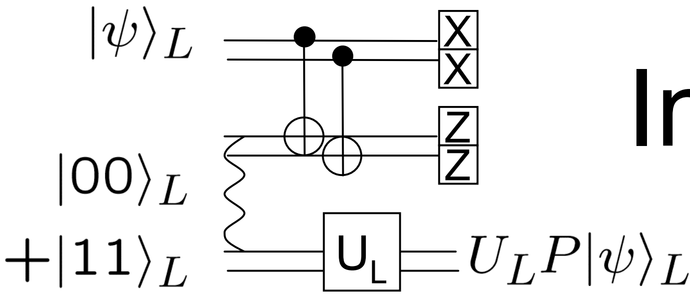
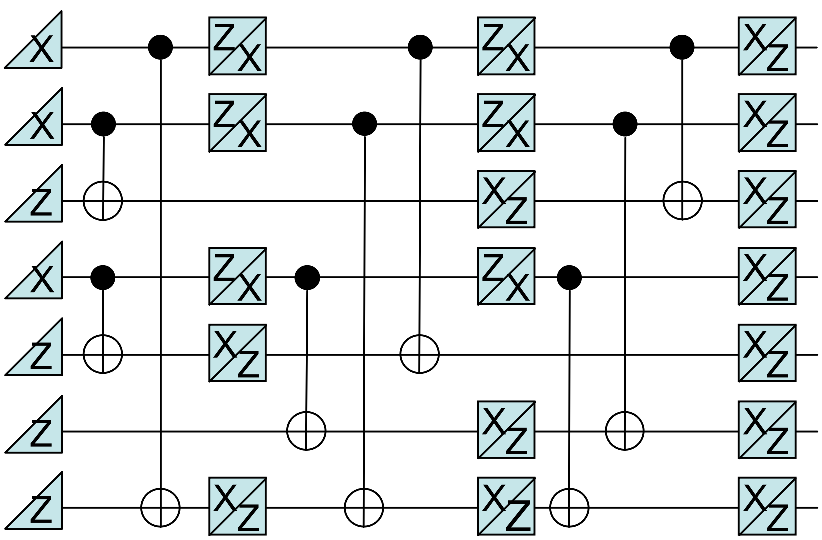
Recent Schemes for Increasing the Quantum Fault-tolerance Threshold

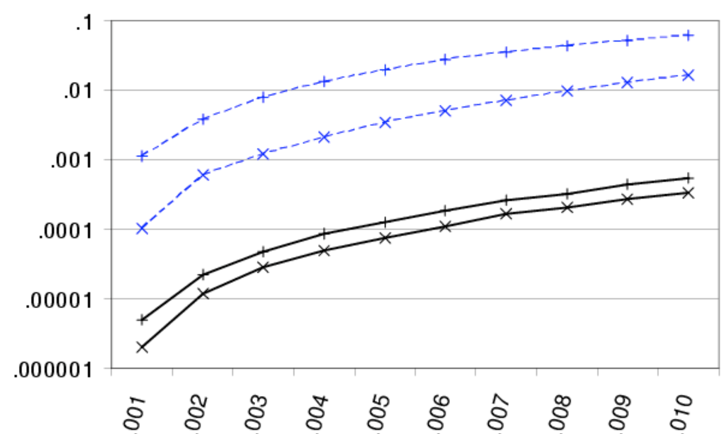


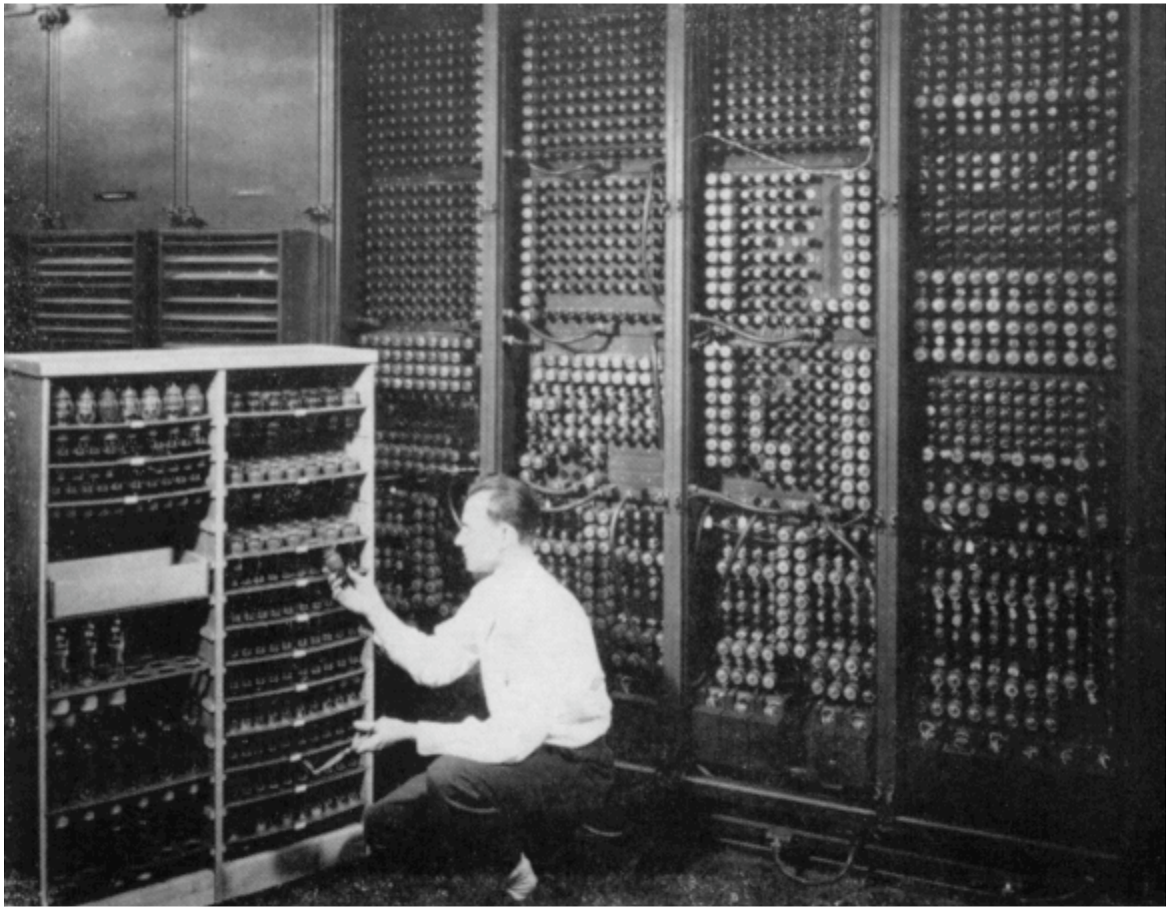
$P \in \{I, X, Y, Z\}^{\otimes n}$

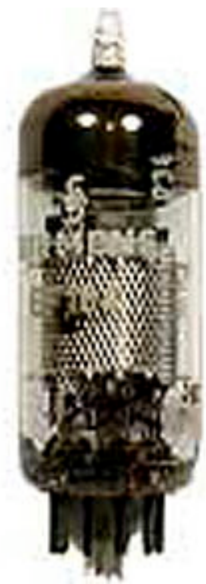
Ben Reichardt

Berkeley







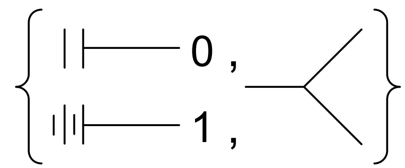


Classical fault-tolerance

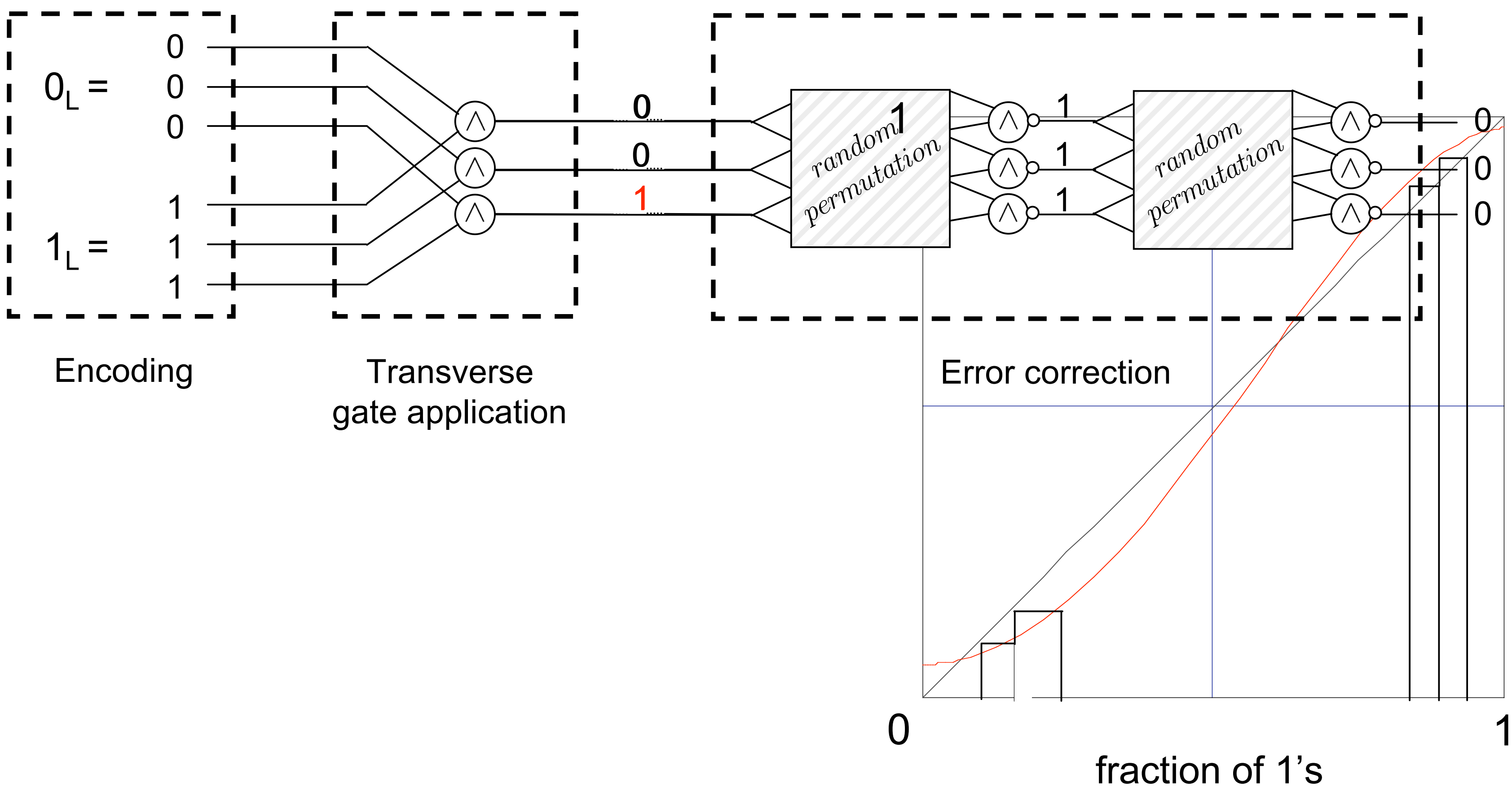
[Von Neumann '56]

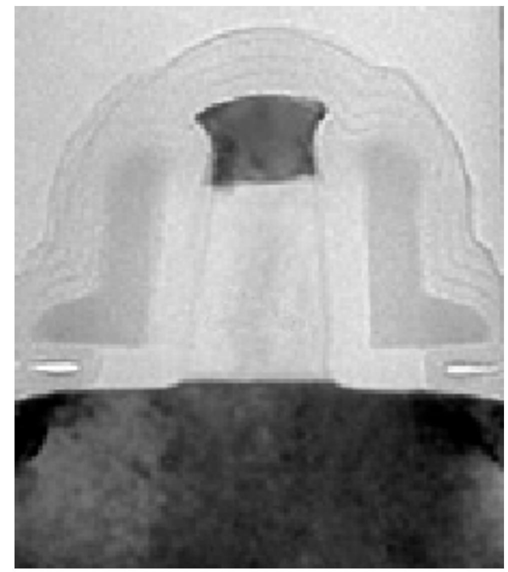
Make fault-tolerant a circuit consisting of a universal set of operations, some faulty:

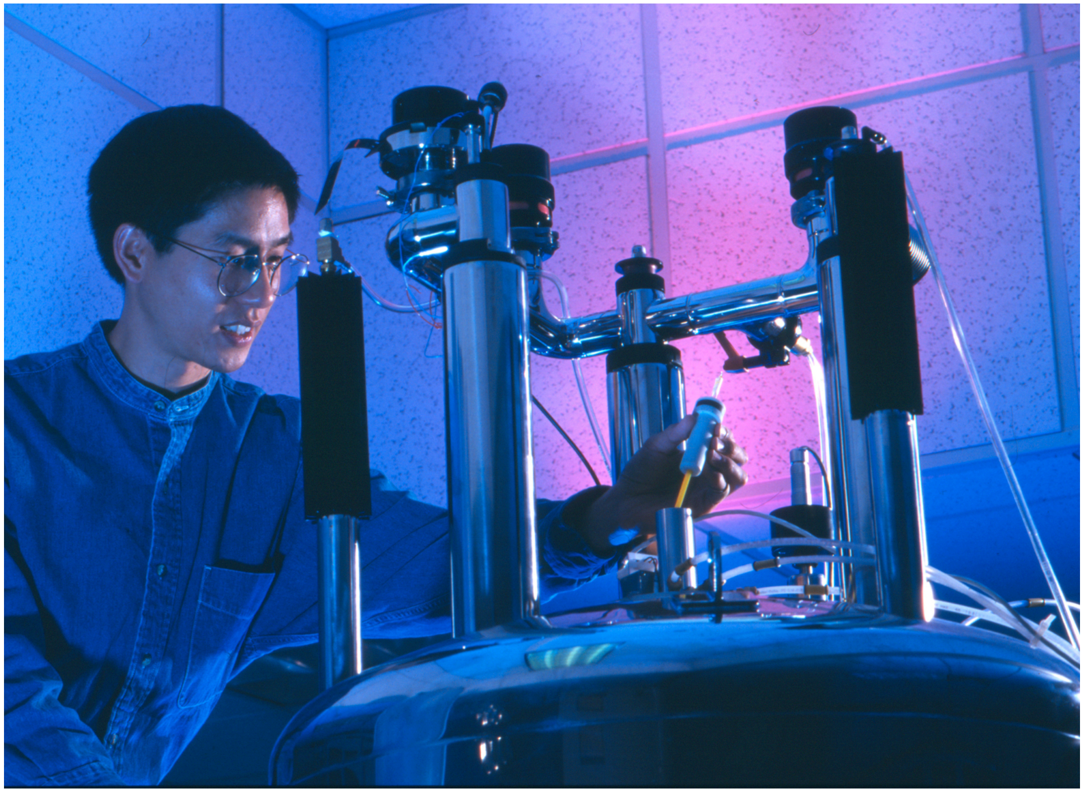
Perfect op's:

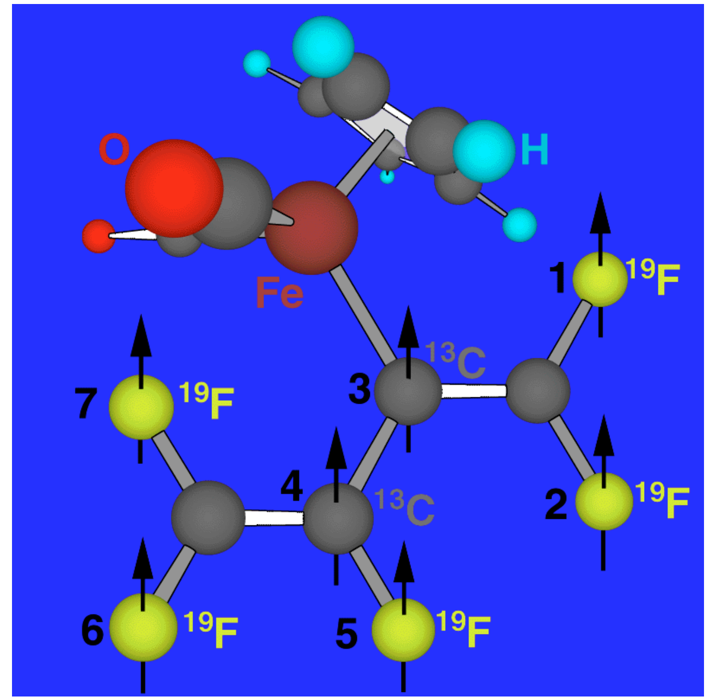


Faulty op's: {AND, NOT}



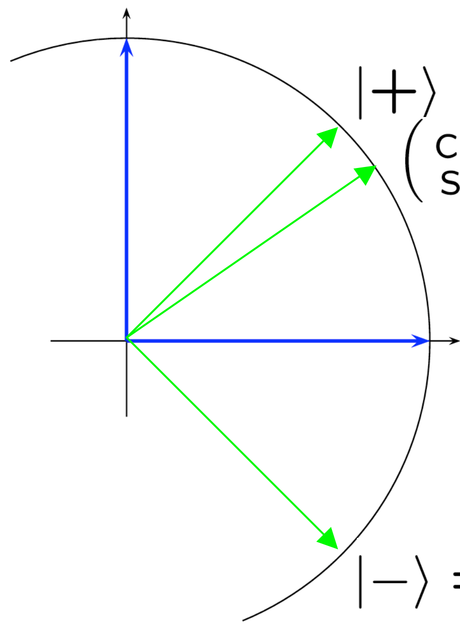



O
H
Fe
1 19F
2 19F
3 13C
4 13C
5 19F
6 19F
7 19F

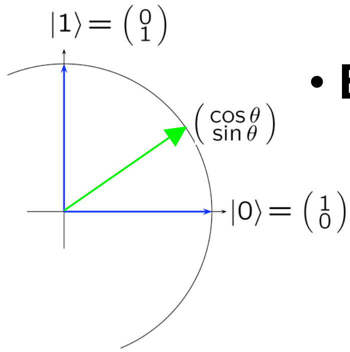
Continuous quantum states

Classical bit

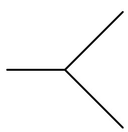
Quantum bit (qubit)



Qu disadvantages

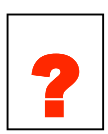


- **Errors inevitable**
  - **- Continuous quantum states**
  - **- Tradeoff between controllability and stability**



- **No cloning: local duplication undefined**

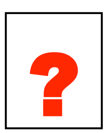
Qu advantage!



- **Can defer exposure of data to operations**

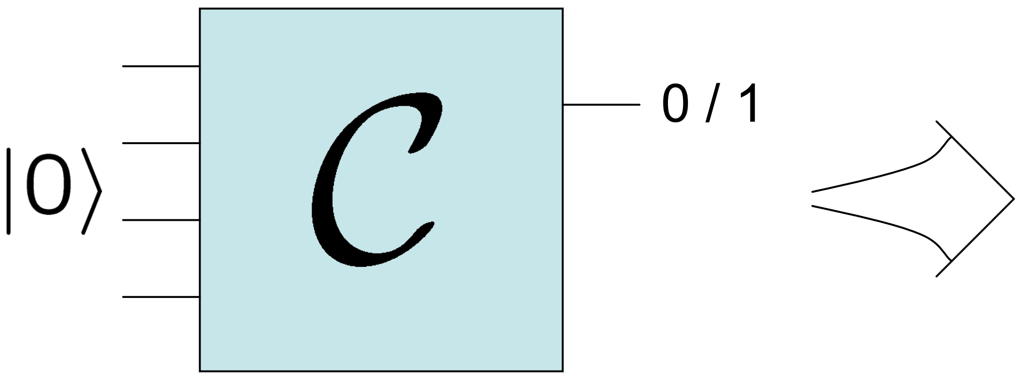
Overview

- **Postselection** — **Improved threshold result** [R ‘04]
  - Modification of standard error correction scheme increases estimated threshold 3x, to almost 1%.
- **Teleportation** — **Knill’s threshold result**
  - Theorem: Threshold for erasure errors is ½ for Bell measurements.
  - Estimated 5-10% threshold for independent depolarizing errors.

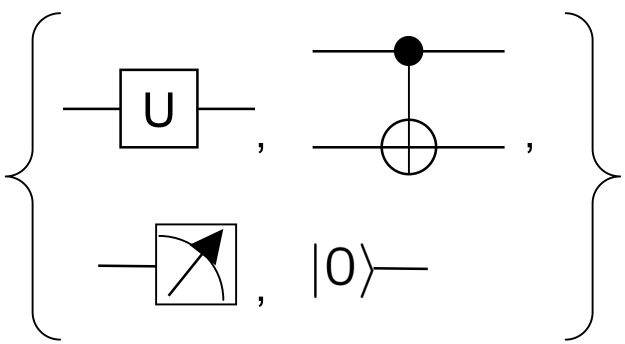


Defer exposure of data to operations

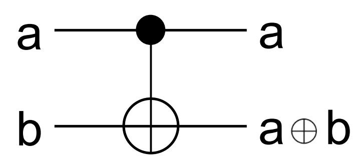
Quantum fault-tolerance problem

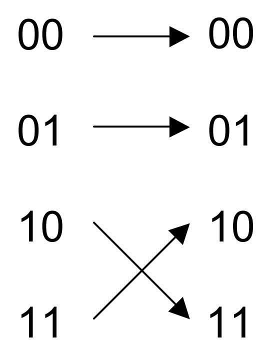


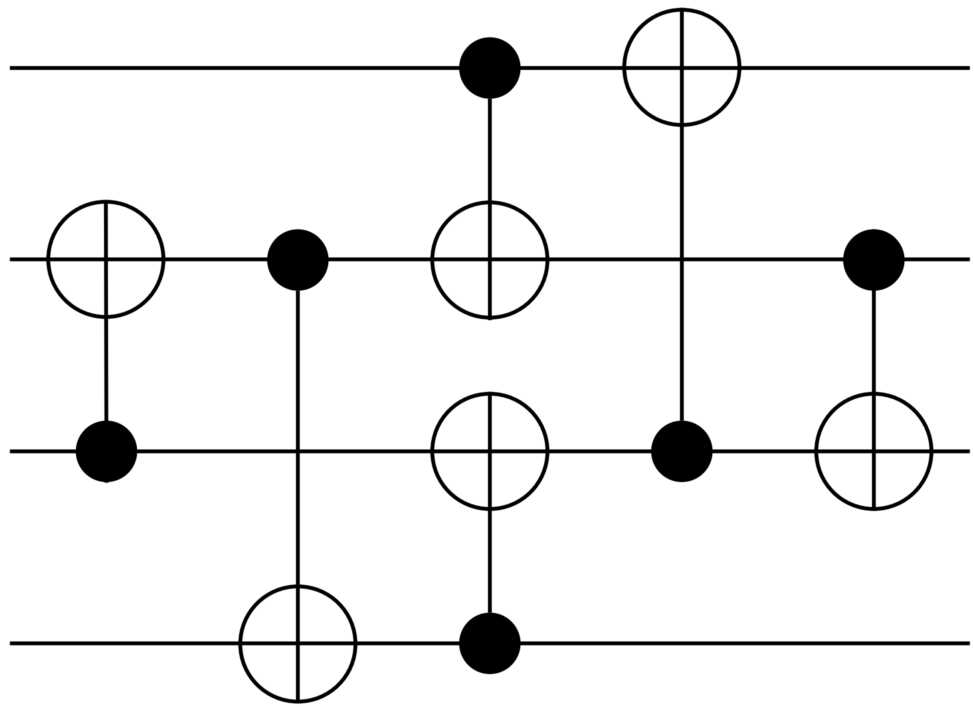
Larger, fault-tolerant
$\mathcal{C}$

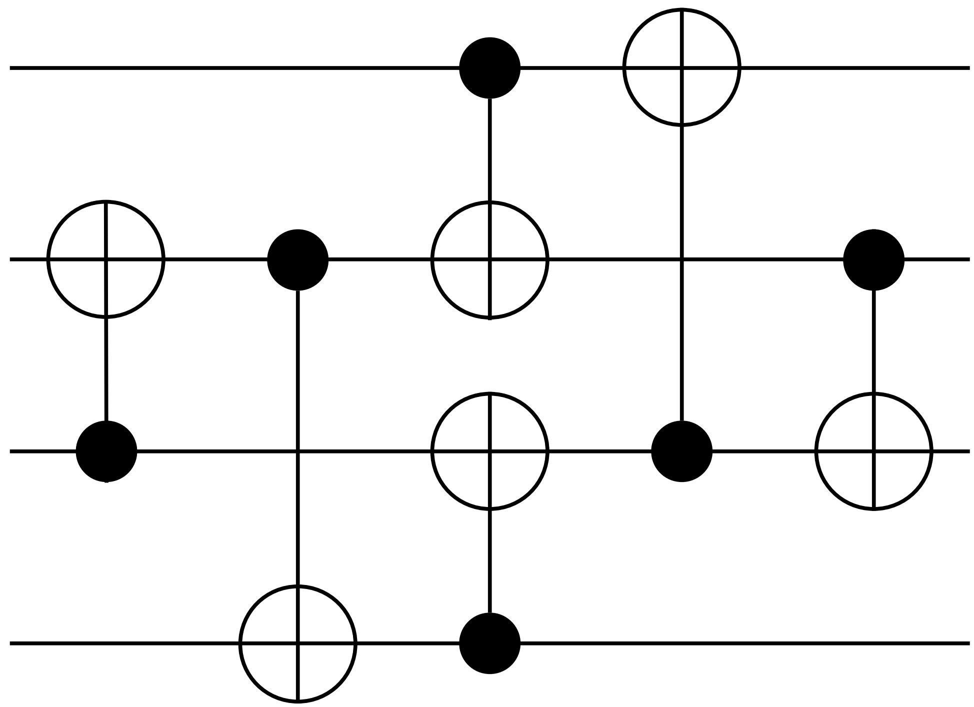


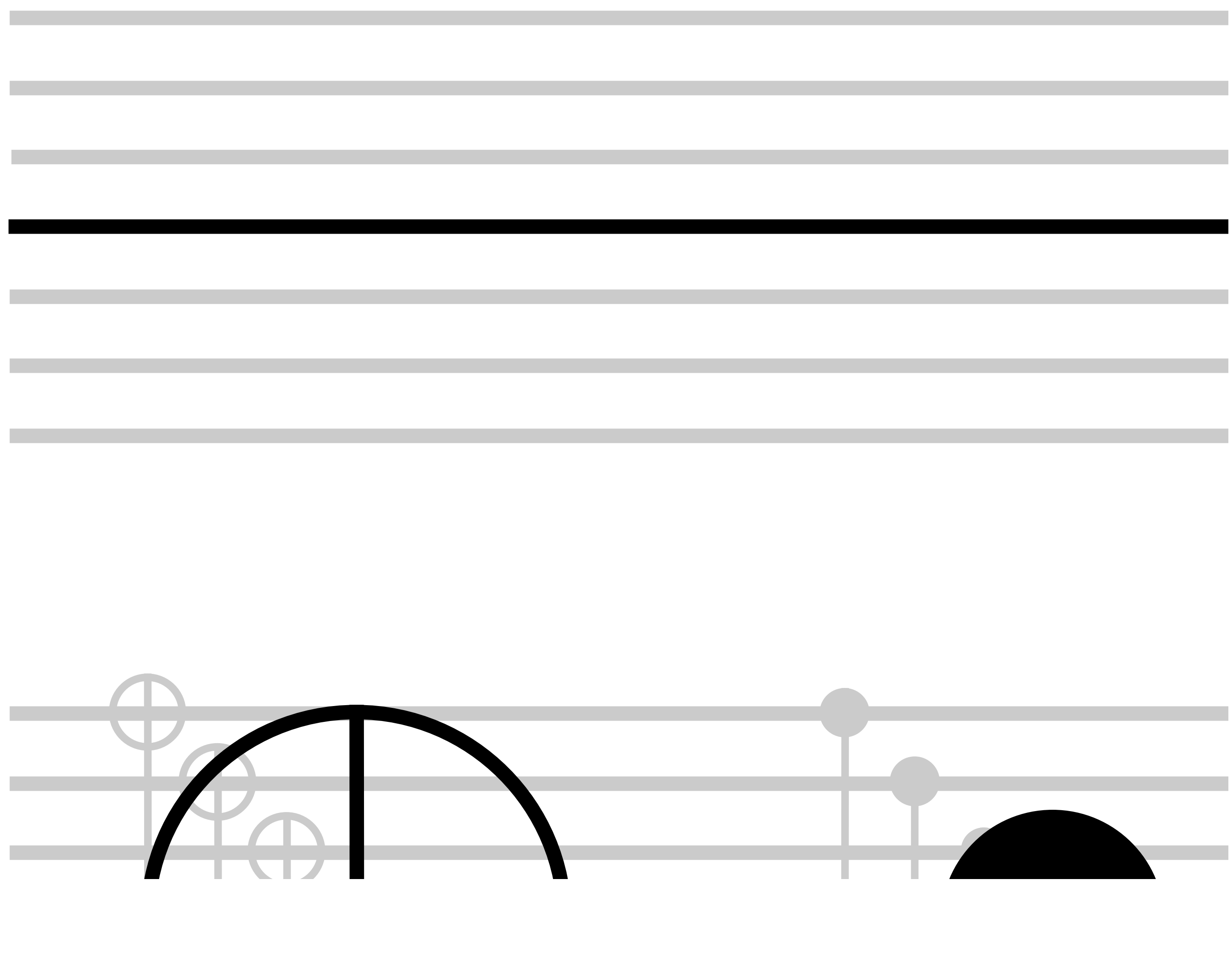
CNOT:

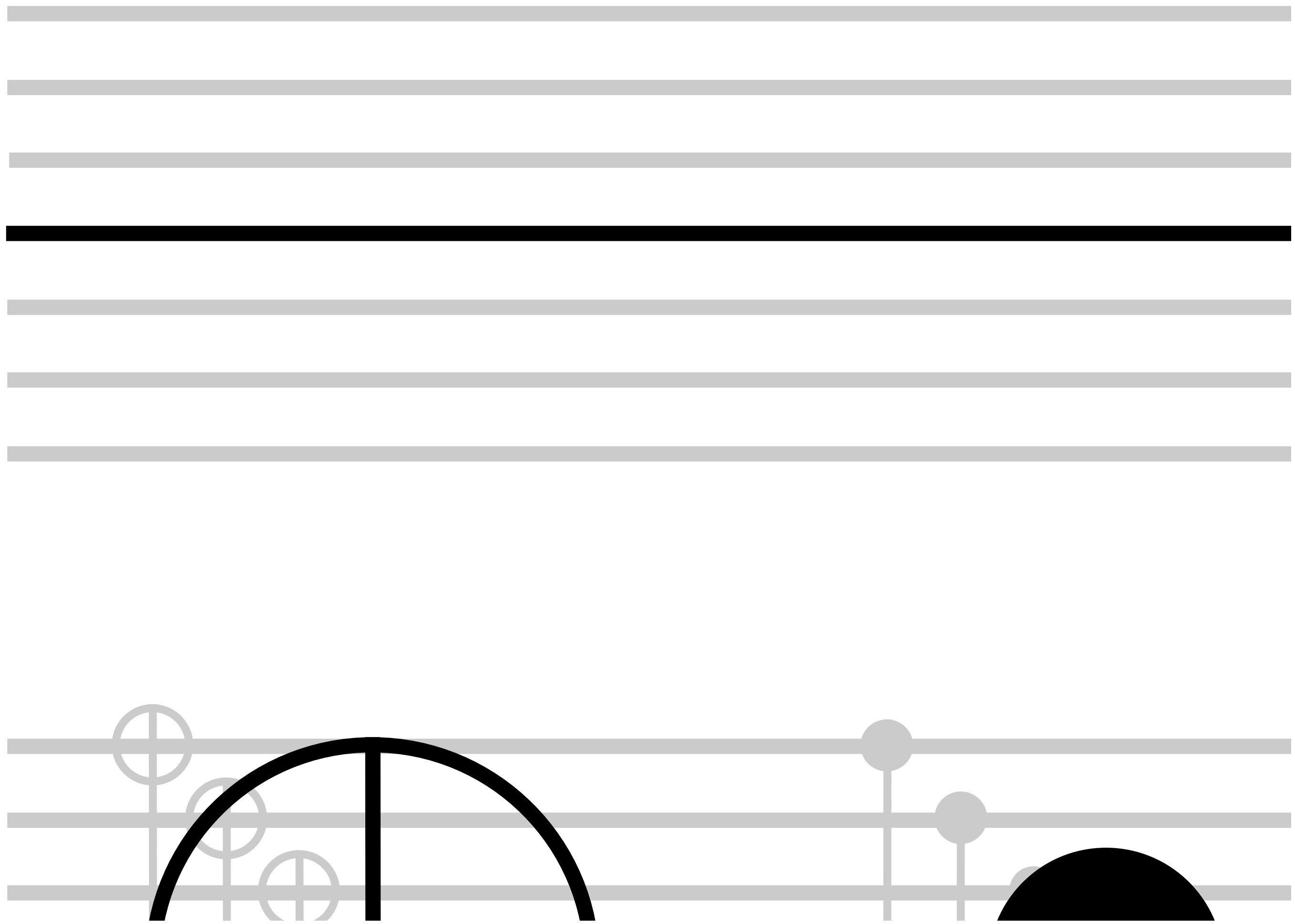


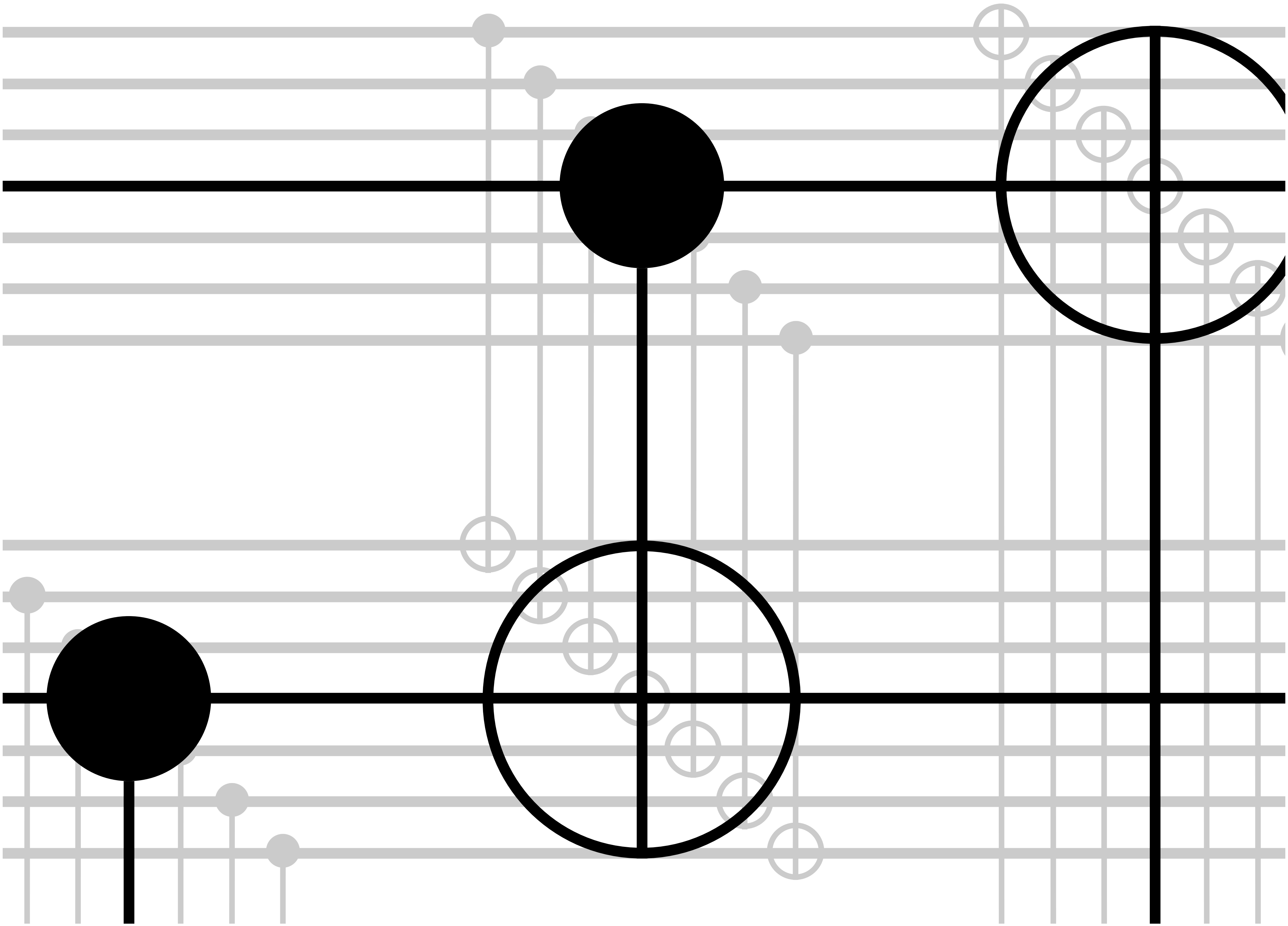


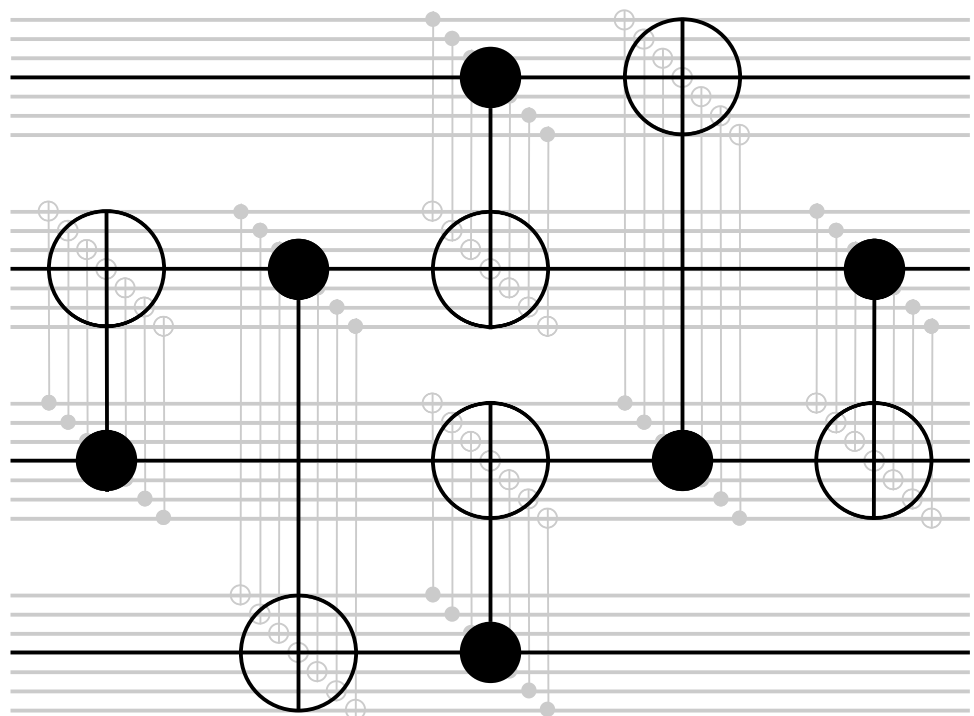










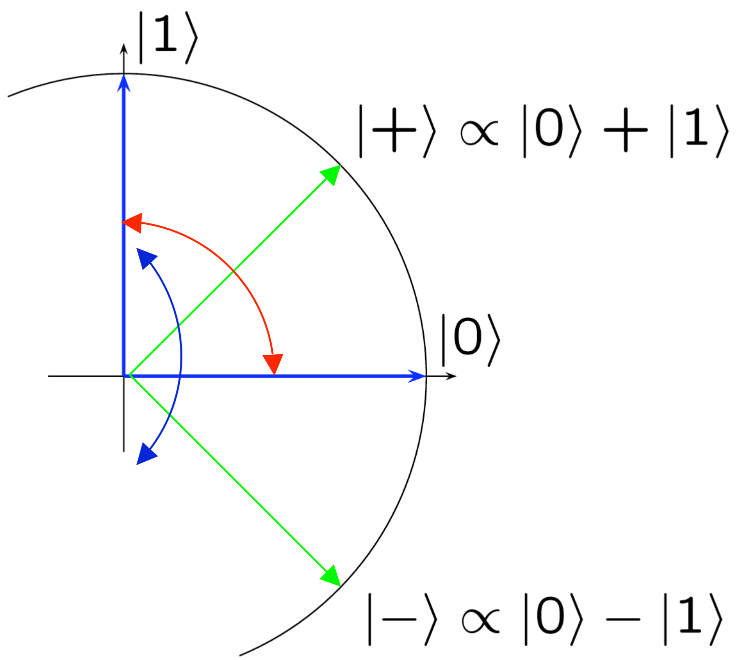


Discrete errors

X bit flips: $|0\rangle \leftrightarrow |1\rangle$

Z phase flips:

$$\begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto -|1\rangle \end{aligned}$$

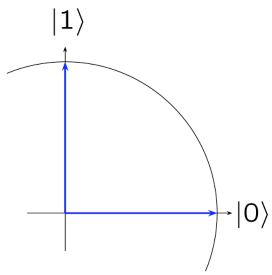


Duality:

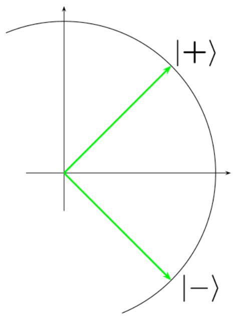
$$\begin{matrix} X & & Z \\ |0\rangle, |1\rangle & \Leftrightarrow & |+\rangle, |-\rangle \end{matrix}$$

Quantum codes

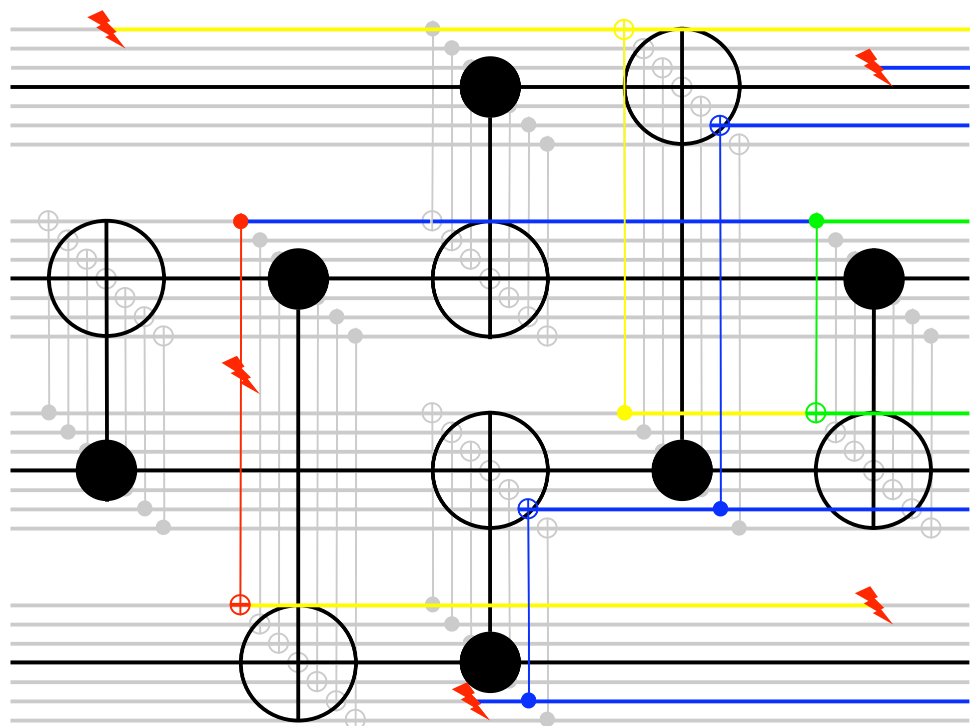
- Classical codewords in the 0/1 basis
  ⇒ Correct bit flip X errors

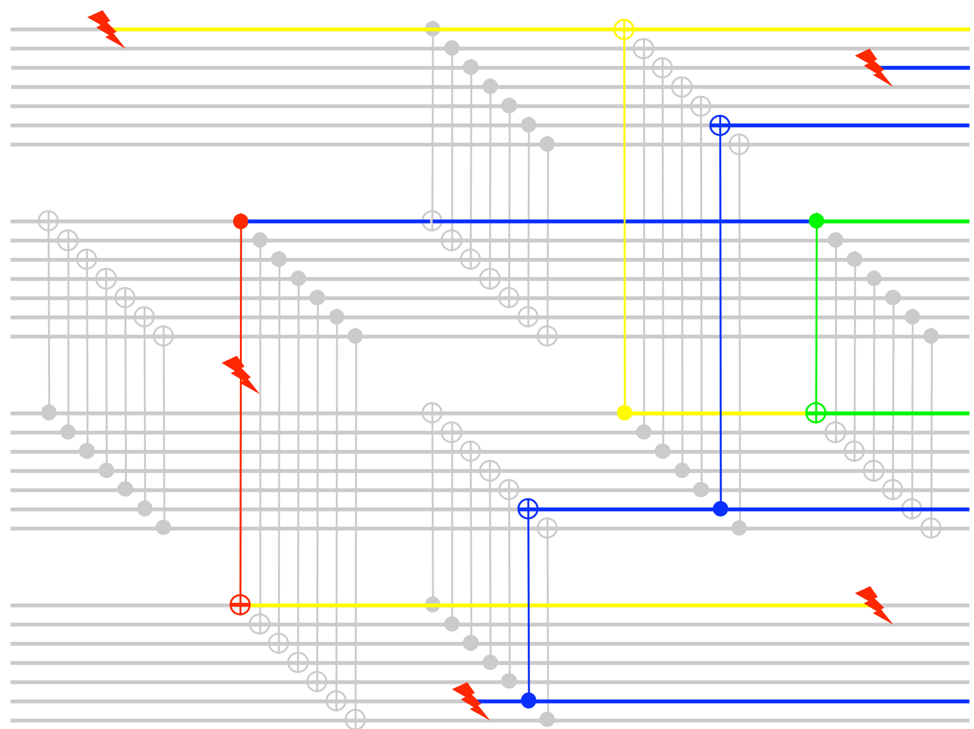


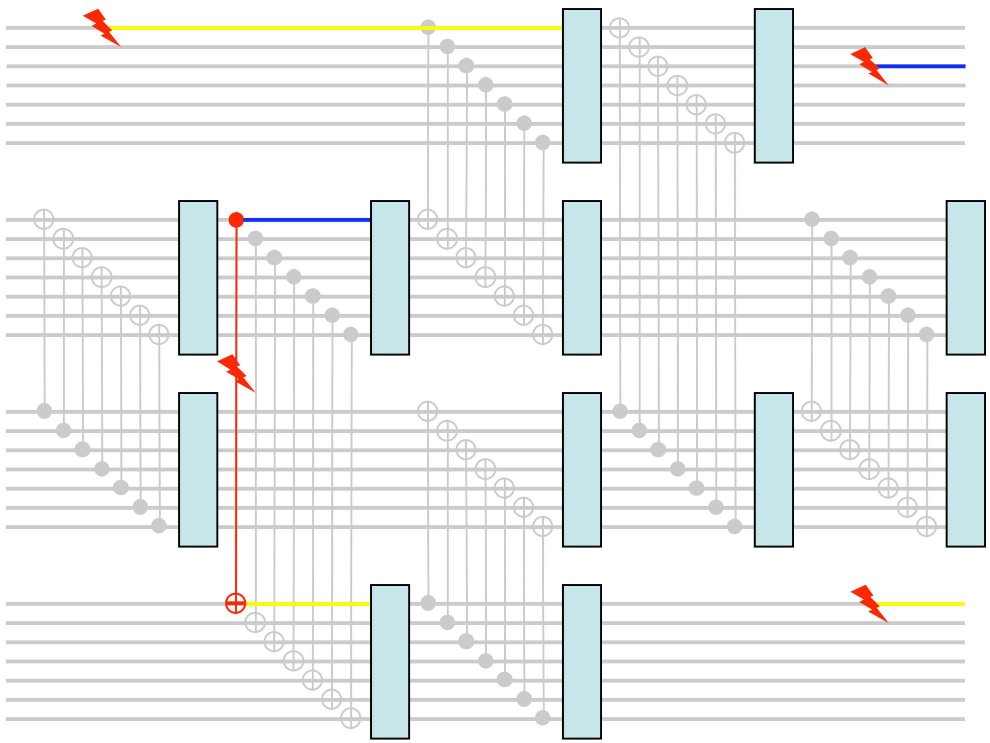
- Classical codewords in the +/- basis
  ⇒ Correct phase flip Z errors



- E.g., quantum [[7,1,3]] code corrects arbitrary error on one qubit
  – Based on classical Hamming [7,4,3] code

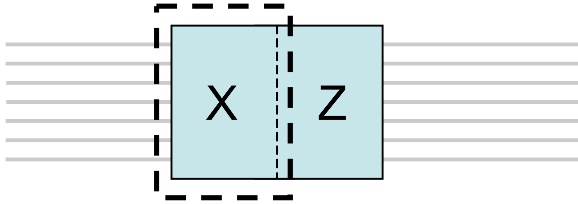




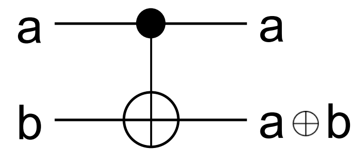


Standard fault-tolerance scheme

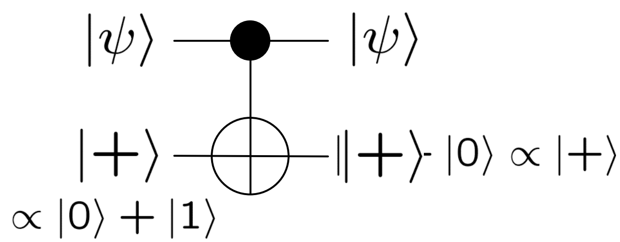
[Steane,…]



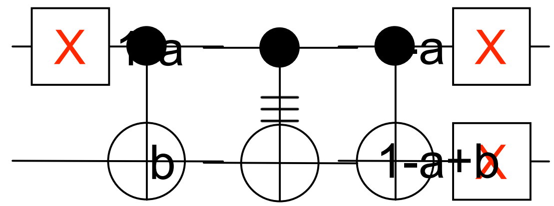
Def: CNOT

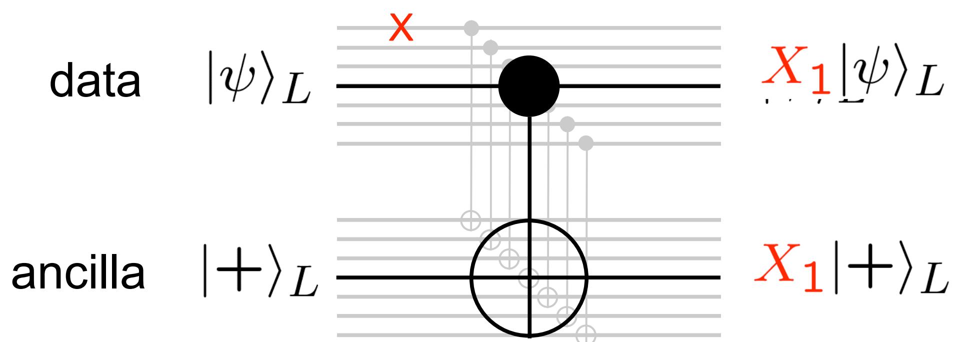


Fact 1:



Fact 2:





Threshold from concatenation

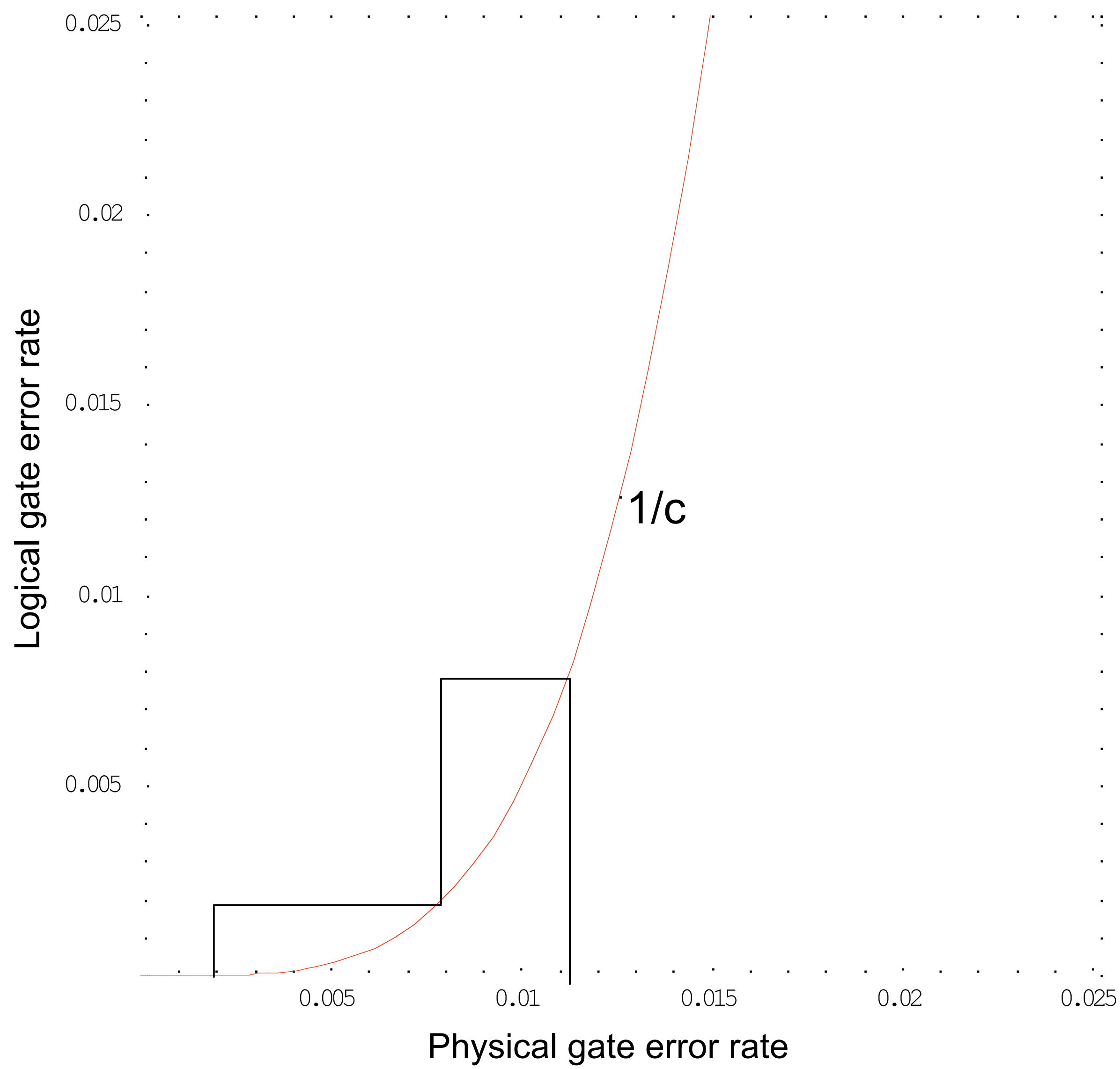
- N gate circuit

  $\Rightarrow$ Want error $\ll$1/N

- [[7, 1, 3]] code only corrects 1 error

Probability of error	Physical bits per logical bit
p	1
$c\,p^2$	7
$\sim p^{2^2}$	7^2
p^{2^3}	7^3

O(log log N) concatenations
O(log N) physical bits / logical



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Improved threshold result [R '04]

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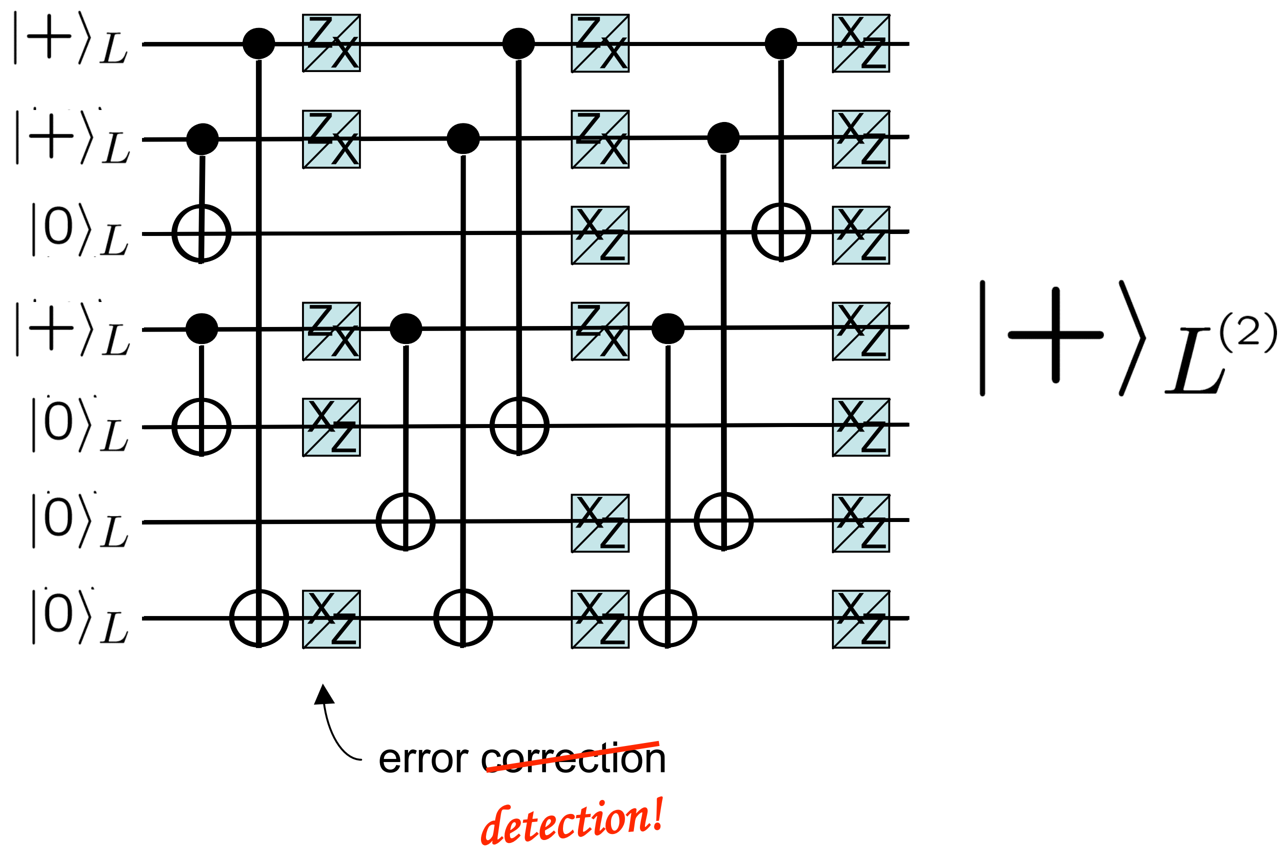
- **Teleportation**

Knill's threshold result

– Theorem: Threshold for erasure errors is ½ for Bell measurements.

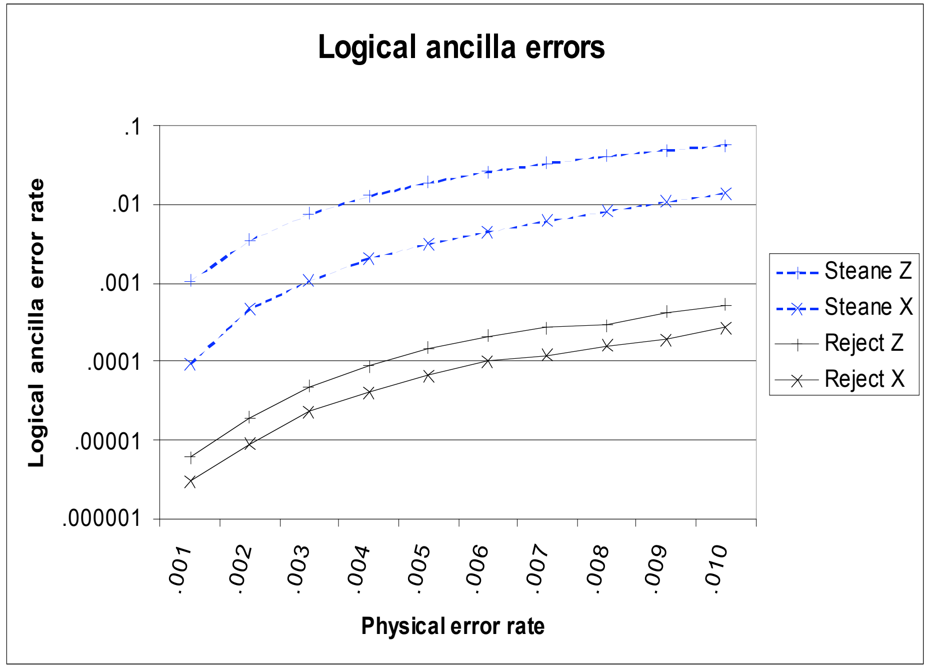
– Estimated 5-10% threshold for independent depolarizing errors.

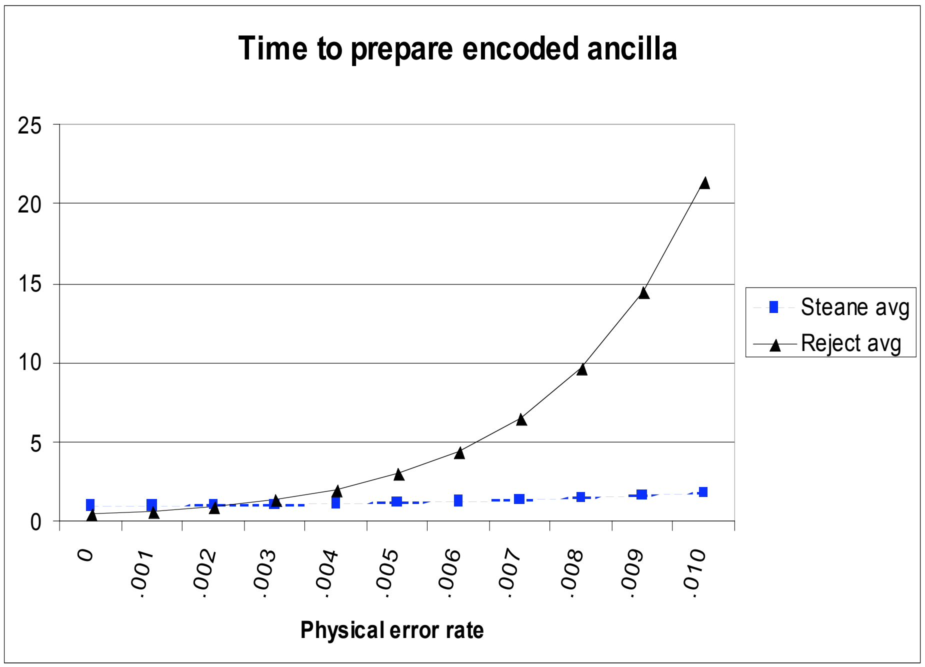
! **Defer exposure of data to operations**

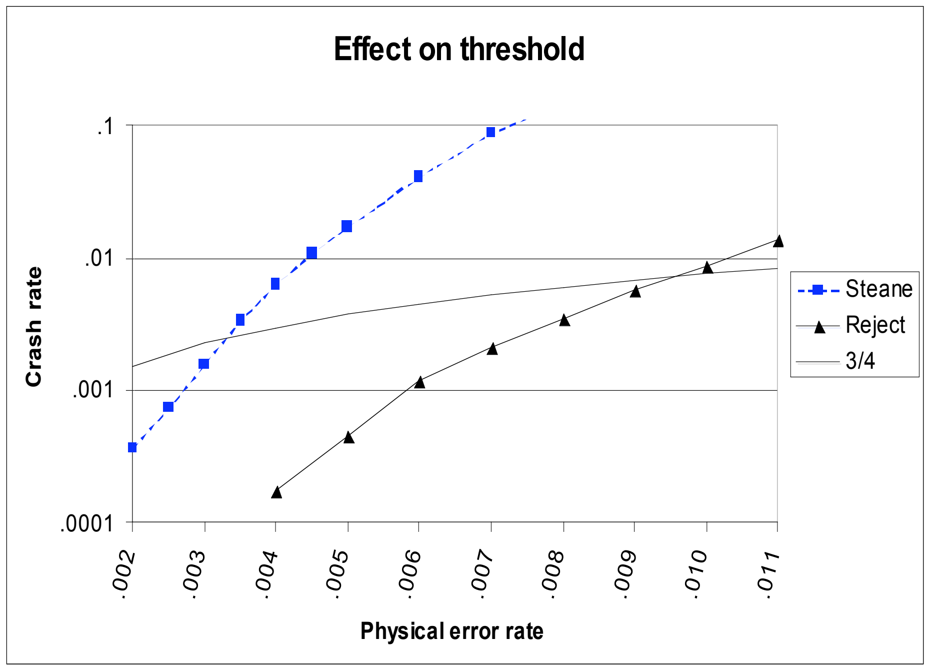

$|+\rangle_L$
$|+\rangle_L$
$|0\rangle_L$
$|+\rangle_L$
$|0\rangle_L$
$|0\rangle_L$
$|0\rangle_L$
$|+\rangle_{L^{(2)}}$
error ~~correction~~
detection!

Effect of **postselection** in ancilla preparation

[R '04]

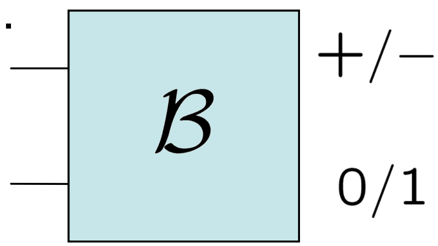




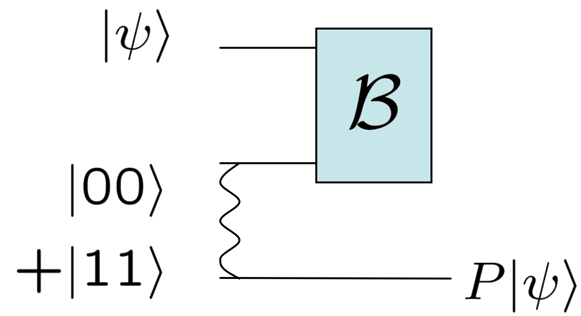


Teleportation: Knill's erasure threshold of ½

Theorem [Knill '03]: Threshold for erasure error is ½ for Bell measurements.



Teleportation



$P \in \{I, X, Y, Z\}$

Teleportation

Alice

$|\psi\rangle$

$|00\rangle + |11\rangle$

$P|\psi\rangle$

$P \in \{I, X, Y, Z\}$

Bob

Teleportation + Computation

Alice

$|\psi\rangle$

$|00\rangle + |11\rangle$

Assume no errors!

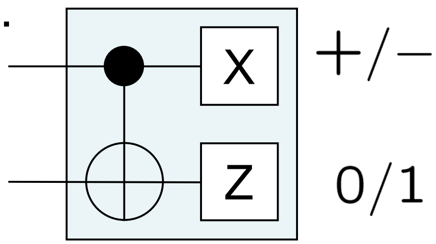
$UP|\psi\rangle$

$^{-1}U^{-1}$

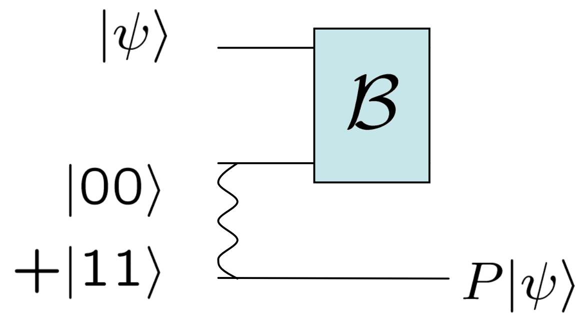
Bob

Teleportation: Knill's erasure threshold of ½

Theorem [Knill '03]: Threshold for erasure error is ½ for Bell measurements.

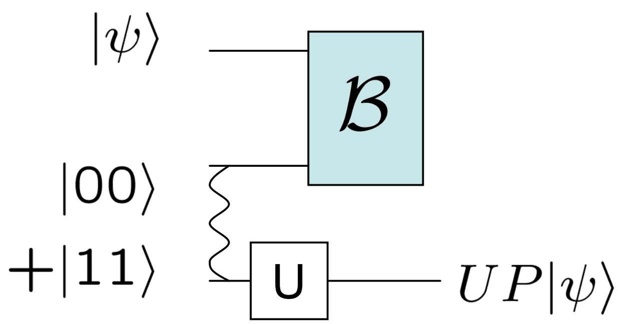


Teleportation



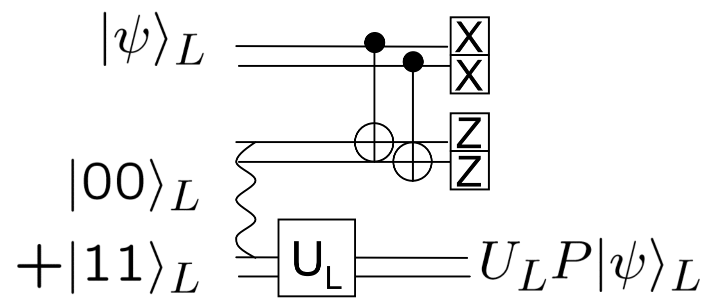
$P \in \{I, X, Y, Z\}$

+ Computation



$UP^{-1}U^{-1}$

+ Fault-tolerance

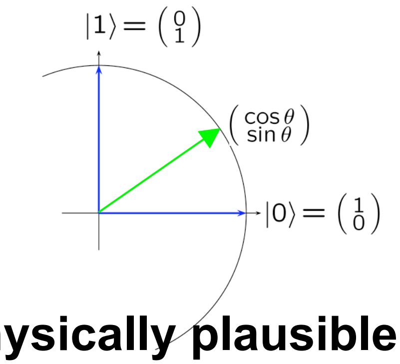


$P \in \{I, X, Y, Z\}^{\otimes n}$

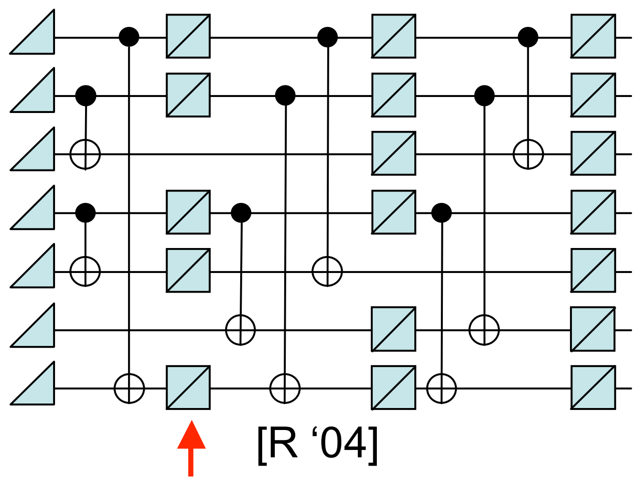
[Knill '04]: Estimated threshold of 5-10%.

Open questions ?

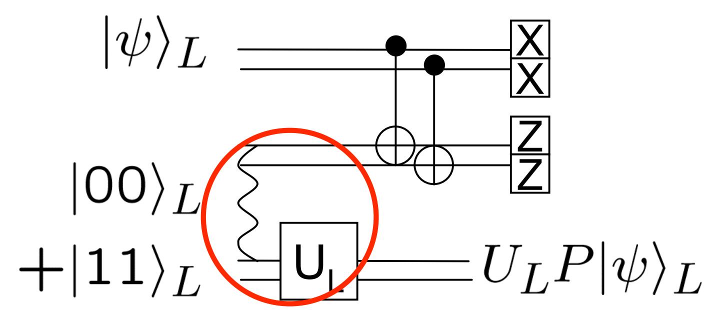
- **Errors inevitable in quantum computers**



- **Fault-tolerance schemes can tolerate physically plausible error rates**



[R ‘04]



[Knill ‘04]

Efficient? **Provable?**