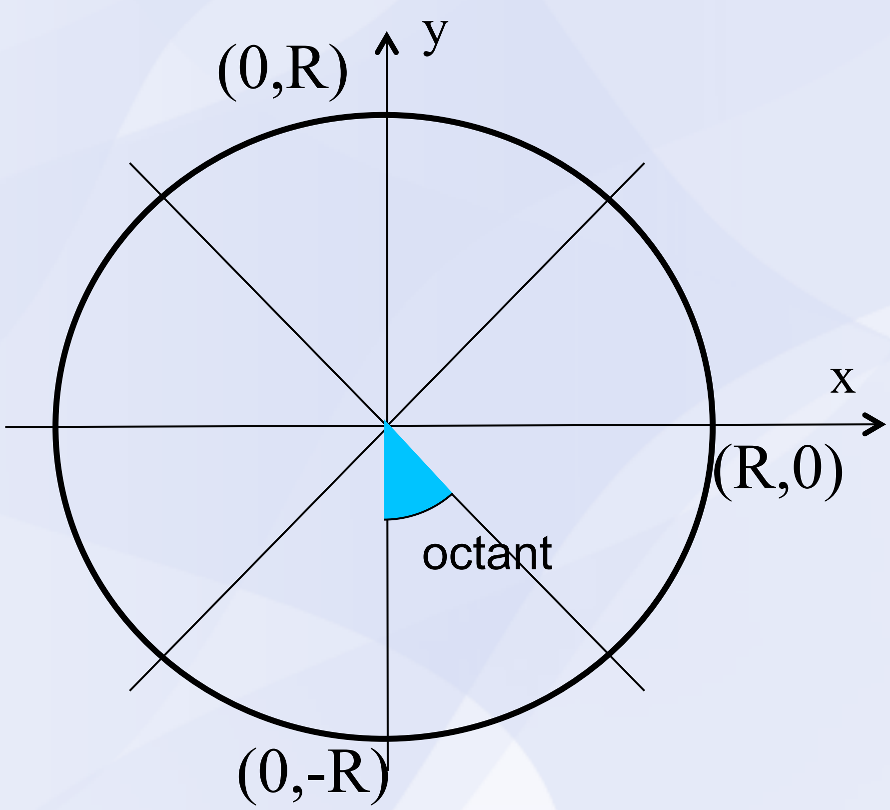
# Bresenham's Circle Algorithm




PowerPoint drawings all modified from [1]

# Due to symmetry:

## Consider only one octant is enough

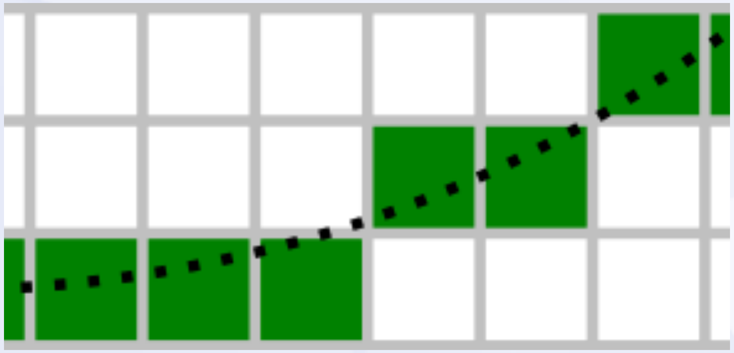


Image cropped from [2]

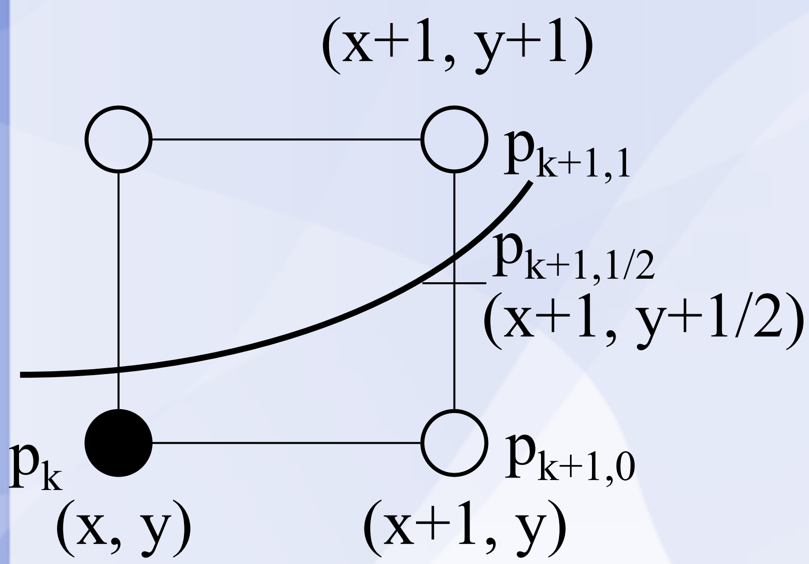
# X,Y,R are integers

$$F(x,y) = x^2 + y^2 - R^2$$

$F(x,y) = 0$ on circle

$F(x,y) < 0$ inside circle

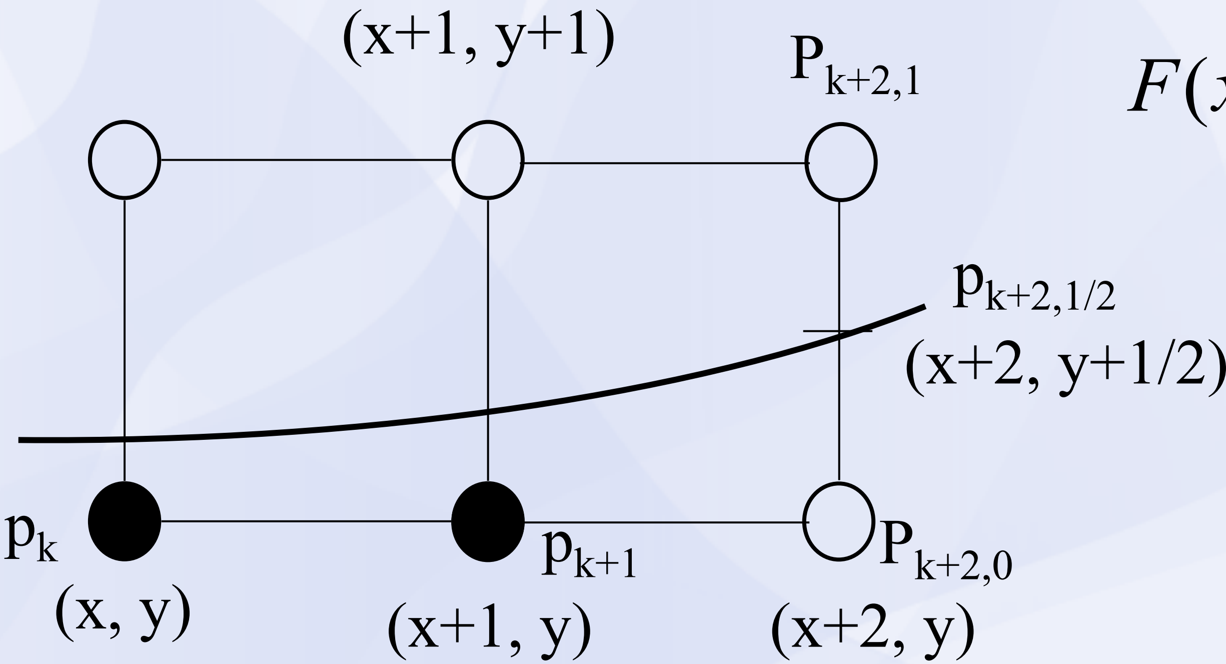
$F(x,y) > 0$ outside circle




$$d_k = F(x+1, y+\frac{1}{2})$$

$d_k<0$: $p_{k+1,1/2}$ in, draw $p_{k+1,0}$

$d_k>0$: $p_{k+1,1/2}$ out, draw $p_{k+1,1}$



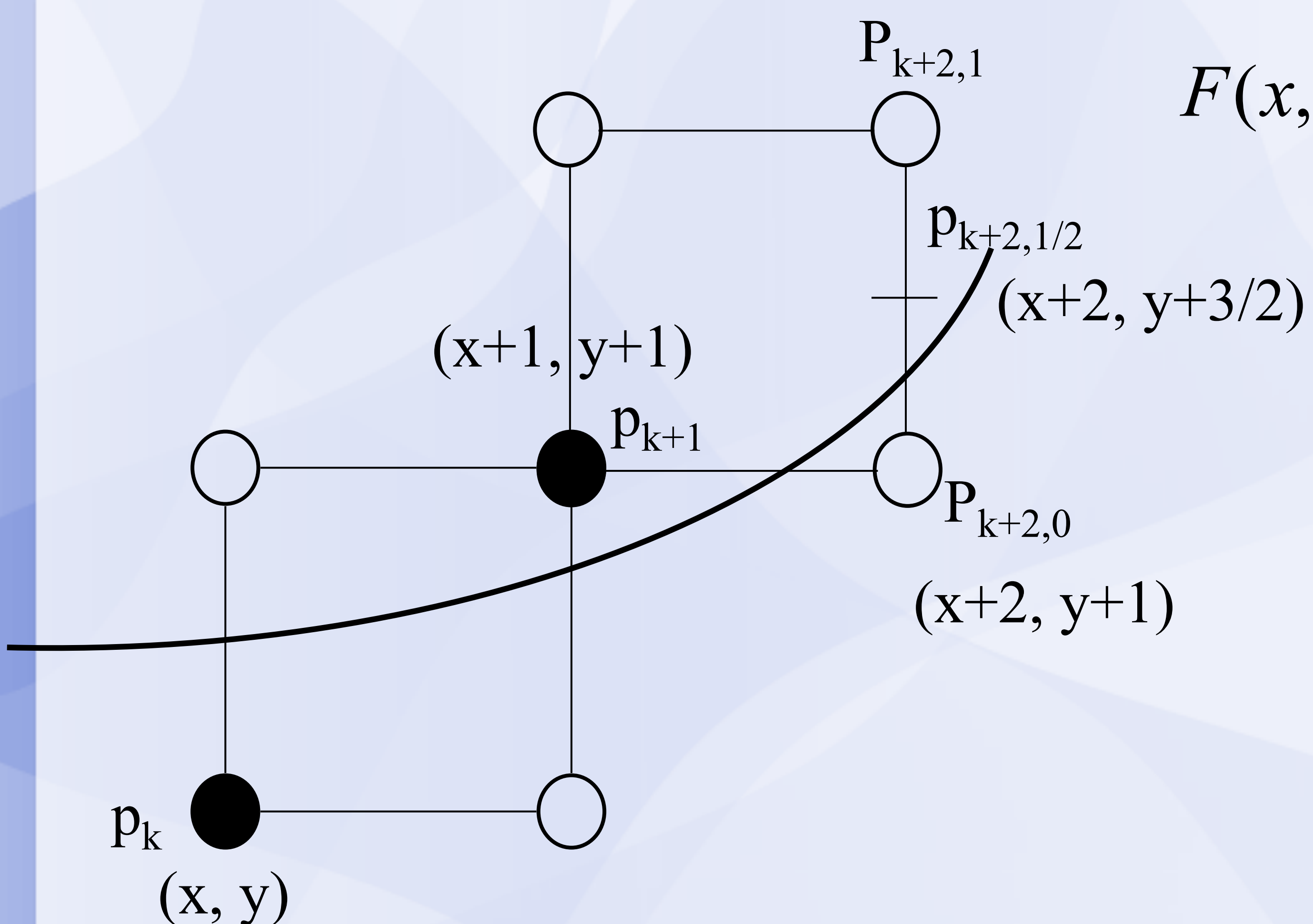

$$F(x, y) = x^2 + y^2 - R^2$$

$d_k<0$:

$$d_k = F(x+1, y+\frac{1}{2})$$

$$d_{k+1} = F(x+2, y+\frac{1}{2}) = d_k + 2x + 3$$

$$F(x, y) = x^2 + y^2 - R^2$$




$d_k>0$: $$d_k = F(x+1, y+\frac{1}{2})$$

$$d_{k+1} = F(x+2, y+\frac{3}{2}) = d_k + 2(x+y) + 5$$

# Initial d: $d_0$

Initial: (0,-R)

$d_0$ = F(1,−R+1/2) = 5/4-R  It's real!

Δd is always integer

$$d_{k+1} = F(x+2, y+\frac{1}{2}) = d_k + 2x + 3$$

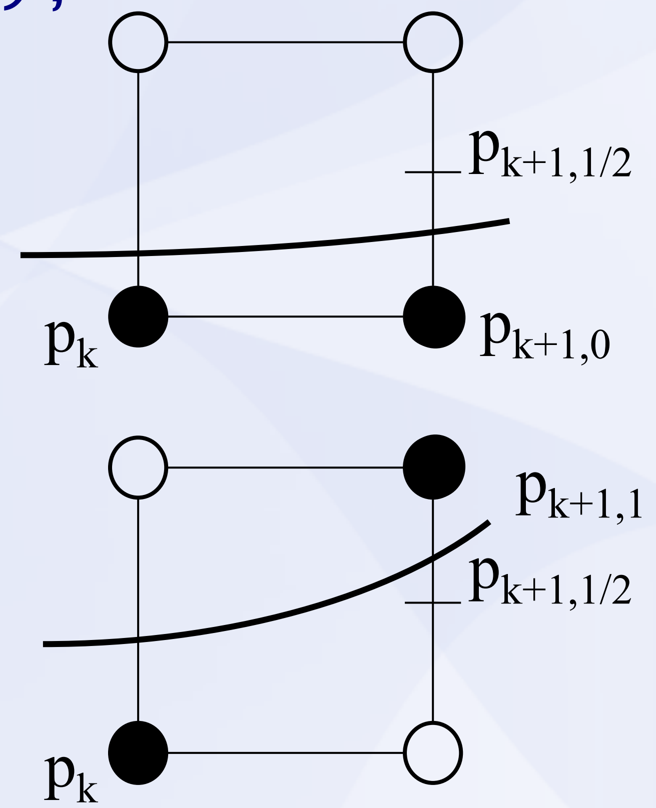
$$d_{k+1} = F(x+2, y+\frac{3}{2}) = d_k + 2(x+y) + 5$$

So $d_k$ always integer + 1/4

We judge $d_k > 0$ => $d_k - ¼ >= 0$

We make: $d_0 = 1 - R$

```
x = 0; y = -R;
d = 1- R; setPixel(x,y);
while (-y > x) {
  if (d >= 0) {
   d += 2x + 3;
   x++;
  } else {
   d += 2(x+y) + 5;
   x++; y++;
  }
  setPixel(x,y);
}
```




Multiplication by 2 can be achieved using left-shift

Also known as midpoint Circle algorithm

Another  way to define d:

$$d_k = [F(x+1, y+1)]^2 - [F(x+1, y)]^2$$

# Reference


[1]: http://ezekiel.vancouver.wsu.edu/~cs442/lectures/raster/circrect/circrect.ppt

[2]: http://en.wikipedia.org/wiki/Midpoint_circle_algorithm

Thanks!