

CSCI 420 Computer Graphics  
Lecture 26

## Visualization

Height Fields and Contours  
Scalar Fields  
Volume Rendering  
Vector Fields  
[Angel Ch. 2.11]

Jernej Barbic  
University of Southern California

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## Scientific Visualization

- Generally do not start with a 3D triangle model
- Must deal with very large data sets
  - MRI, e.g. 512 x 512 x 200 = 50MB points
  - Visible Human 512 x 512 x 1734 = 433 MB points
- Visualize both real-world and simulation data
- User interaction
- Automatic search for relevant data

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## Types of Data


- Scalar fields (3D volume of scalars)
  - E.g., x-ray densities (MRI, CT scan)
- Vector fields (3D volume of vectors)
  - E.g., velocities in a wind tunnel
- Tensor fields (3D volume of tensors [matrices])
  - E.g., stresses in a mechanical part
- Static or dynamic through time

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## Height Field


- Visualizing an explicit function

$z = f(x,y)$



- Adding contour curves

$g(x,y) = c$

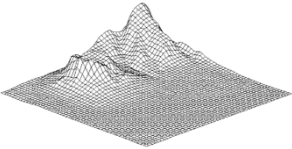


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## Meshes

- Function is sampled (given) at  $x_i, y_j, 0 \leq i, j \leq n$
- Assume equally spaced
 
$$x_i = x_0 + i\Delta x \quad z_{ij} = f(x_i, y_j)$$

$$y_j = y_0 + j\Delta y$$
- Generate quadrilateral or triangular mesh
- [Assignment 1]



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## Contour Curves

- Recall: implicit curve  $f(x,y) = 0$
- $f(x,y) < 0$  inside,  $f(x,y) > 0$  outside
- Here: contour curve at  $f(x,y) = c$
- Implicit function  $f$  sampled at regular intervals for  $x,y$ 

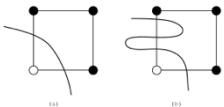
$$x_i = x_0 + i\Delta x$$

$$y_j = y_0 + j\Delta y$$
- How can we draw the curve?

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### Marching Squares

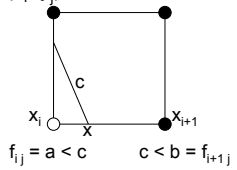
- Sample function  $f$  at every grid point  $x_i, y_j$
- For every point  $f_{ij} = f(x_i, y_j)$  either  $f_{ij} \leq c$  or  $f_{ij} > c$
- Distinguish those cases for each corner  $x$ 
  - White:  $f_{ij} \leq c$
  - Black:  $f_{ij} > c$
- Now consider cases for curve
- Assume "smooth"



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### Interpolating Intersections

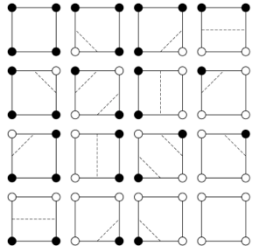
- Approximate intersection
  - Midpoint between  $x_i, x_{i+1}$  and  $y_j, y_{j+1}$
  - Better: interpolate
- If  $f_{ij} = a$  is closer to  $c$  than  $b = f_{i+1j}$  then intersection is closer to  $(x_i, y_j)$ :
 
$$\frac{x - x_i}{x_{i+1} - x} = \frac{c - a}{b - c}$$
- Analogous calculation for  $y$  direction



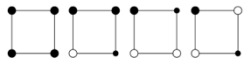
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### Cases for Vertex Labels

16 cases for vertex labels

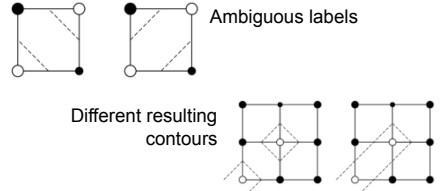


4 unique cases modulo symmetries

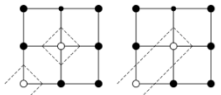


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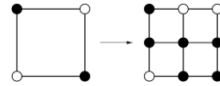
### Ambiguities of Labelings



Different resulting contours



Resolution by subdivision (if such higher resolution available / possible)



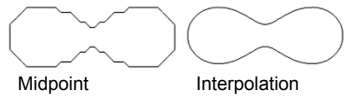
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### Marching Squares Examples

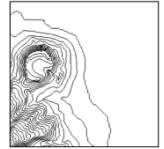
- Ovals of Cassini, 50 x 50 grid

$$f(x,y) = (x^2 + y^2 + a^2)^2 - 4a^2x^2 - b^4$$

$a = 0.49, b = 0.5$



Midpoint      Interpolation



Contour plot of Honolulu data

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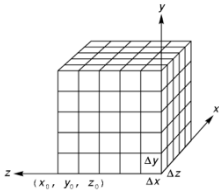
### Outline

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- Vector Fields

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### Scalar Fields

- Volumetric data sets
- Example: tissue density
- Assume again regularly sampled
  - $x_i = x_0 + i\Delta x$
  - $y_j = y_0 + j\Delta y$
  - $z_k = z_0 + k\Delta z$
- Represent as voxels



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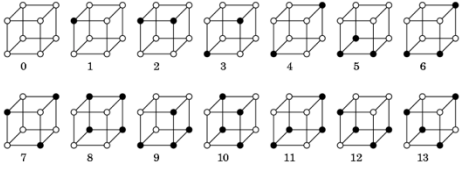
### Isosurfaces

- $f(x,y,z)$  represents volumetric data set
- Two rendering methods
  - Isosurface rendering
  - Direct volume rendering (use all values [next])
- Isosurface given by  $f(x,y,z) = c$
- Recall implicit surface  $g(x, y, z)$ :
  - $g(x, y, z) < 0$  inside
  - $g(x, y, z) = 0$  surface
  - $g(x, y, z) > 0$  outside
- Generalize right-hand side from 0 to c

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### Marching Cubes

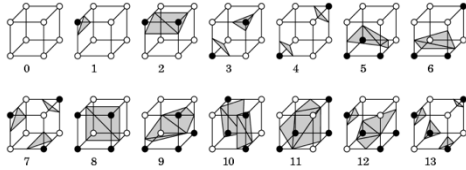
- Display technique for isosurfaces
- 3D version of marching squares
- 14 cube labelings (after elimination of symmetries)



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### Marching Cube Tessellations

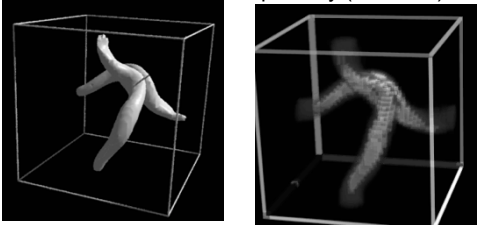
- Generalize marching squares, just more cases
- Interpolate as in 2D
- Ambiguities similar to 2D



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### Volume Rendering

- Sometimes isosurfaces are unnatural or do not give sufficient information
- Use all voxels and transparency ( $\alpha$ -values)



Ray-traced isosurface
Volume rendering

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### Surface vs. Volume Rendering

• 3D model of surfaces	• Scalar field in 3D
• Convert to triangles	• Convert it to RGBA values
• Draw primitives	• Render volume “directly”
• Lose or disguise data	• See data as given
• Good for opaque objects	• Good for complex objects

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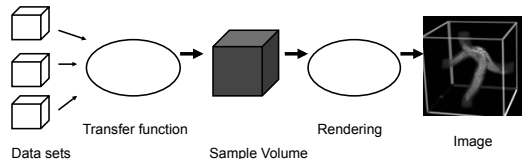
### Sample Applications

- Medical
  - Computed Tomography (CT)
  - Magnetic Resonance Imaging (MRI)
  - Ultrasound
- Engineering and Science
  - Computational Fluid Dynamic (CFD)
  - Aerodynamic simulations
  - Meteorology
  - Astrophysics

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### Volume Rendering Pipeline

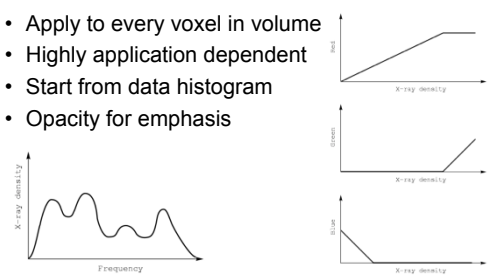
- Transfer function: converts input data set to colors and opacities
  - Example input: 256 x 256 x 256 x 8 bytes = 128 MB
  - Convert to 24 bit color, 8 bit opacity



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### Transfer Functions

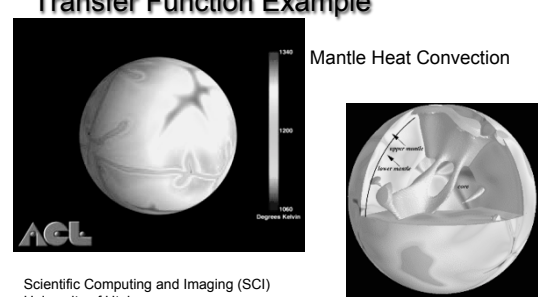
- Transform scalar data values to RGBA values
- Apply to every voxel in volume
- Highly application dependent
- Start from data histogram
- Opacity for emphasis



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### Transfer Function Example

Mantle Heat Convection



Scientific Computing and Imaging (SCI)  
University of Utah

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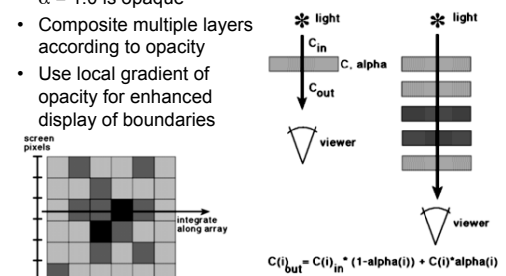
### Volume Ray Casting

- Three volume rendering techniques
  - Volume ray casting
  - Splatting
  - 3D texture mapping
- Ray Casting
  - Integrate color through volume
  - Consider lighting (surfaces?)
  - Use regular x,y,z data grid when possible
  - Finite elements when necessary (e.g., ultrasound)
  - 3D-rasterize geometrical primitives

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### Accumulating Opacity

- $\alpha = 1.0$  is opaque
- Composite multiple layers according to opacity
- Use local gradient of opacity for enhanced display of boundaries




$$C(i)_{out} = C(i)_{in} * (1 - \alpha(i)) + C(i) * \alpha(i)$$

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### Trilinear Interpolation

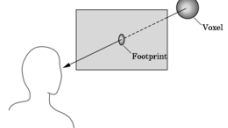
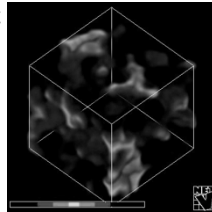
- Interpolate to compute RGBA away from grid
- Nearest neighbor yields blocky images
- Use trilinear interpolation
- 3D generalization of bilinear interpolation



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### Splatting

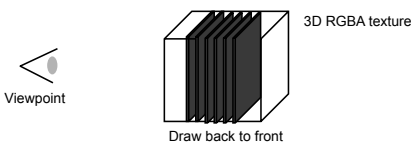
- Alternative to ray tracing
- Assign shape to each voxel (e.g., Gaussian)
- Project onto image plane (splat)
- Draw voxels back-to-front
- Composite ( $\alpha$ -blend)

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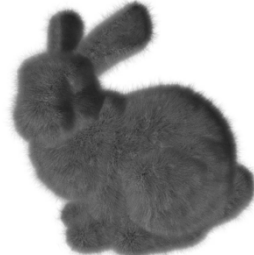
### 3D Textures

- Alternative to ray tracing, splatting
- Build a 3D texture (including opacity)
- Draw a stack of polygons, back-to-front
- Efficient if supported in graphics hardware
- Few polygons, much texture memory



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### Example: 3D Textures



Emil Praun' 01  
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### Example: 3D Textures

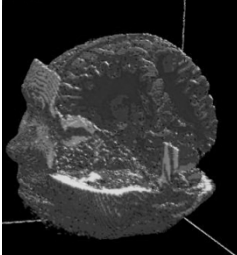
Emil Praun' 01



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### Other Techniques

- Use CSG for cut-aways



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### Acceleration of Volume Rendering

- Basic problem: Huge data sets
- Must program for locality (cache)
- Divide into multiple blocks if necessary
  - Example: marching cubes
- Use error measures to stop iteration
- Exploit parallelism

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### Outline

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- Vector Fields

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### Vector Fields

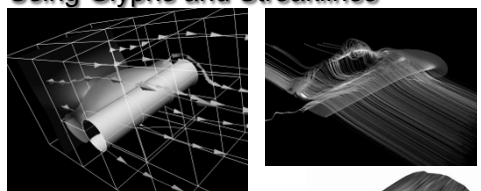
- Visualize vector at each (x,y,z) point
  - Example: velocity field
  - Example: hair
- Hedgehogs
  - Use 3D directed line segments (sample field)
  - Orientation and magnitude determined by vector
- Animation
  - Use for still image
  - Particle systems



Blood flow in human carotid artery

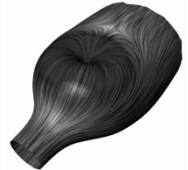
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### Using Glyphs and Streaklines



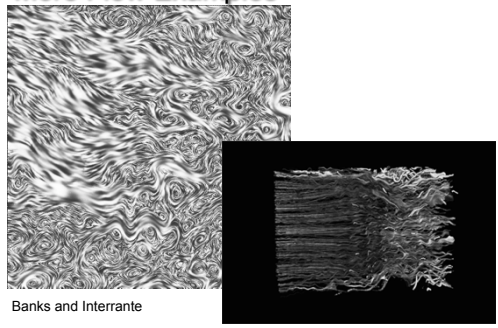
Glyphs for air flow  
University of Utah

Glyph = marker (for example, an arrow) used for data visualization



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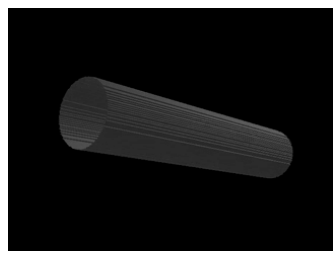
### More Flow Examples



Banks and Interrante

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### Example: Jet Shockwave



P. Sutton  
University of Utah

<http://www.sci.utah.edu/>

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## Summary

- Height Fields and Contours
- Scalar Fields
  - Isosurfaces
  - Marching cubes
- Volume Rendering
  - Volume ray tracing
  - Splatting
  - 3D Textures
- Vector Fields
  - Hedgehogs
  - Animated and interactive visualization

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