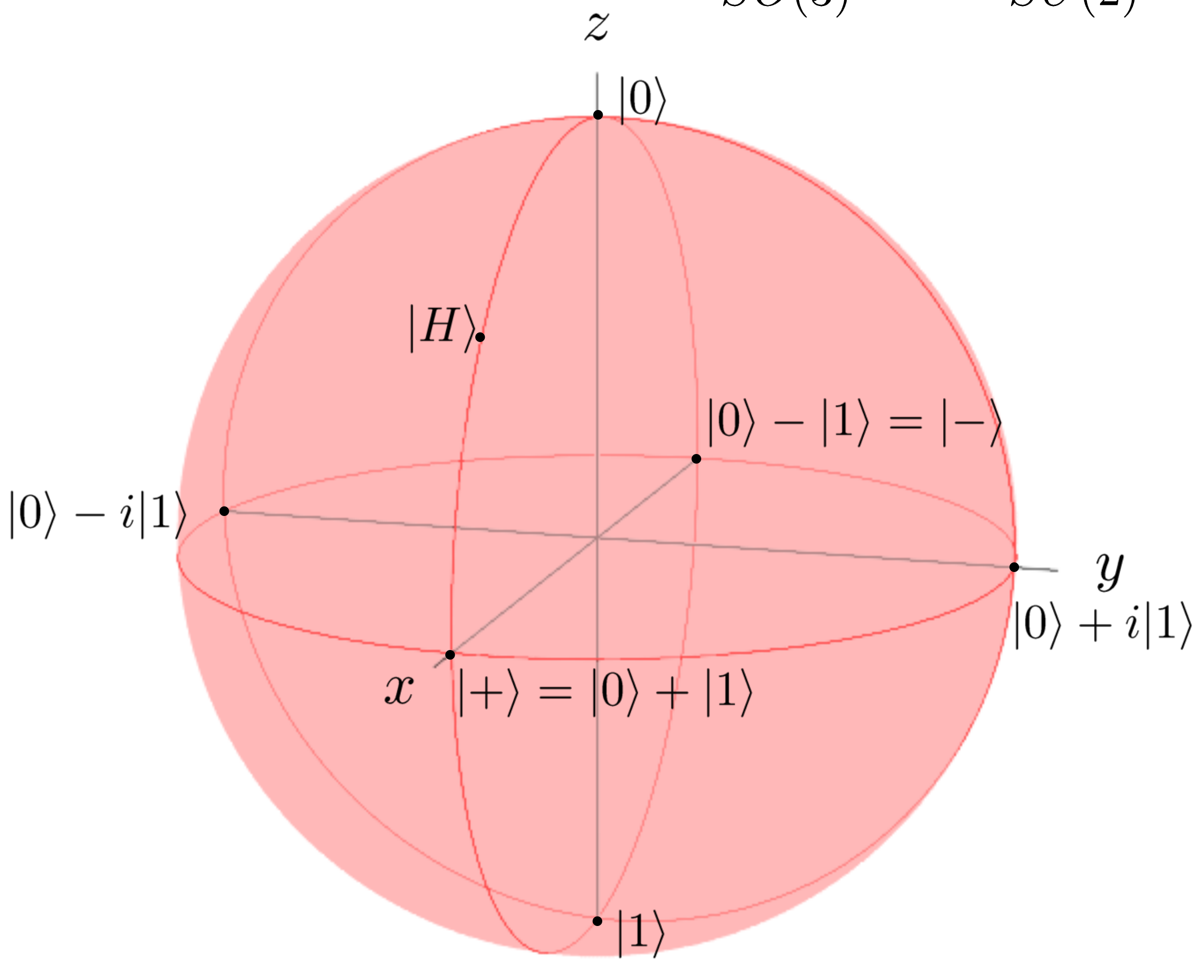
Open question: For which (single qubit) mixed states ρ does
stab ops + prepare ρ ! universality ?

Bloch sphere

$$(x, y, z) \leftrightarrow \frac{1}{2}(I + xX + yY + zZ)$$

$$r = 1 \leftrightarrow \text{pure states}$$

$$\underset{SO(3)}{\text{rotations}} \leftrightarrow \underset{SU(2)}{\text{unitaries}}$$



"Proof of Fact 2"

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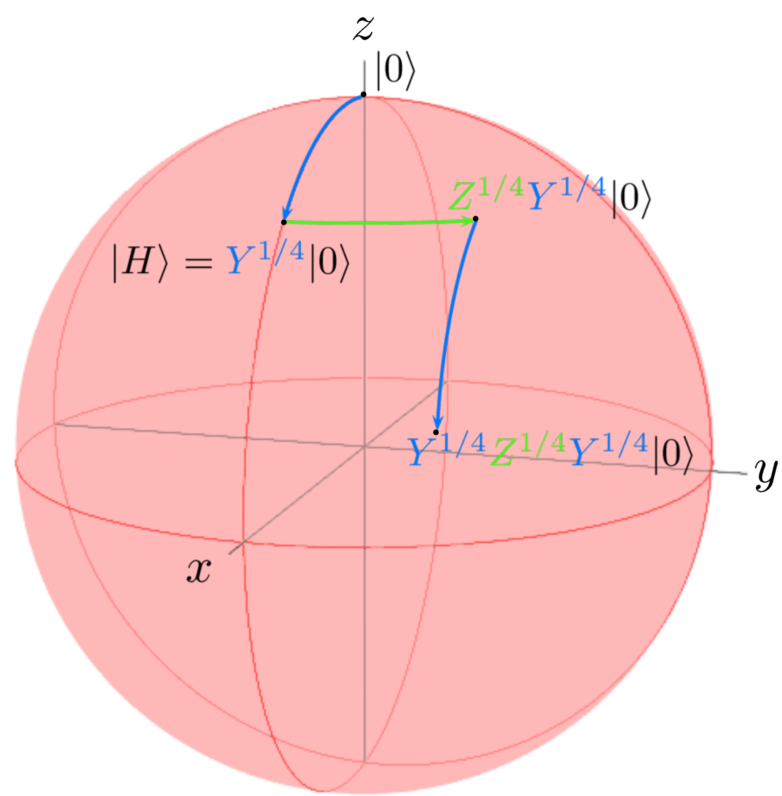

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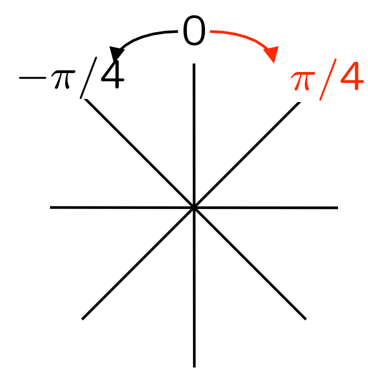
$$(\alpha|0\rangle + \beta|1\rangle)\,(|0\rangle + e^{i\pi/4}|1\rangle)$$

$$= \alpha|00\rangle + \beta e^{i\pi/4}|11\rangle$$
$$+\alpha e^{i\pi/4}|01\rangle + \beta|10\rangle$$



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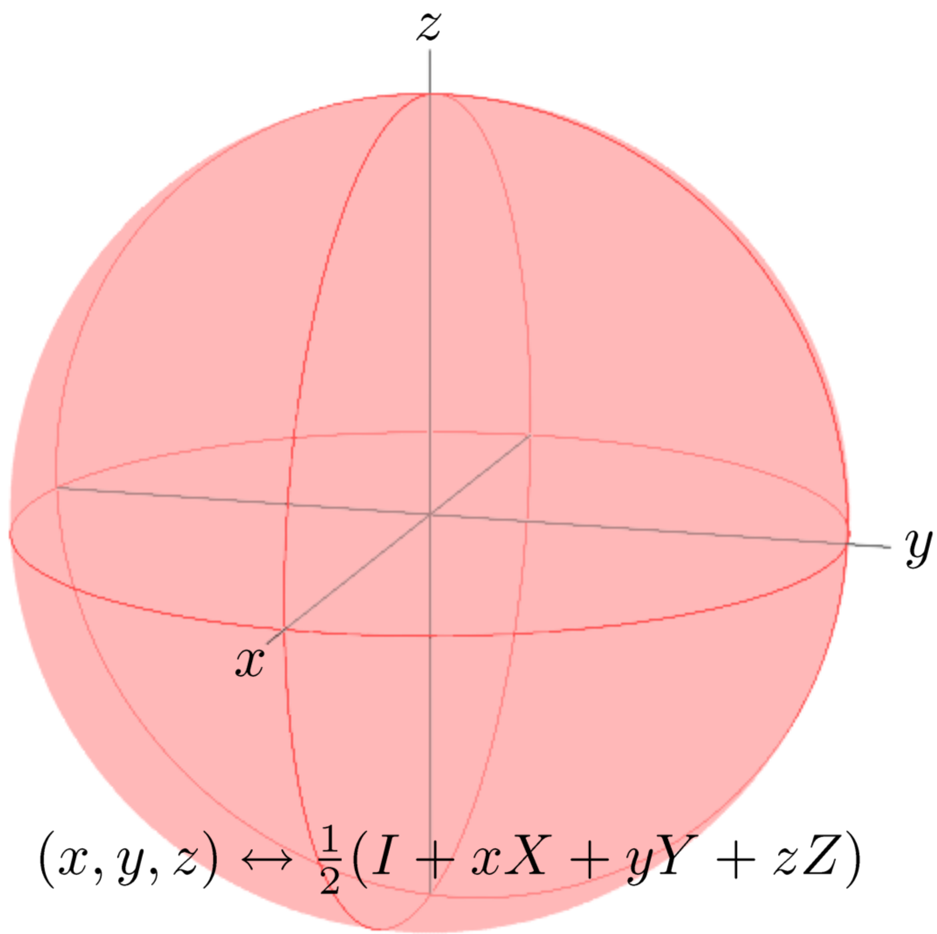

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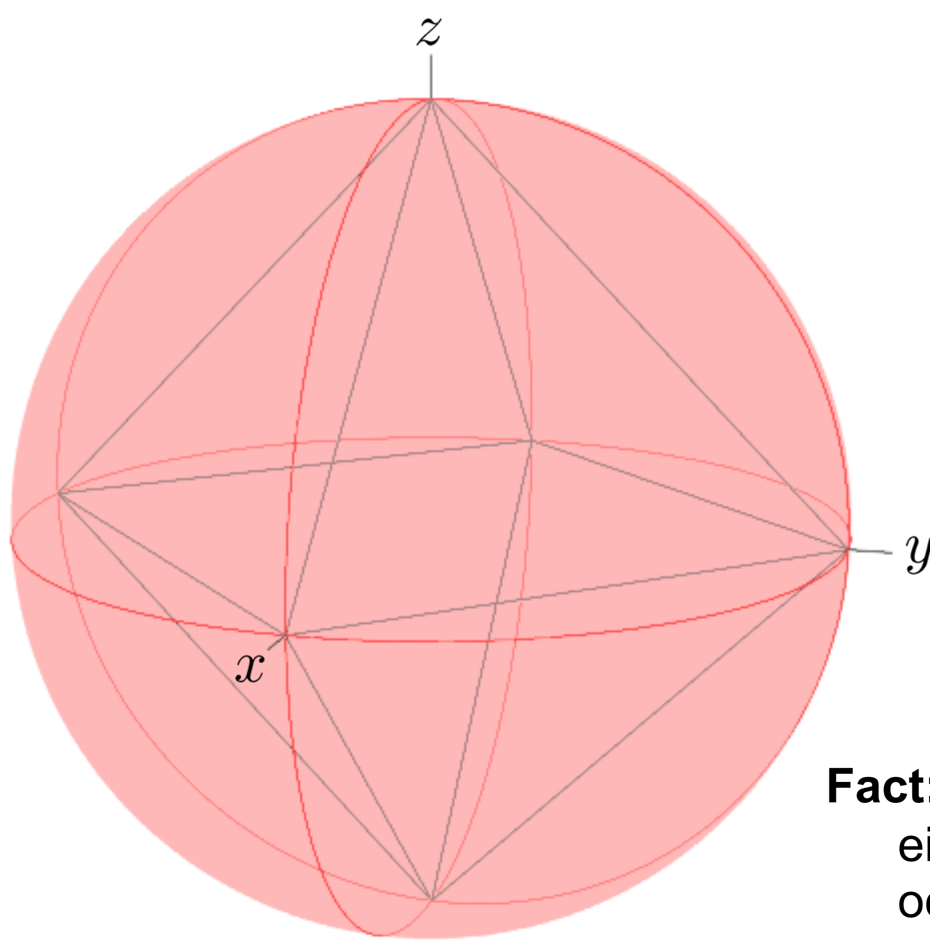

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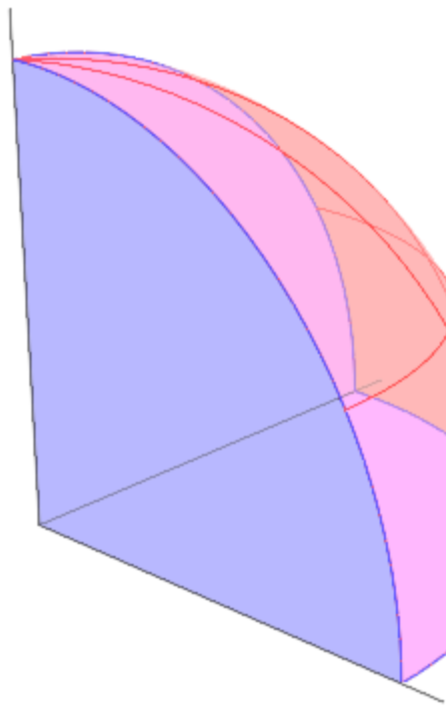

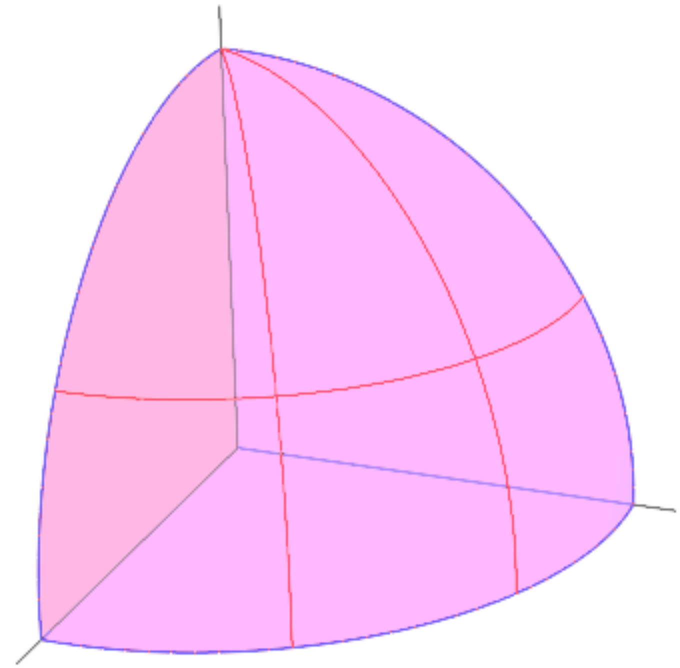

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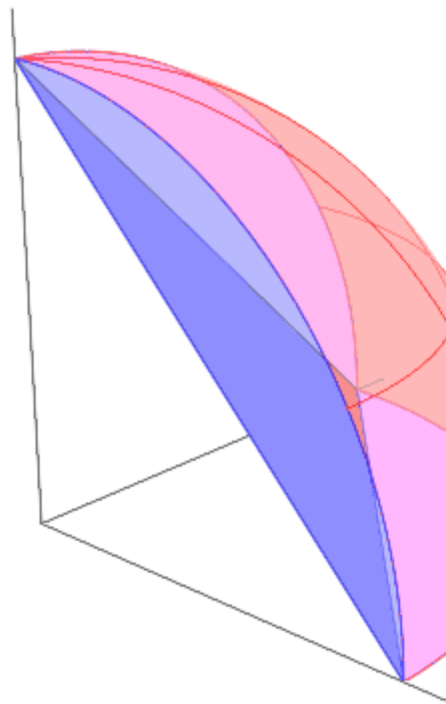

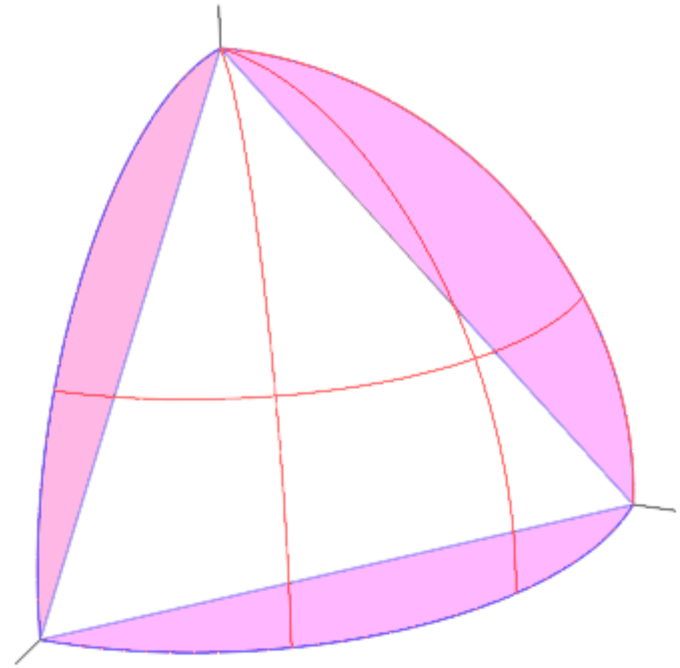

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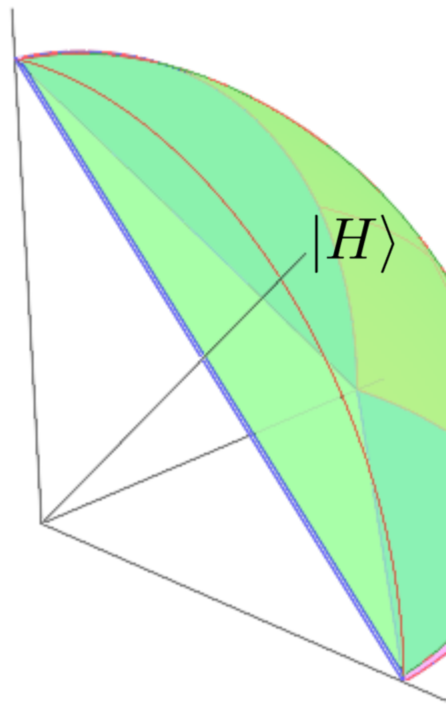

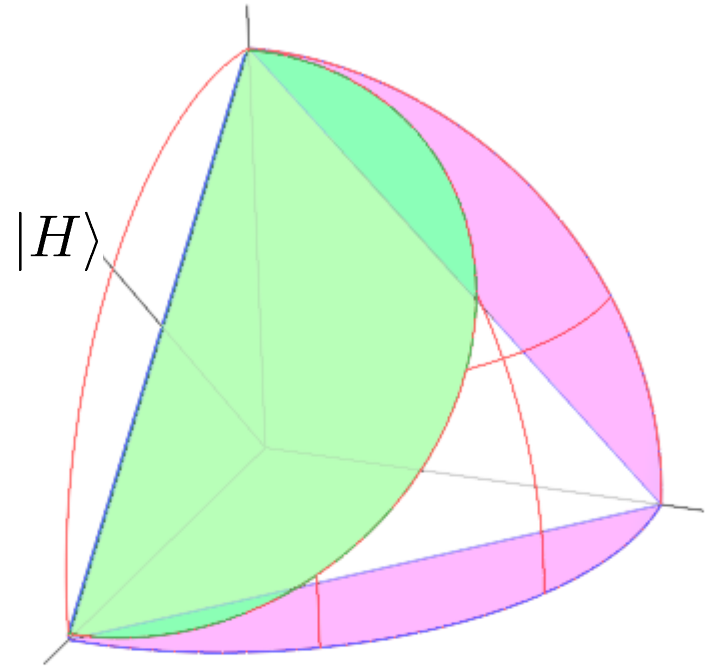

[Bravyi-Kitaev '04, Knill '04] Yes for $|H\rangle$ w/ <14.2% error

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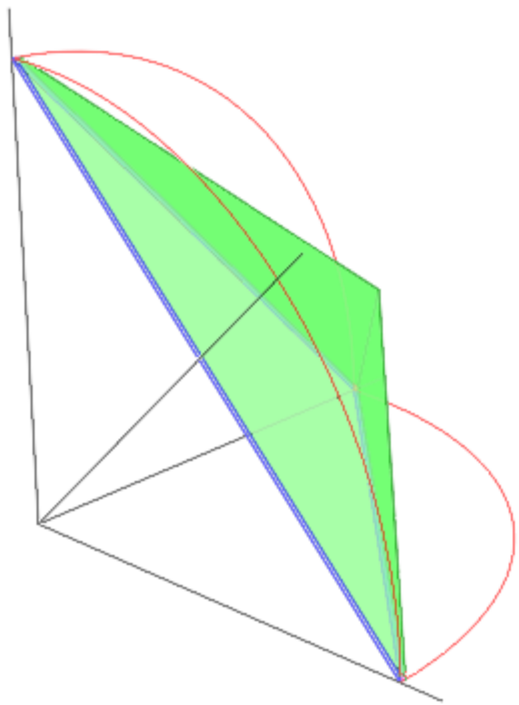

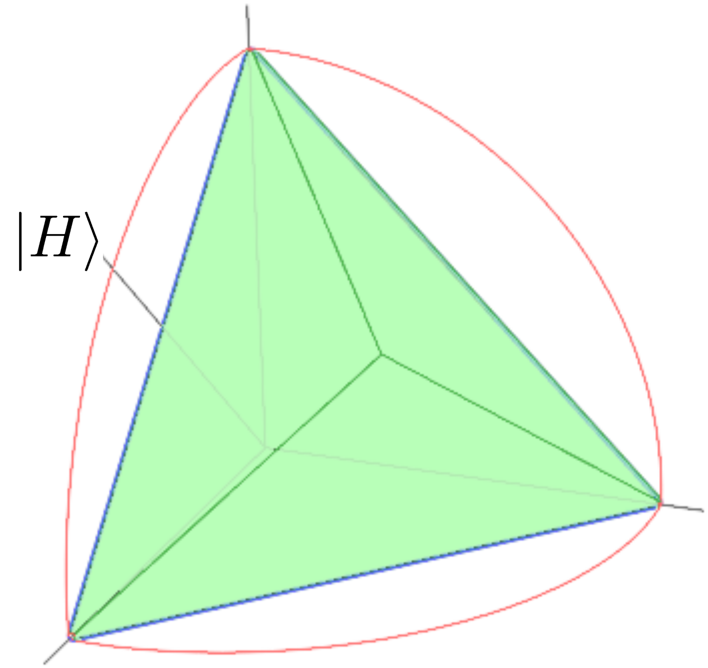

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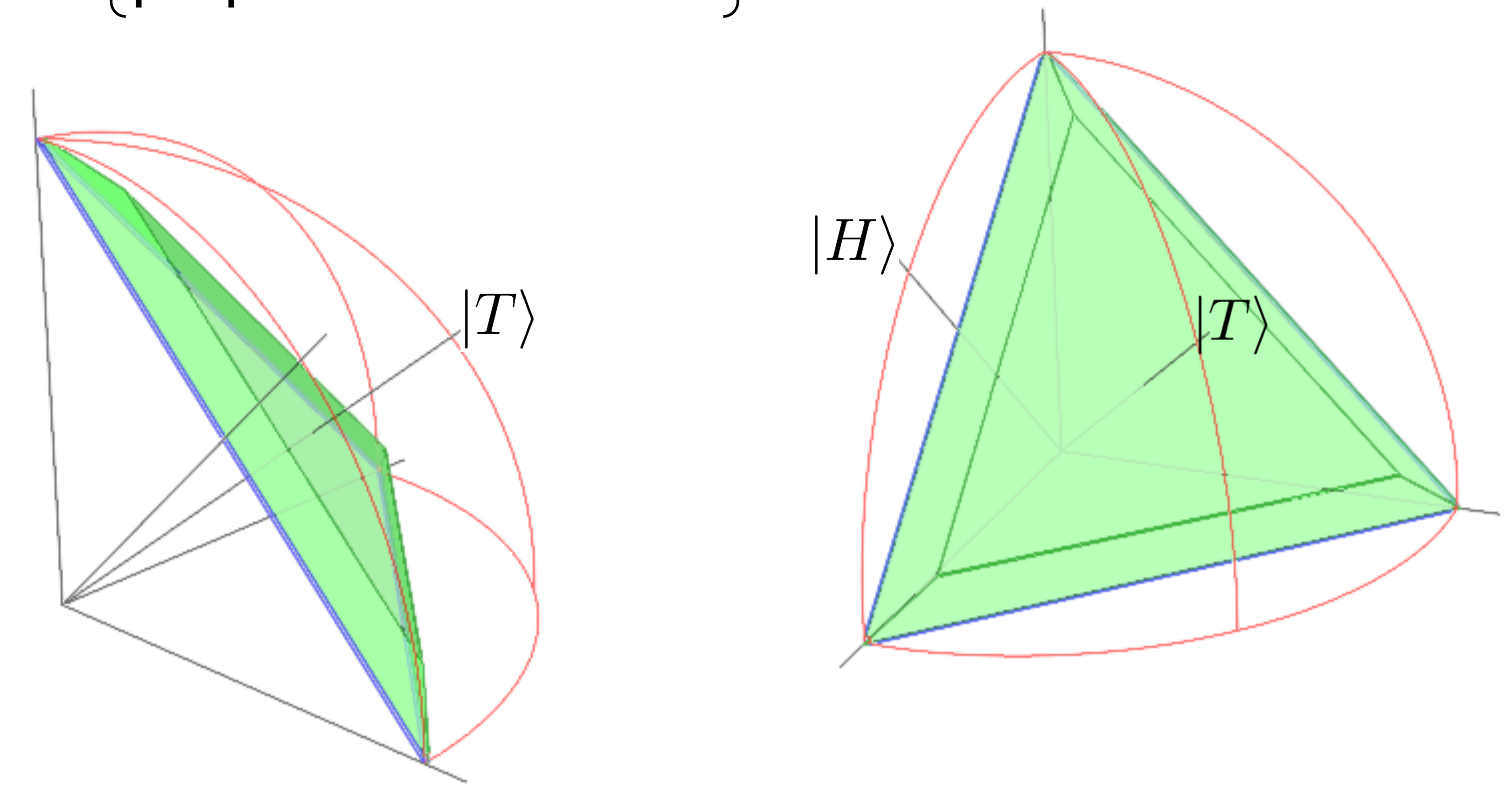

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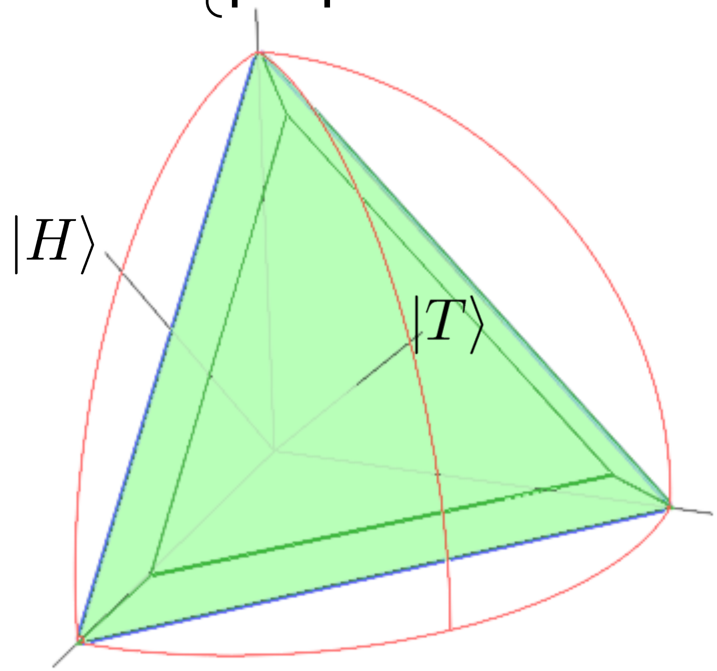

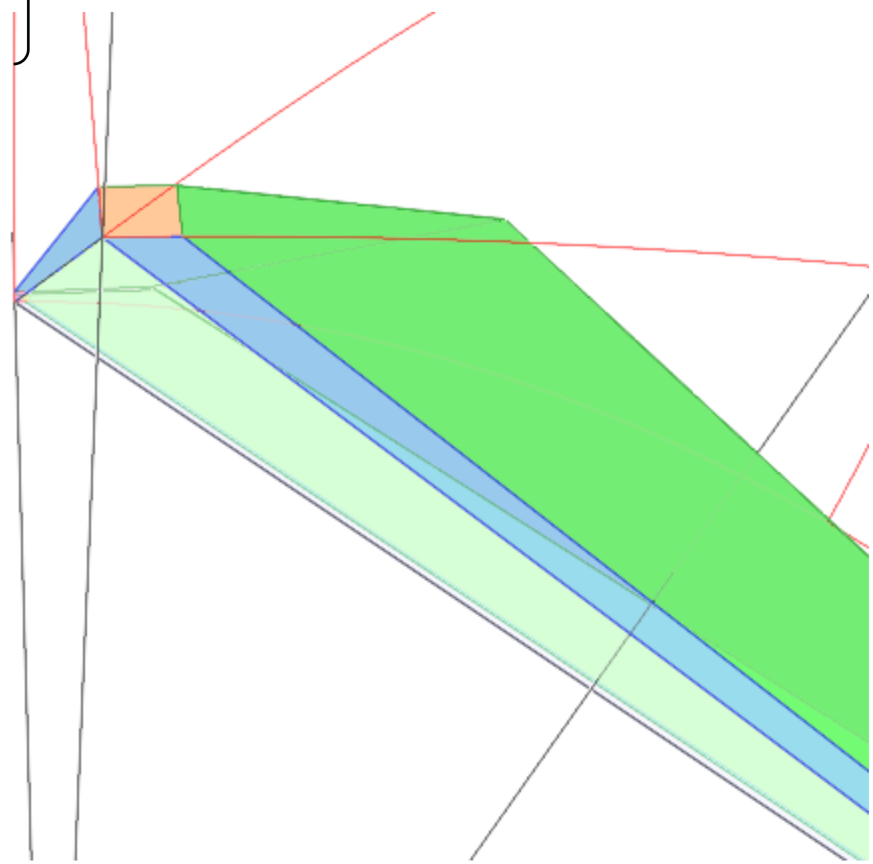

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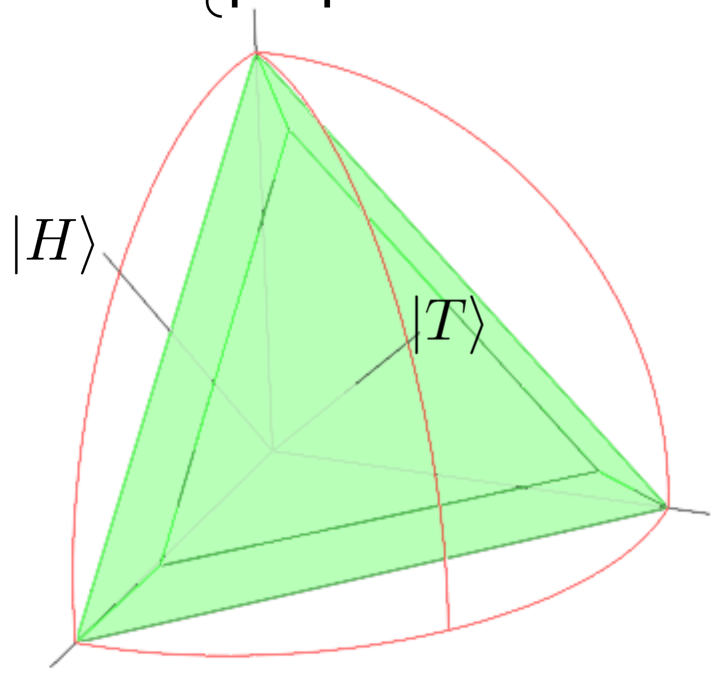

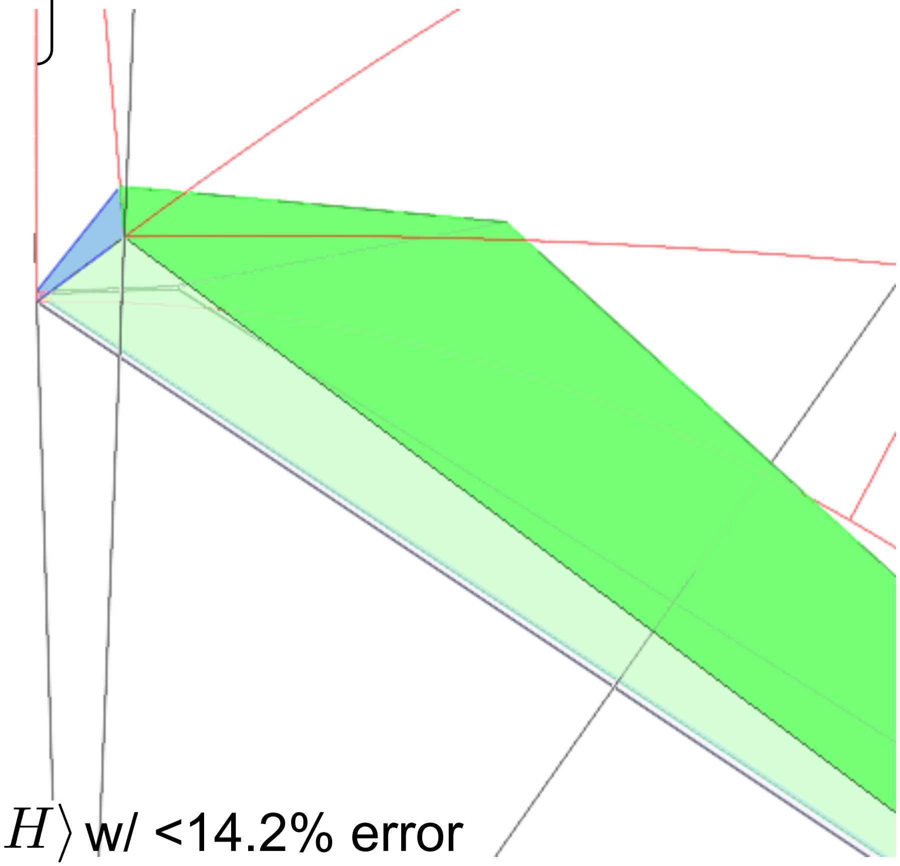

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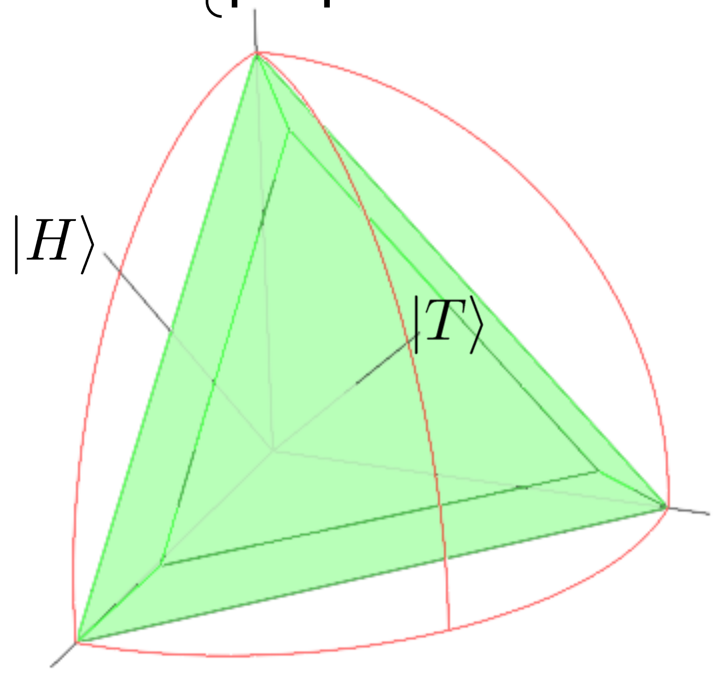

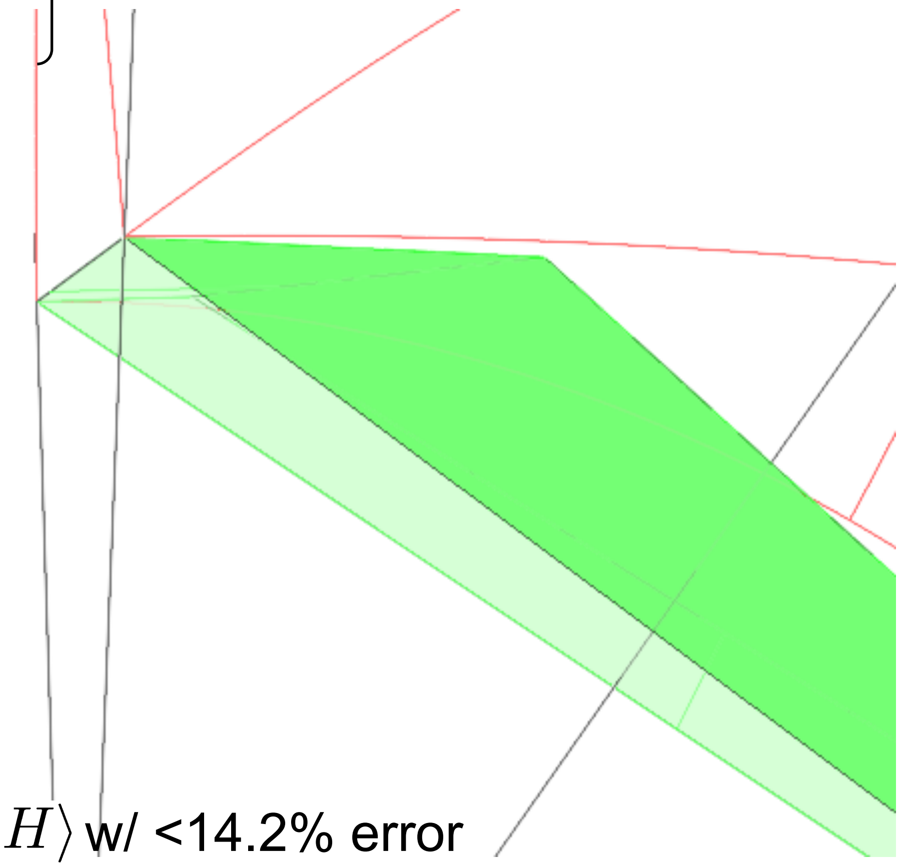

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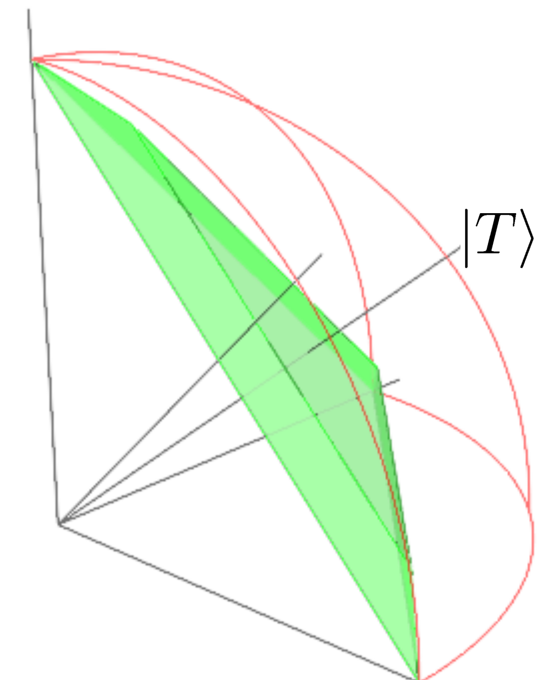

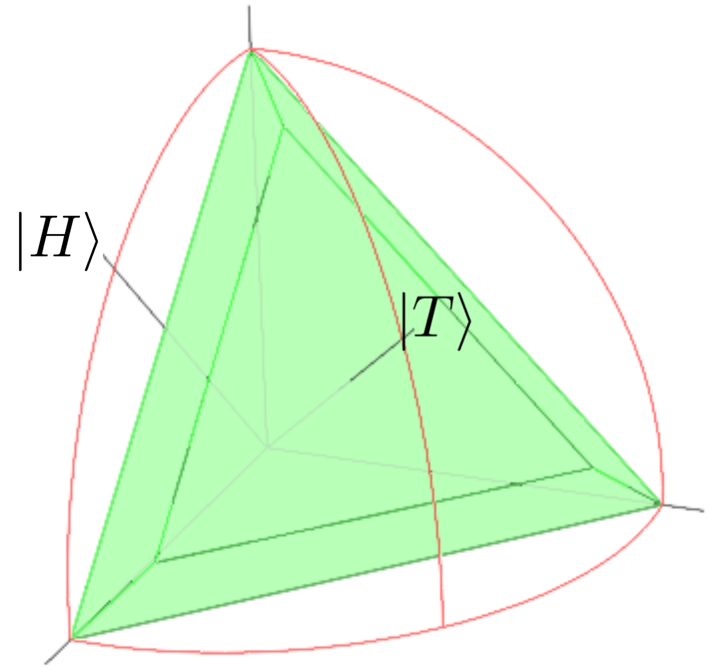

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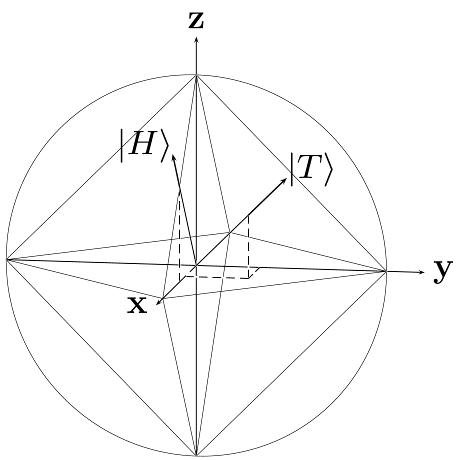
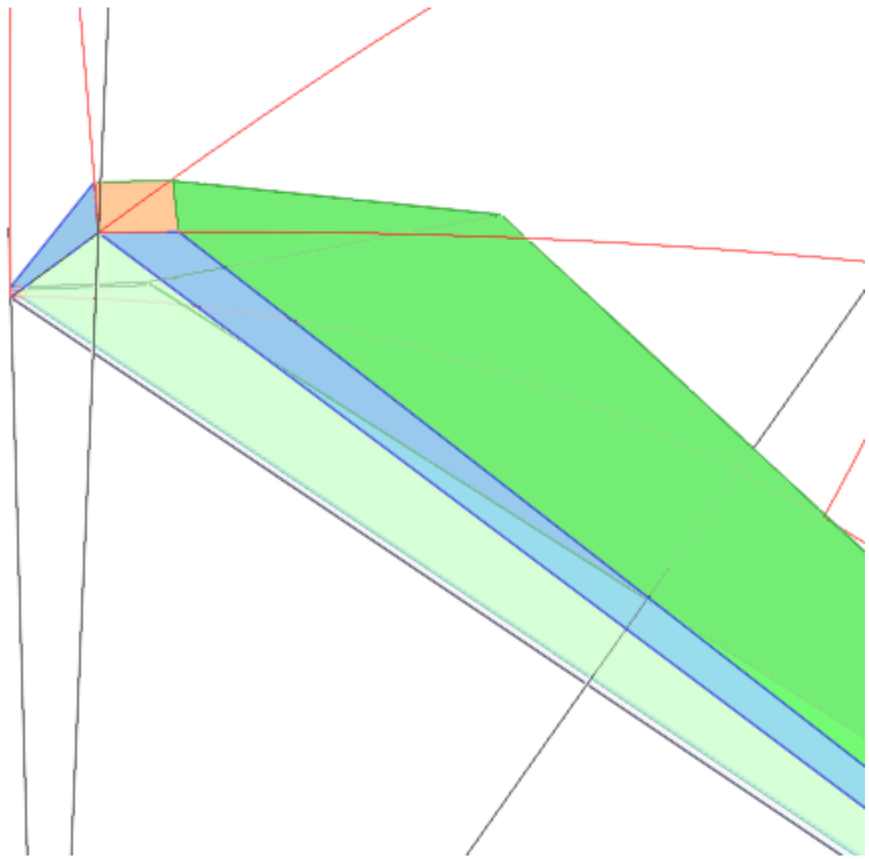
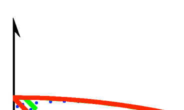
Universality from single-qubit pure states

Lemma: [R'05] Stabilizer ops + prepare $|\psi\rangle$ any single-qubit pure state not a Pauli eigenstate gives quantum universality.

Proof:







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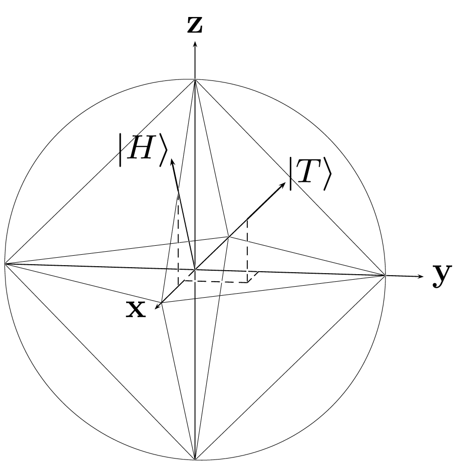
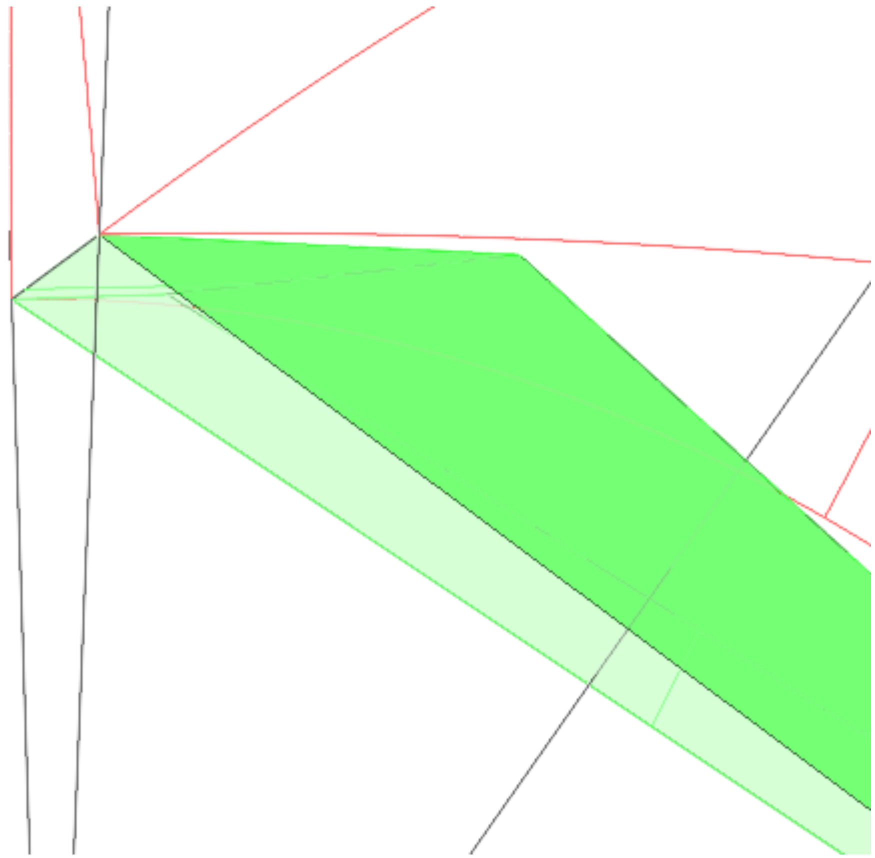
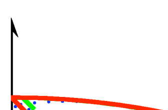






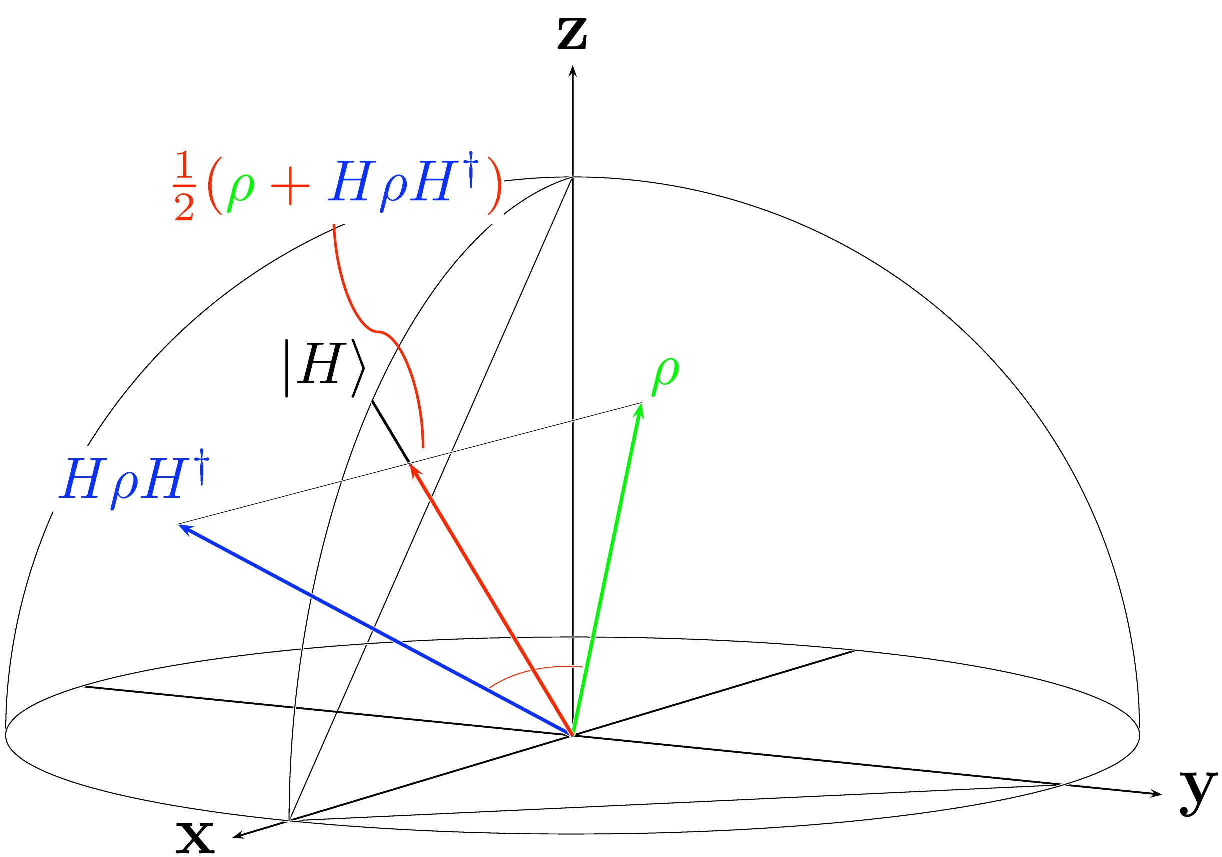
Improved distillation procedure

1. With equal probabilities ½, apply H to ρ .

) Assume ρ lies along H axis: $\rho = \frac{1}{2}(I + x(X+Z))$

$$= \frac{1}{2}\begin{pmatrix} 1+x & 1+x \\ 1+x & 1-x \end{pmatrix}$$

$x =$



$$\rho = \frac{1}{2}(I + x(X+Z)) = \frac{1}{2}\begin{pmatrix} 1+x & 1+x \\ 1+x & 1-x \end{pmatrix}$$

$$\rho = (1-p)|H_+\rangle\langle H_+| + p|H_-\rangle\langle H_-|$$

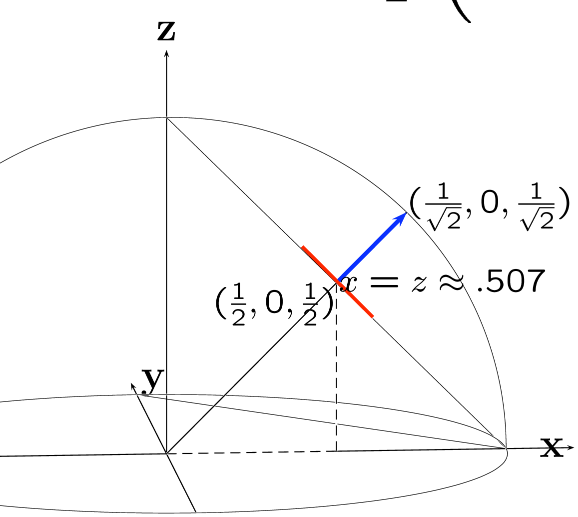
Improved distillation procedure

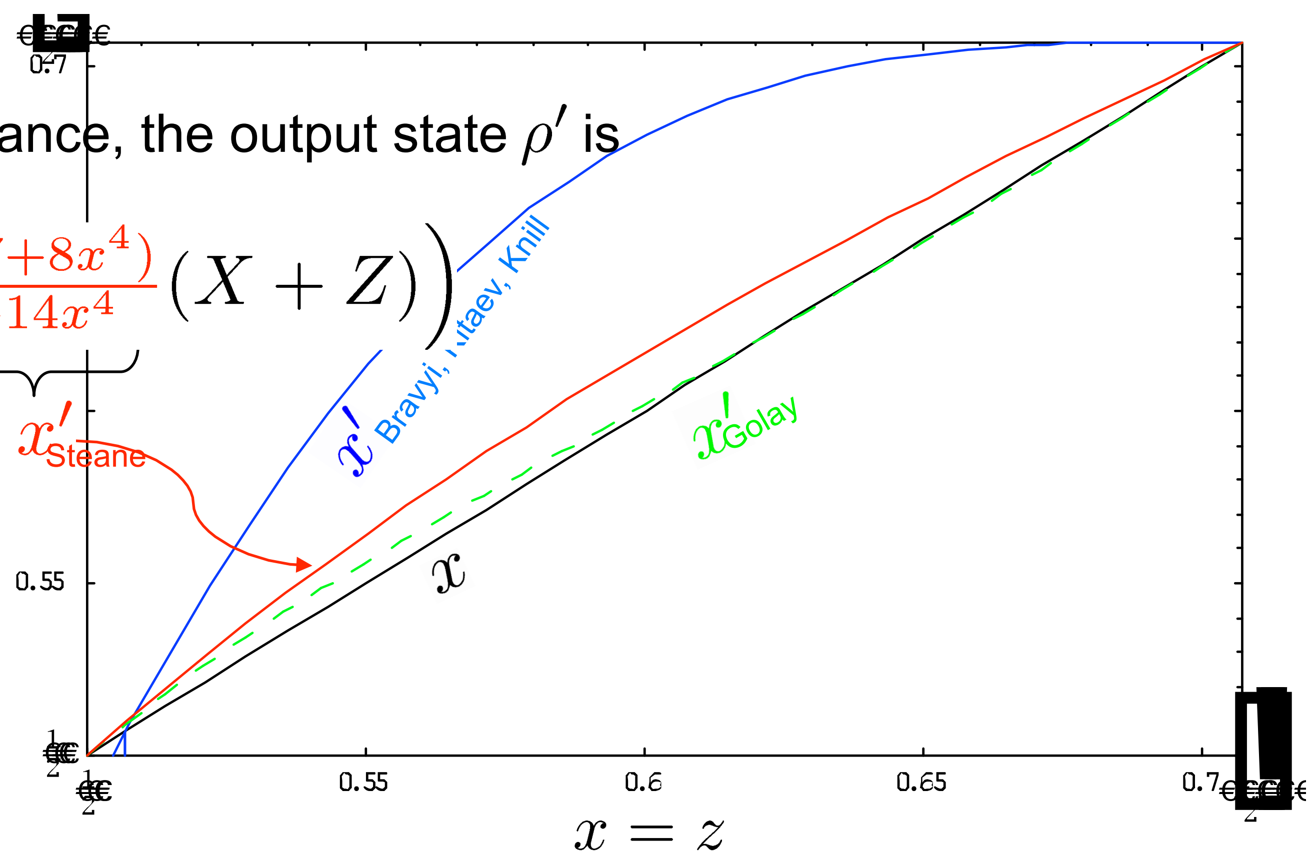
1. Symmetrize ρ into $\rho = \frac{1}{2}(I + x(X+Z)) = \frac{1}{2}\begin{pmatrix} 1+x & 1+x \\ 1+x & 1-x \end{pmatrix}$.

2. Take 7 copies of ρ. Decode according to the [[7,1,3]] Steane/Hamming quantum code, rejecting if errors detected.

3. Conditioned on acceptance, the output state ρ' is

$$\rho' = \frac{1}{2}\left(I + \underbrace{\frac{x^3(7+8x^4)}{1+14x^4}}_{x'_{\text{Steane}}}(X+Z)\right)$$





Proof of improved distillation procedure

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+x & 1+x \\ 1+x & 1-x \end{pmatrix}$$

$$\rho' = \begin{pmatrix} \langle 0_L|\rho^{\otimes n}|0_L\rangle & \langle 0_L|\rho^{\otimes n}|1_L\rangle \\ \langle 1_L|\rho^{\otimes n}|0_L\rangle & \langle 1_L|\rho^{\otimes n}|1_L\rangle \end{pmatrix} / \mathrm{tr}$$

For a CSS code in which $X_L = X^{-n}$, $Z_L = Z^{-n}$,

$$|0_L\rangle = \frac{1}{\sqrt{|C|}} \sum_{a \in C} |a\rangle \qquad |1_L\rangle = X_L|0_L\rangle$$

where C is the set of codewords for a classical code.

Thus $\langle 0_L|\rho^{\otimes n}|0_L\rangle \propto \sum_{a,b \in C} \langle a|\rho^{\otimes n}|b\rangle$.

E.g. $\langle 0001111|\rho^{\otimes 7}|0110011\rangle = (\rho_{00})^1(\rho_{01})^2(\rho_{10})^2(\rho_{11})^2$.

Generally, $$\langle a|\rho^{\otimes n}|b\rangle = \rho_{00}^{n-\frac{1}{2}(|a|+|b|+|a\oplus b|)} \rho_{01}^{\frac{1}{2}(-|a|+|b|+|a\oplus b|)} \rho_{10}^{\frac{1}{2}(|a|-|b|+|a\oplus b|)} \rho_{11}^{\frac{1}{2}(|a|+|b|-|a\oplus b|)}$$

$\rho = (\rho_{00} \quad \rho_{01}$

Proof of Theorem

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Universality from **multi**-qubit pure states

Theorem: [R'05] Stabilizer ops + prepare $|\psi\rangle$ any pure state not a stabilizer state gives quantum universality.

Proof: 9 sequence of Clifford unitaries and postselected Pauli measurements which reduces $|\psi\rangle$ down to a single-qubit pure state which is not a Pauli eigenstate.

By induction, true for n=1.

$$|\psi\rangle = \alpha|0\rangle|\psi_0\rangle + \beta|1\rangle|\psi_1\rangle$$

with α, $\beta \neq 0$, $|\psi_0\rangle$ and $|\psi_1\rangle$ stabilizer states (else apply induction).

By applying Clifford unitaries, w.l.o.g. $|\psi_0\rangle = |0^{n-1}\rangle$.

$$\cdots\ \cdots\ \cdots\ |\psi\rangle = \alpha|0\rangle|0^{n-1}\rangle + \beta|1\rangle|+^{n-1}\rangle$$

But $\alpha|0\rangle + \frac{\beta}{2^{(n-1)/2}}|1\rangle,\ \frac{\alpha}{2^{(n-1)/2}}|0\rangle + \beta|1\rangle$ can't both be stabilizer states!

Universality via **Magic states distillation**

Theorem: [R, '04] Stabilizer operations
+ Prepare $|H\rangle$ w/ < $\frac{1}{2}(1-\frac{1}{\sqrt{2}})$ error $\Rightarrow$ Universality.

Fact: Stabilizer operations
+ Any other single-qubit unitary
$\Rightarrow$ Universality.

Appl. 2: Stabilizer op. fault-tolerance
$\rightsquigarrow$ Universal fault-tolerance.

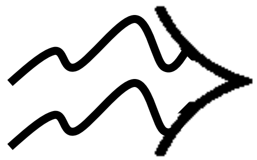
Corollary: Stabilizer operations + (ability to prepare repeatedly
any pure state which is not a stabilizer state) gives universality.

Application to fault-tolerant computing

[Knill, quant-ph/0404104]

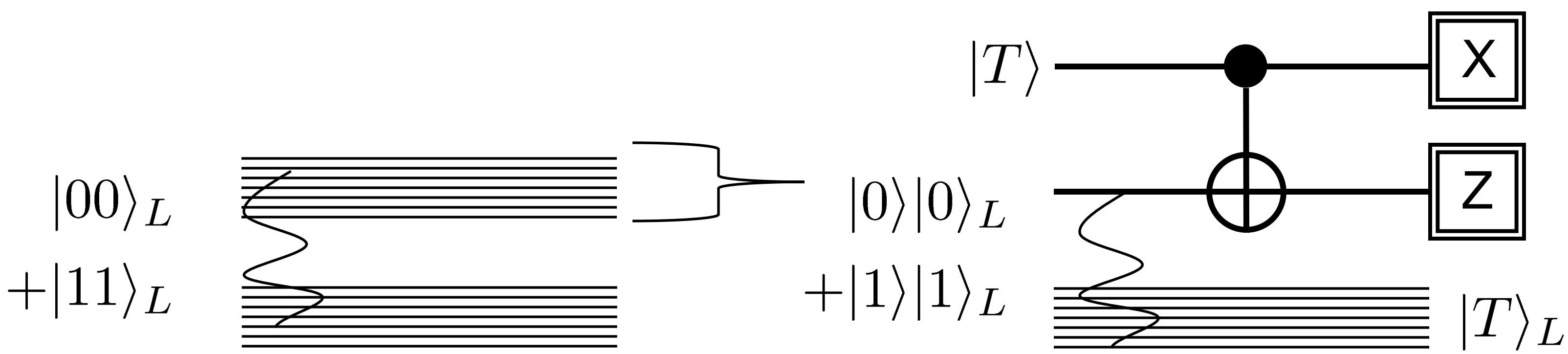
Given scheme for fault-tolerantly applying stabilizer circuits, extend it to a universal fault-tolerant scheme.

Universal fault-tolerance



Stabilizer op. fault-tolerance

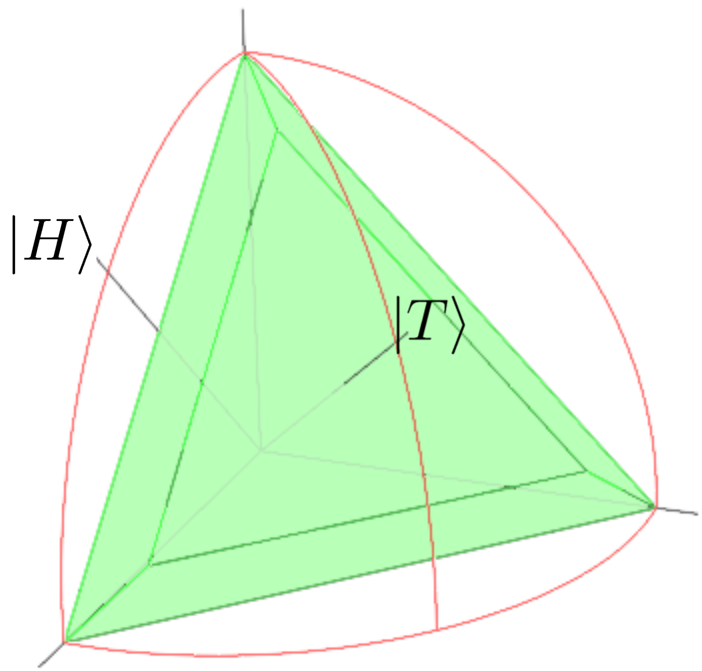
E.g., Knill's scheme has threshold of 5-10% for fault-tolerant stabilizer operations, and the same threshold for fault-tolerant universal operations.



Open questions

Fact: Any mixture of Pauli eigenstates (points in octahedron) is classically simulable. $\Rightarrow$ Universality from $|H\rangle$ w/ <$\frac{1}{2}(1-\frac{1}{\sqrt{2}})$ error is tight.

Open: Is stabilizer operations + (ability to prepare repeatedly single-qubit mixed state ρ) universal for all ρ outside the octahedron?



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ρ

Open: Is stabilizer operations + (ability to prepare repeatedly single-qubit mixed state ρ) universal for all ρ outside the octahedron?

Open: What about perturbations to the states ρ? What about asymmetries? What if we only have fidelity lower bound? Can we characterize stable fixed points for stabilizer codes?

Open: Can we give a provable reduction of fault-tolerance to problem of preparing stabilizer states with independent errors?

Definitions

Def: Pauli group $\mathcal{P} = \left\{ s\,\mu_1 \otimes \cdots \otimes \mu_n : \begin{array}{r} s \in \{\pm 1, \pm i\}, \\ \mu_i \in \{I, X, Y, Z\} \end{array} \right\}$

Def: Clifford group $\mathcal{C} = \text{Normalizer}(\mathcal{P}) \subset U_{2^n}$

i.e., $cpc^\dagger \in \mathcal{P} \quad \forall c \in \mathcal{C}, p \in \mathcal{P}$

generated by

Gates	Conjugation action
$CNOT$	$X \otimes I \to X \otimes X$ $Z \otimes I \to Z \otimes I$ $I \otimes X \to I \otimes X$ $I \otimes Z \to Z \otimes Z$
$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$X \leftrightarrow Z$
$P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$	$X \to Y \to -X$

Stabilizer operations

Clifford group $\mathcal{C} = \langle CNOT, H, P \rangle$

+ prepare / measure Pauli operator eigenstates

Fact: Circuit consisting only of stabilizer operations can be efficiently classically simulated.

Operation	State	Stabilizer $S = \{M \in \mathcal{P} : M\|\psi\rangle = \|\psi\rangle\}$
1. prepare $\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	$\langle X \rangle$
2. prepare $\|1\rangle$	$\frac{1}{\sqrt{2}}(\|01\rangle + \|11\rangle)$	$\langle X \otimes I, I \otimes -Z \rangle$
3. CNOT$_{1,2}$	$\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$	$\langle XX, -ZZ \rangle$

$$\begin{array}{ccccccc} X & \otimes & I & \rightarrow & X & \otimes & X \\ Z & \otimes & I & \rightarrow & Z & \otimes & I \\ I & \otimes & X & \rightarrow & I & \otimes & X \\ I & \otimes & Z & \rightarrow & Z & \otimes & Z \end{array}$$

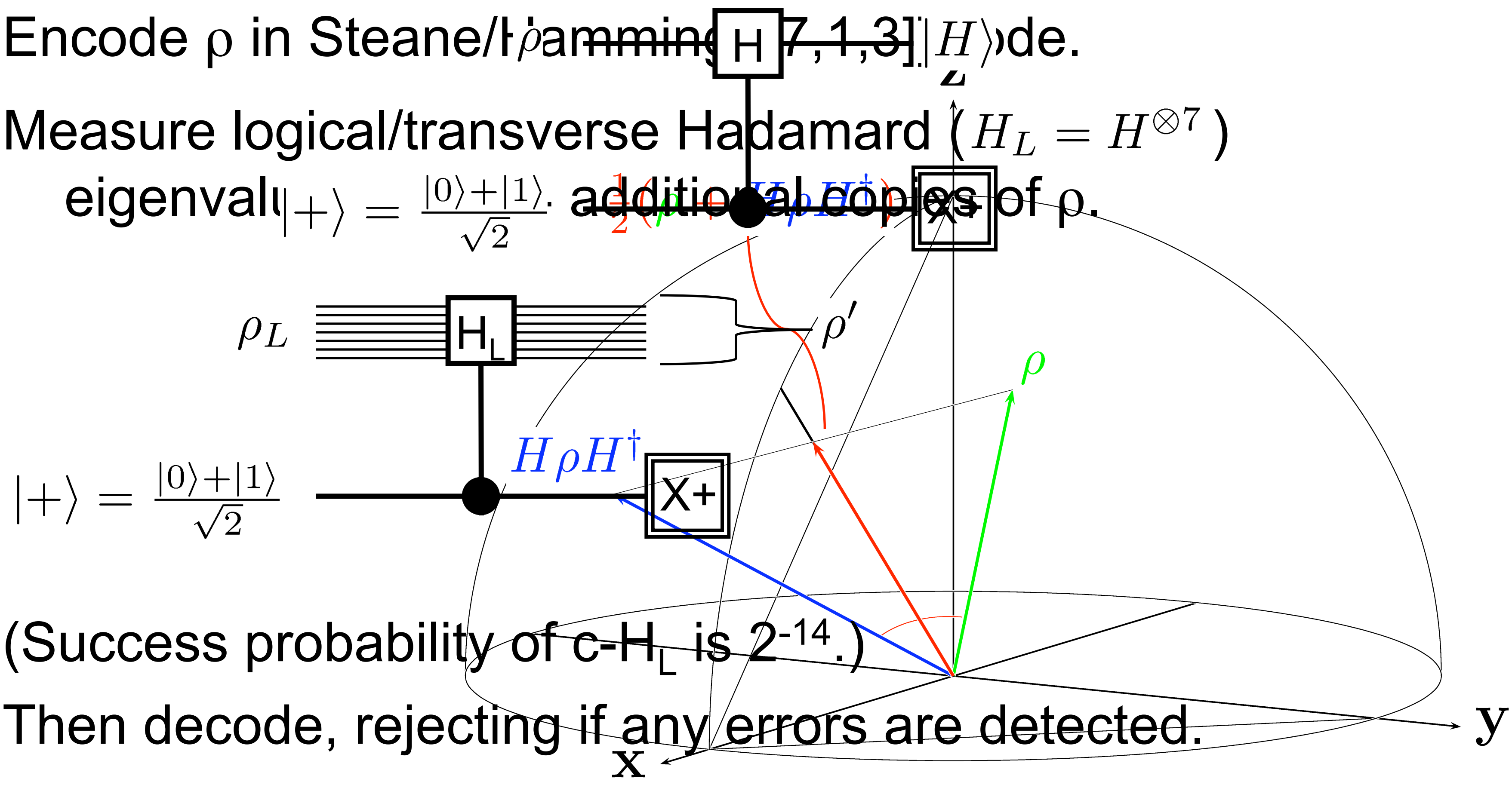
Knill's method for H-distillation: c-H_L

With equal probabilities ½, apply H to ρ;) assume ρ lies along H axis: $\rho = (1-p)|H,+\rangle\langle H,+| + p|H,-\rangle\langle H,-|$

$$= \tfrac{1}{2}(I + x(X+Z)) = \tfrac{1}{2}\begin{pmatrix} 1+x & 1+x \\ 1+x & 1-x \end{pmatrix}, \quad x = \frac{1-2p}{\sqrt{2}} .$$

Encode ρ in Steane/Hamming [7,1,3] code.

Measure logical/transverse Hadamard ($H_L = H^{\otimes 7}$) eigenval[illegible] additional copies of ρ.

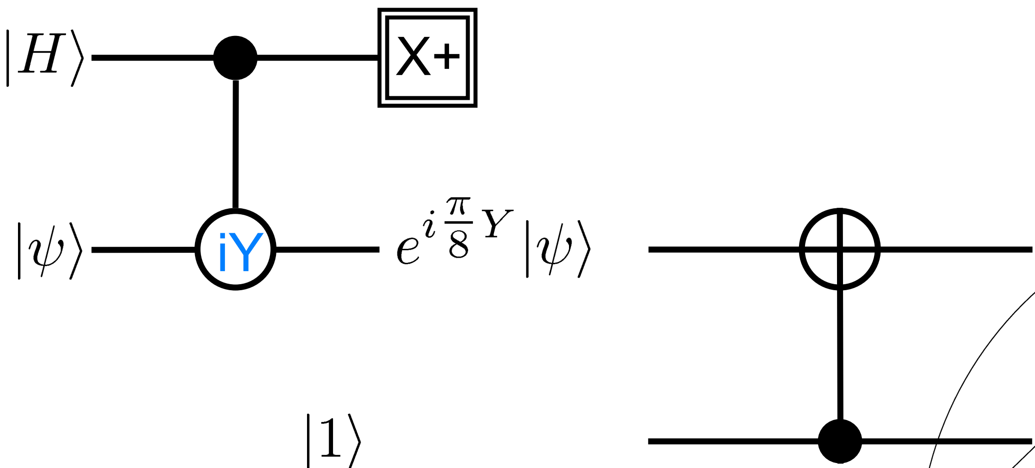


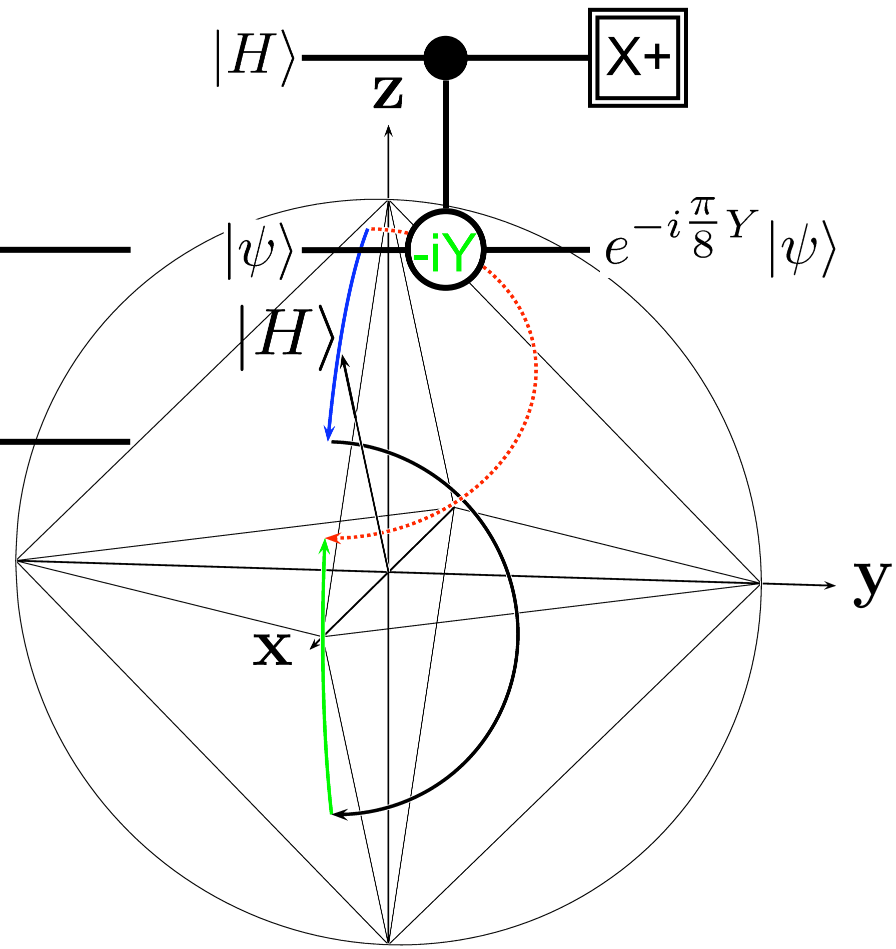
(Success probability of c-H_L is 2^{-14}.)

Then decode, rejecting if any errors are detected.

Knill's method for H distillation: c-H

$$|H\rangle \propto (1+\sqrt{2})|0\rangle + |1\rangle$$





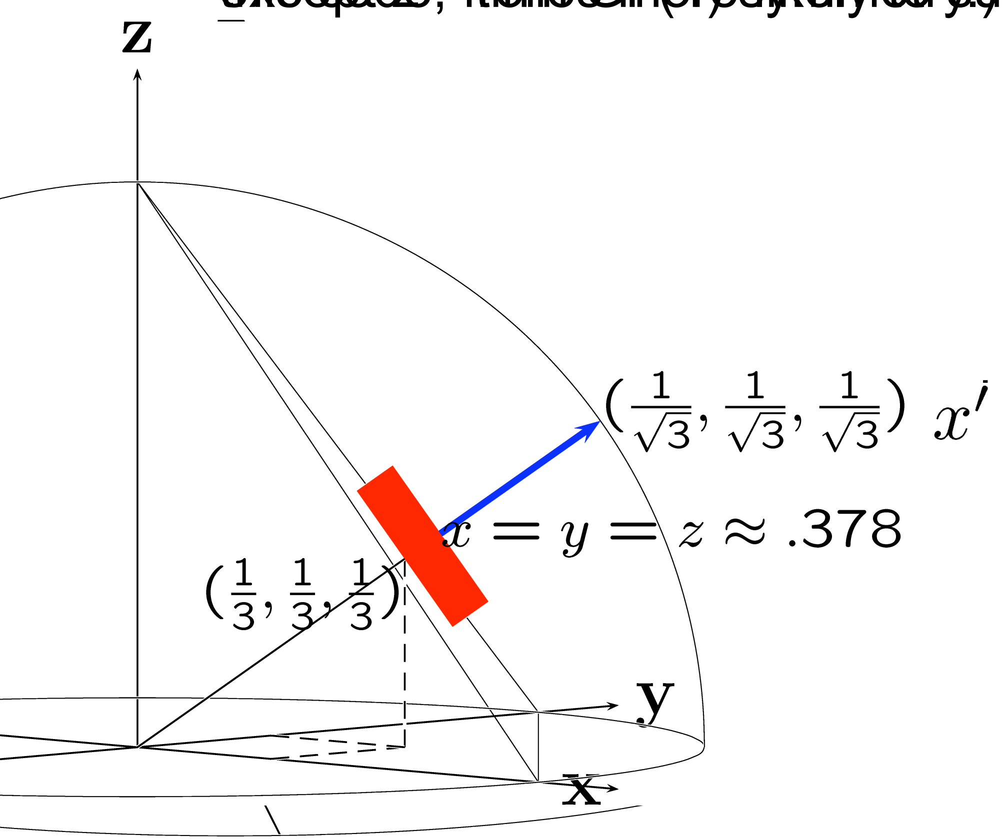
Using two copies of $|H\rangle$, get controlled-Hadamard with probability ¼.

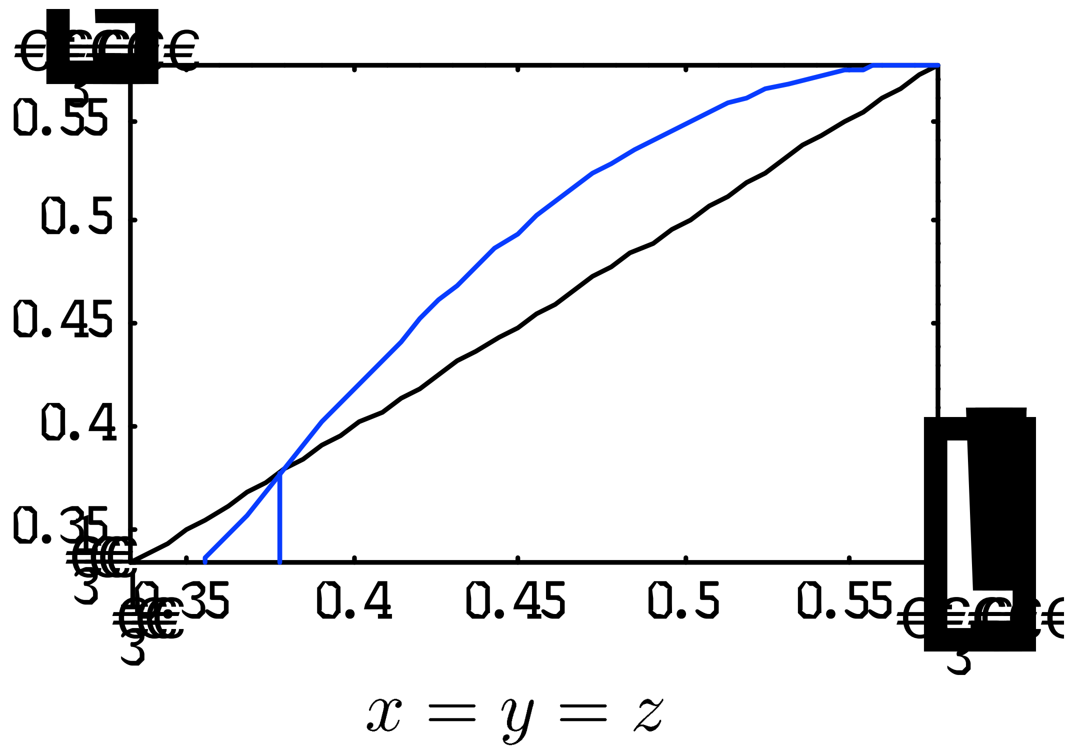
Bravyi & Kitaev's equivalent distillation procedure & T-distillation procedure

Idea: Choose n-qubit code C. Take n copies of ρ, and decode C, rejecting if any errors are detected (i.e., project onto logical subspace) to leave ρ'. Recurse, using n copies of ρ'…

T-distillation: Use a particular 15-qubit code, based on a generalization of the Hamming code. Procedure is exactly equivalent to Knill's procedure, except 2^{14} times more likely to succeed.)

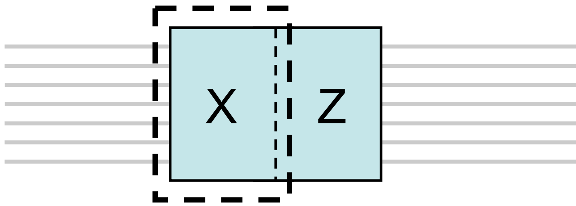
H-distillation: Symmetrize ρ in the T direction, by applying with equal prob. 1/3 either I, T (X!Y!Z!X), or T^2. Use the [[5,1,3]] code. (Note that $T = T^5$ for this code, from GF(4) symmetry.)



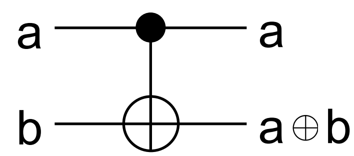


Standard fault-tolerance scheme

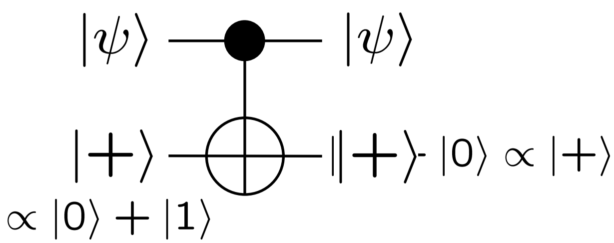
[Steane,...]



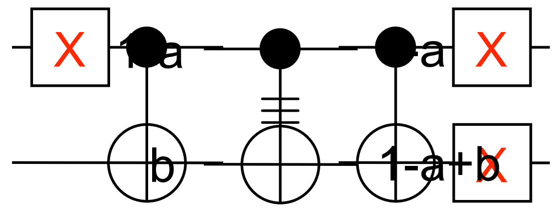
Def: CNOT

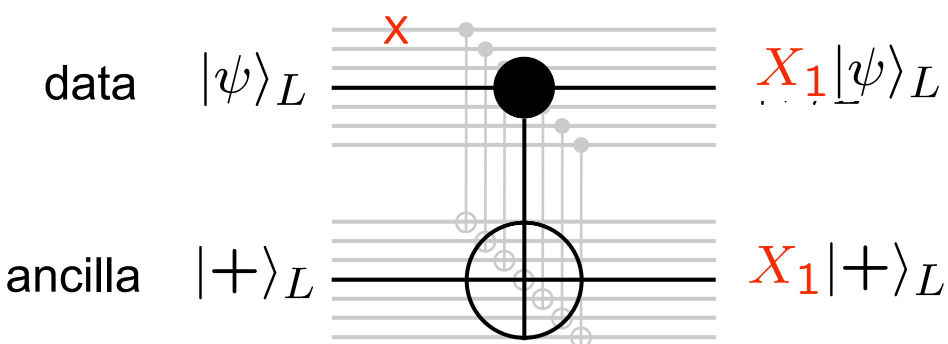


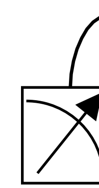
Fact 1:



Fact 2:







Threshold from concatenation

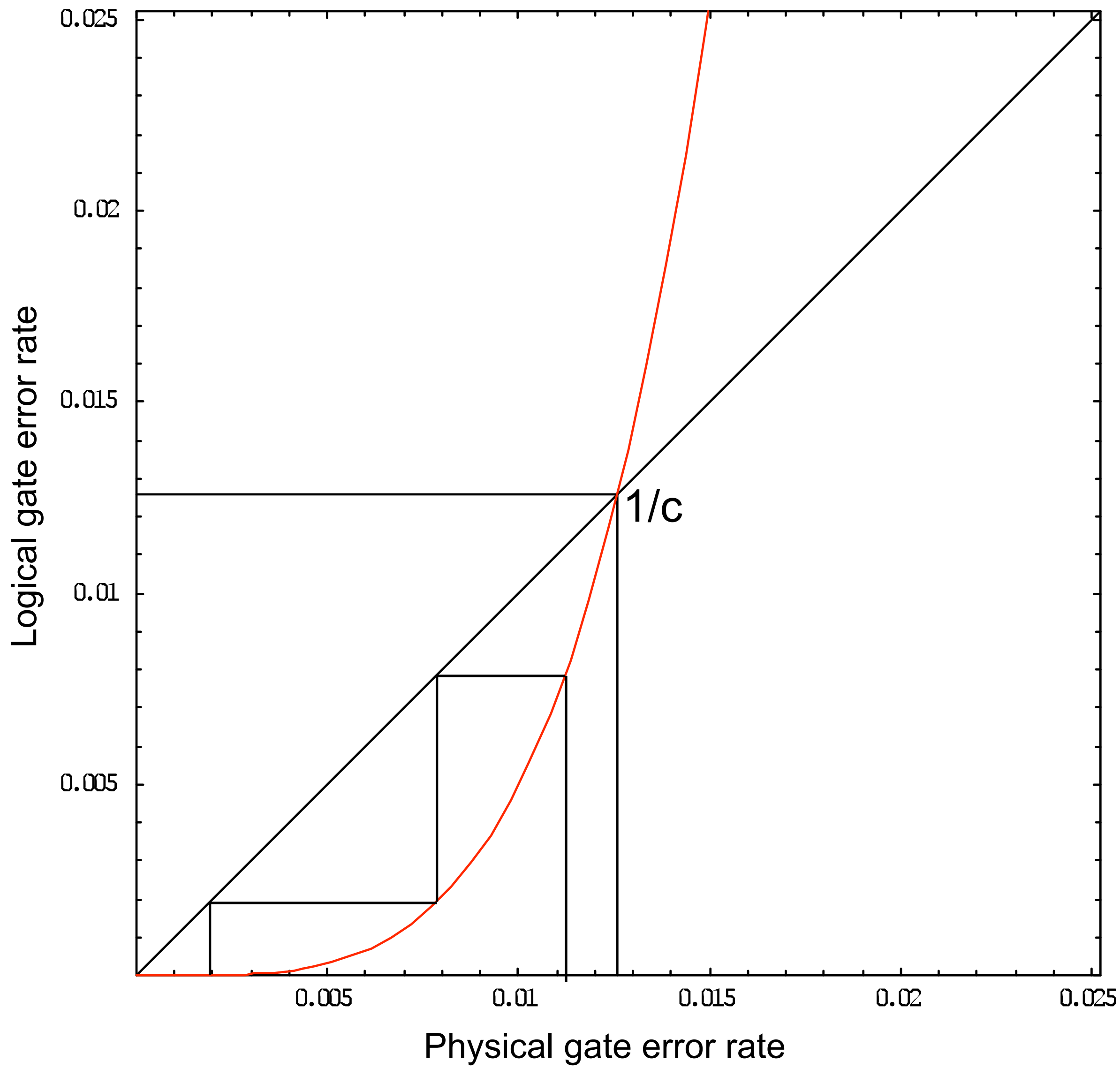
- N gate circuit

  $\Rightarrow$ Want error $\ll 1/N$

- [[7, 1, 3]] code only corrects 1 error

Probability of error	Physical bits per logical bit
p	1
$c\,p^2$	7
$\sim p^{2^2}$	7^2
p^{2^3}	7^3

O(log log N) concatenations
O(log N) physical bits / logical

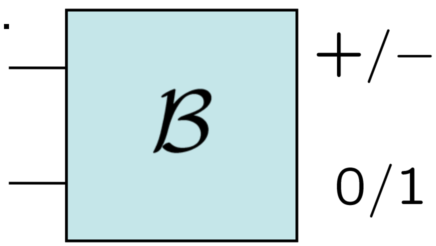


ph

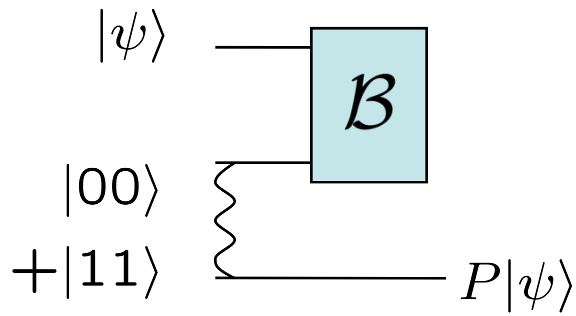
$1/c^{1/t}$

Teleportation: Knill's erasure threshold of ½

Theorem [Knill '03]: Threshold for erasure error is ½ for Bell measurements.



Teleportation



$P \in \{I, X, Y, Z\}$

Teleportation

Alice

$|\psi\rangle$ —

$|0\rangle|0\rangle + |1\rangle|1\rangle$

$P|\psi\rangle$

$P \in \{I, X, Y, Z\}$

$+|$

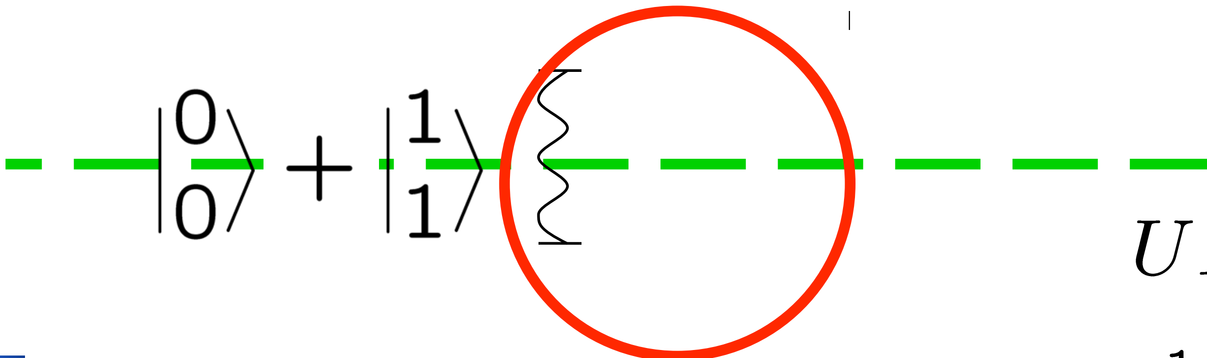
$+|$

Bob

Teleportation + Computation

Alice

$|\psi\rangle$ —



$|00\rangle + |11\rangle$

Assume no errors!

$UP|\psi\rangle$

$^{-1}U^{-1}$

$+|$

$+|$

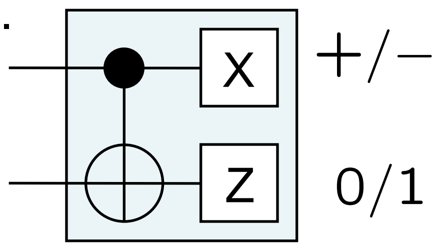
$|0$

$+|1$

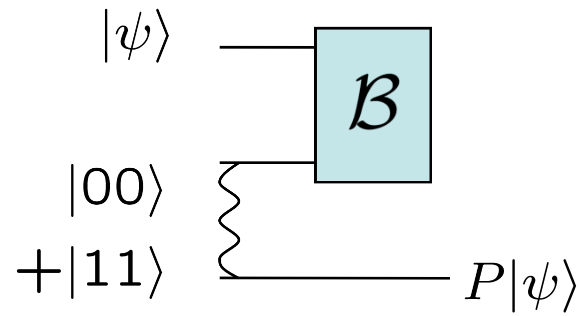
Bob

Teleportation: Knill's erasure threshold of ½

Theorem [Knill '03]: Threshold for erasure error is ½ for Bell measurements.

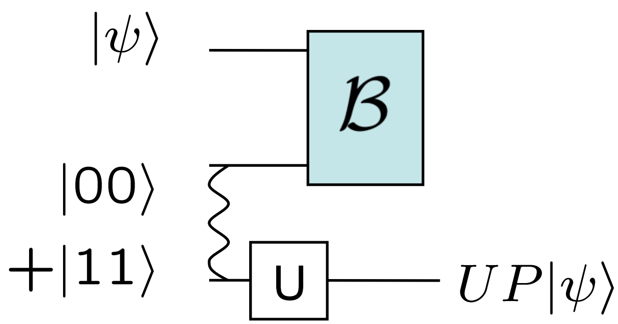


Teleportation



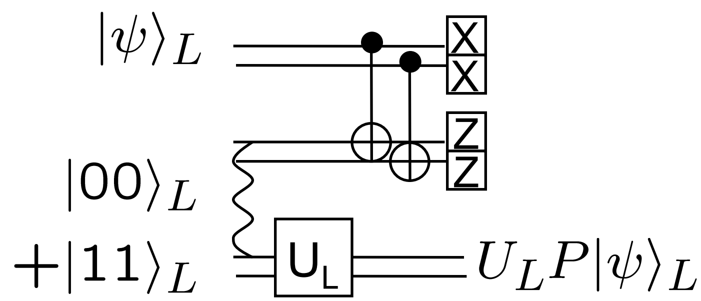
$P \in \{I, X, Y, Z\}$

+ Computation



$UP^{-1}U^{-1}$

+ Fault-tolerance

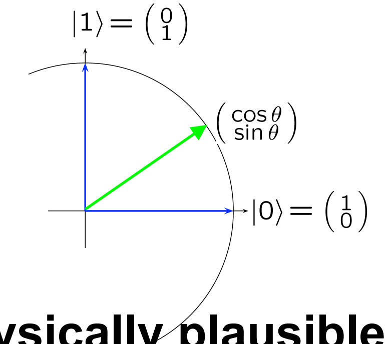


$P \in \{I, X, Y, Z\}^{\otimes n}$

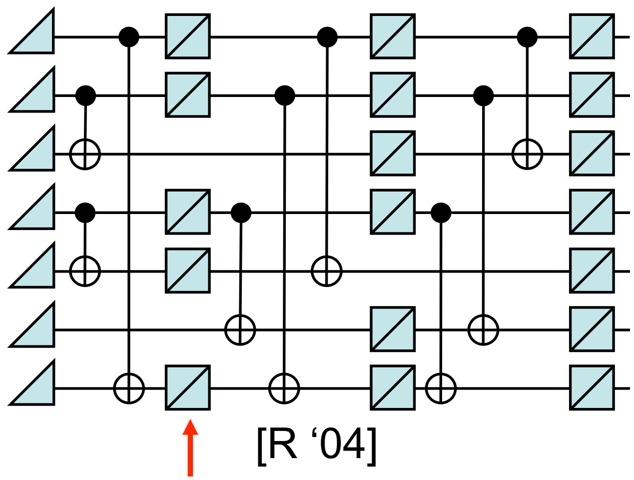
[Knill '04]: Estimated threshold of 5-10%.

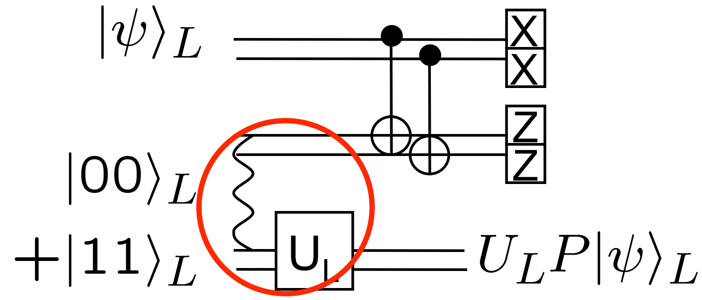
Open questions

- **Errors inevitable in quantum computers**



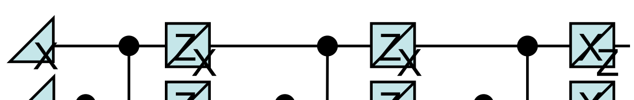
- **Fault-tolerance schemes can tolerate physically plausible error rates**


[R '04]



[Knill '04]

Efficient? **Provable?**



Improved threshold result [R '04]