

CSCI 420 Computer Graphics
Lecture 7


Polygon Meshes and Implicit Surfaces

Polygon Meshes
Implicit Surfaces
Constructive Solid Geometry
[Angel Ch. 12.1-12.3]

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Modeling Complex Shapes



Source: Wikipedia

- An equation for a sphere is possible, but how about an equation for a telephone, or a face?
- Complexity is achieved using simple pieces
 - polygons, parametric surfaces, or implicit surfaces
- Goals
 - Model *anything* with arbitrary precision (in principle)
 - Easy to build and modify
 - Efficient computations (for rendering, collisions, etc.)
 - Easy to implement (a minor consideration...)

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What do we need from shapes in Computer Graphics?

- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

No single technique solves all problems!

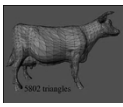
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Shape Representations

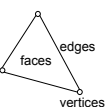
Polygon Meshes
Parametric Surfaces
Implicit Surfaces

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Polygon Meshes



- Any shape can be modeled out of polygons
 - if you use enough of them...
- Polygons with how many sides?
 - Can use triangles, quadrilaterals, pentagons, ... n-gons
 - Triangles are most common.
 - When > 3 sides are used, ambiguity about what to do when polygon nonplanar, or concave, or self-intersecting.
- Polygon meshes are built out of
 - *vertices* (points)
 - *edges* (line segments between vertices)
 - *faces* (polygons bounded by edges)



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Polygon Models in OpenGL

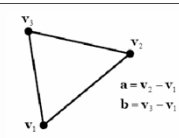
- for faceted shading


```
glNormal3fv(n);
glBegin(GL_POLYGONS);
glVertex3fv(vert1);
glVertex3fv(vert2);
glVertex3fv(vert3);
glEnd();
```
- for smooth shading


```
glBegin(GL_POLYGONS);
glNormal3fv(normal1);
glVertex3fv(vert1);
glNormal3fv(normal2);
glVertex3fv(vert2);
glNormal3fv(normal3);
glVertex3fv(vert3);
glEnd();
```

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Normals



Triangle defines unique plane


- can easily compute normal

$$\mathbf{n} = \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}$$

- depends on vertex orientation!
- clockwise order gives $\mathbf{n}' = -\mathbf{n}$

Vertex normals less well defined


- can average face normals
- works for smooth surfaces
- but not at sharp corners
- think of a cube



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Where Meshes Come From

- Specify manually
 - Write out all polygons
 - Write some code to generate them
 - Interactive editing: move vertices in space
- Acquisition from real objects
 - Laser scanners, vision systems
 - Generate set of points on the surface
 - Need to convert to polygons



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Data Structures for Polygon Meshes

- Simplest (but dumb)
 - float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
 - redundant: each vertex stored multiple times
- Vertex List, Face List
 - List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
 - List of triangles, each a triple of vertex id's (or pointers) topological (connectivity, adjacency) info only

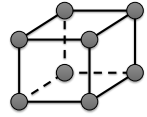
Fine for many purposes, but finding the faces adjacent to a vertex takes O(F) time for a model with F faces. Such queries are important for topological editing.
- Fancier schemes:
 - Store more topological info so adjacency queries can be answered in O(1) time.
 - *Winged-edge data structure* – edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).

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A File Format for Polygon Models: OBJ

```

# OBJ file for a 2x2x2 cube
v -1.0 1.0 1.0      - vertex 1
v -1.0 -1.0 1.0    - vertex 2
v 1.0 -1.0 1.0     - vertex 3
v 1.0 1.0 1.0
v -1.0 1.0 -1.0
v -1.0 -1.0 -1.0
v 1.0 -1.0 -1.0
v 1.0 1.0 -1.0
f 1 2 3 4
f 8 7 6 5
f 4 3 7 8
f 5 1 4 8
f 5 6 2 1
f 2 6 7 3
            
```



Syntax:

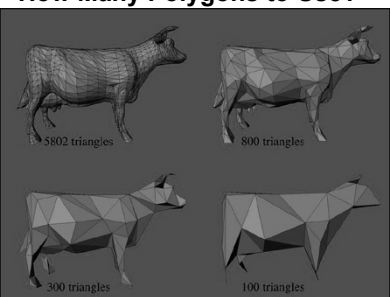
v x y z - a vertex at (x,y,z)

f v₁ v₂ ... v_n - a face with vertices v₁, v₂, ... v_n

anything - comment

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How Many Polygons to Use?



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Why Level of Detail?

- Different models for near and far objects
- Different models for rendering and collision detection
- Compression of data recorded from the real world

We need automatic algorithms for reducing the polygon count without

- losing key features
- getting artifacts in the silhouette
- popping

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Problems with Triangular Meshes?

- Need a lot of polygons to represent smooth shapes
- Need a lot of polygons to represent detailed shapes
- Hard to edit
- Need to move individual vertices
- Intersection test? Inside/outside test?

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Shape Representations

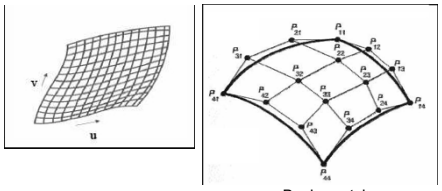
Polygon Meshes
Parametric Surfaces
Implicit Surfaces

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Parametric Surfaces

$p(u,v) = [x(u,v), y(u,v), z(u,v)]$

– e.g. plane, cylinder, bicubic surface, swept surface



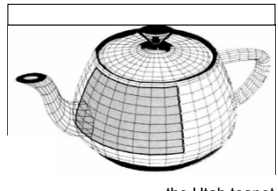
Bezier patch

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Parametric Surfaces

$p(u,v) = [x(u,v), y(u,v), z(u,v)]$

– e.g. plane, cylinder, bicubic surface, swept surface



the Utah teapot

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Parametric Surfaces

Why better than polygon meshes?

- Much more compact
- More convenient to control --- just edit control points
- Easy to construct from control points

What are the problems?

- Work well for smooth surfaces
- Must still split surfaces into discrete number of patches
- Rendering times are higher than for polygons
- Intersection test? Inside/outside test?

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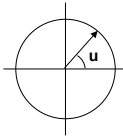
Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces

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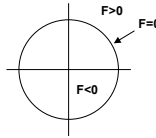
Two Ways to Define a Circle

Parametric



$x = f(u) = r \cos(u)$
 $y = g(u) = r \sin(u)$

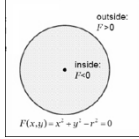
Implicit



$F(x,y) = x^2 + y^2 - r^2$

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Implicit Surfaces



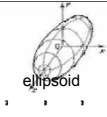
- well defined inside/outside
- polygons and parametric surfaces do not have this information
- Computing is hard: implicit functions for a cube? telephone?

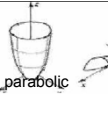
- **Implicit surface:** $F(x,y,z) = 0$
 - e.g. plane, sphere, cylinder, quadric, torus, blobby models
 - sphere with radius r : $F(x,y,z) = x^2 + y^2 + z^2 - r^2 = 0$
 - terrible for iterating over the surface
 - great for intersections, inside/outside test

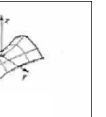
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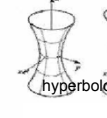
Quadric Surfaces

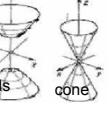
$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0$

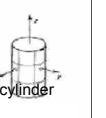

ellipsoid


parabolic


hyperboloids

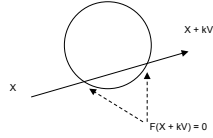

hyperboloids


cone

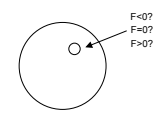

cylinder

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What Implicit Functions are Good For



Ray - Surface Intersection Test



Inside/Outside Test

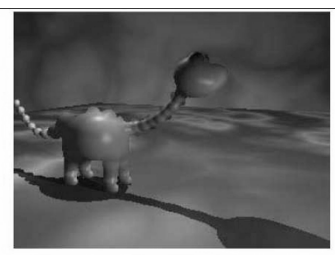
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Surfaces from Implicit Functions

- Constant Value Surfaces are called (depending on whom you ask):
 - constant value surfaces
 - level sets
 - isosurfaces
- Nice Feature: you can add them! (and other tricks)
 - this merges the shapes
 - When you use this with spherical exponential potentials, it's called *Blobs*, *Metaballs*, or *Soft Objects*. Great for modeling animals.

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Blobby Models



by Brian Wyvill, <http://www.cpsc.ucalgary.ca/~blob/>

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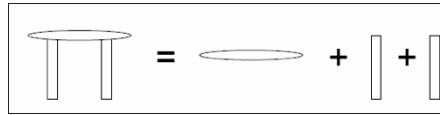
How to draw implicit surfaces?

- It's easy to ray trace implicit surfaces
 - because of that easy intersection test
- Volume Rendering can display them
- Convert to polygons: the Marching Cubes algorithm
 - Divide space into cubes
 - Evaluate implicit function at each cube vertex
 - Do root finding or linear interpolation along each edge
 - Polygonize on a cube-by-cube basis

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Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together



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Constructive Solid Geometry (CSG)

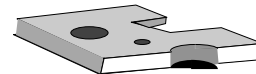
union	difference	intersection
		
the merger of two objects into one	the subtraction of one object from another	the portion common to both objects

Source: Wikipedia

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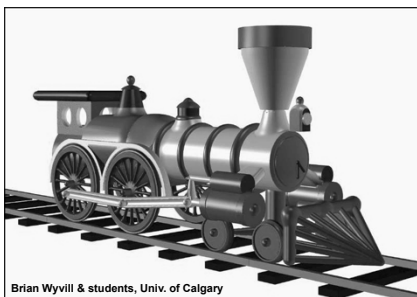
Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together
- This is sensible for objects that are actually made that way (human-made, particularly machined objects)



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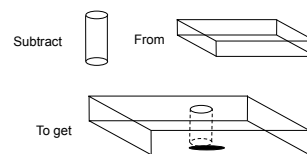
A CSG Train



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Negative Objects

- Use point-by-point boolean functions
- remove a volume by using a negative object
 - e.g. drill a hole by subtracting a cylinder



$$\text{Inside}(\text{BLOCK}-\text{CYL}) = \text{Inside}(\text{BLOCK}) \text{ And Not}(\text{Inside}(\text{CYL}))$$

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Set Operations

- UNION: $\text{Inside}(A) \parallel \text{Inside}(B)$
 - Join A and B
- INTERSECTION: $\text{Inside}(A) \ \&\& \ \text{Inside}(B)$
 - Chop off any part of A that sticks out of B
- SUBTRACTION: $\text{Inside}(A) \ \&\& \ (! \ \text{Inside}(B))$
 - Use B to Cut A

Examples:

- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.

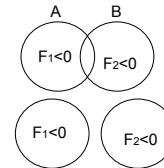
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Implicit Functions for Booleans

- Recall the implicit function for a solid: $F(x,y,z) < 0$
- Boolean operations are replaced by arithmetic:
 - MAX replaces AND (intersection)
 - MIN replaces OR (union)
 - MINUS replaces NOT(unary subtraction)

Thus

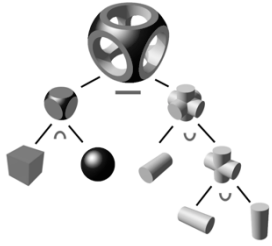
- $F(\text{Intersect}(A,B)) = \text{MAX}(F(A), F(B))$
- $F(\text{Union}(A,B)) = \text{MIN}(F(A), F(B))$
- $F(\text{Subtract}(A,B)) = \text{MAX}(F(A), -F(B))$



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CSG Trees

- Set operations yield tree-based representation



Source: Wikipedia

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Implicit Surfaces

- Good for smoothly blending multiple components
- Clearly defined solid along with its boundary
- Intersection test and Inside/outside test are easy
- Need to polygonize to render --- expensive
- Interactive control is not easy
- Fitting to real world data is not easy
- Always smooth

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Summary

- Polygonal Meshes
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry

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