

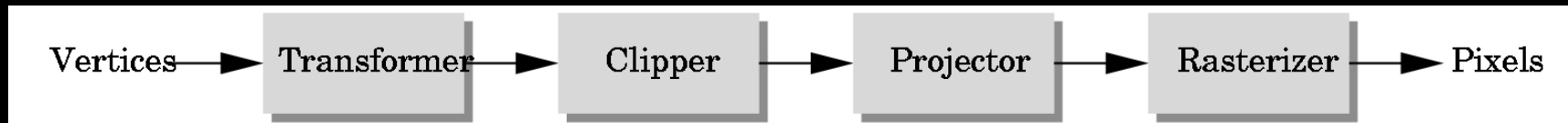
CSCI 420 Computer Graphics  
Lecture 13

# Clipping

Line Clipping  
Polygon Clipping  
Clipping in Three Dimensions  
[Angel Ch. 6]

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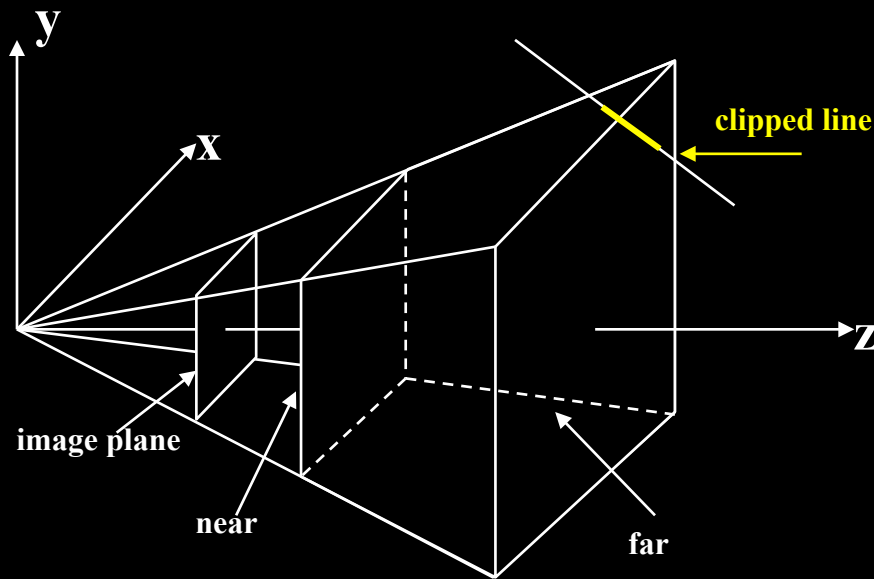
# The Graphics Pipeline, Revisited



- Must eliminate objects that are outside of viewing frustum
- **Clipping**: object space (eye coordinates)
- **Scissoring**: image space (pixels in frame buffer)
  - most often less efficient than clipping
- We will first discuss **2D clipping** (for simplicity)
  - OpenGL uses 3D clipping

# Clipping Against a Frustum

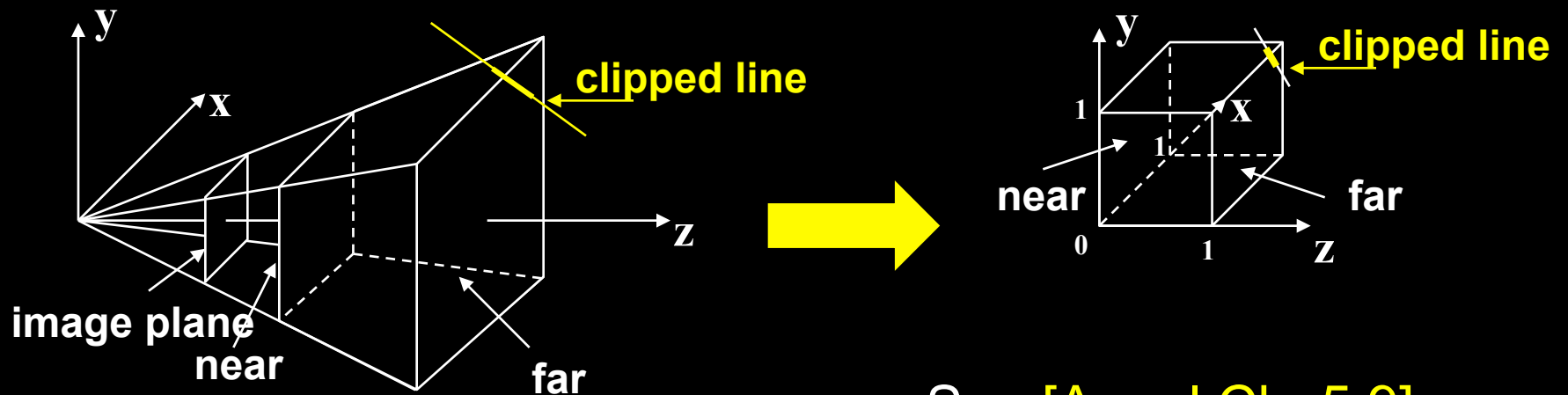
- General case of frustum (truncated pyramid)



- Clipping is tricky because of frustum shape

# Perspective Normalization

- Solution:
  - Implement perspective projection by **perspective normalization** and orthographic projection
  - Perspective normalization is a homogeneous transformation



See [Angel Ch. 5.9]

# The Normalized Frustum

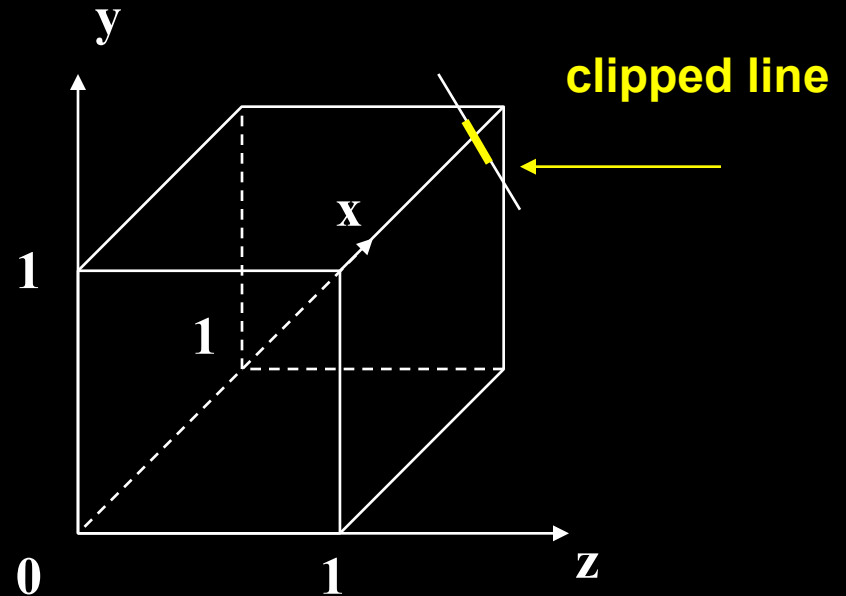
- OpenGL uses  $-1 \leq x, y, z \leq 1$  (others possible)
- Clip against resulting cube
- Clipping against arbitrary (programmer-specified) planes requires more general algorithms and is more expensive

# The Viewport Transformation

- Transformation sequence again:
  1. **Camera**: From object coordinates to eye coords
  2. **Perspective normalization**: to clip coordinates
  3. **Clipping**
  4. **Perspective division**: to normalized device coords.
  5. **Orthographic projection** (setting  $z_p = 0$ )
  6. **Viewport transformation**: to screen coordinates
- Viewport transformation can distort
  - Solution: pass the correct window aspect ratio to `gluPerspective`

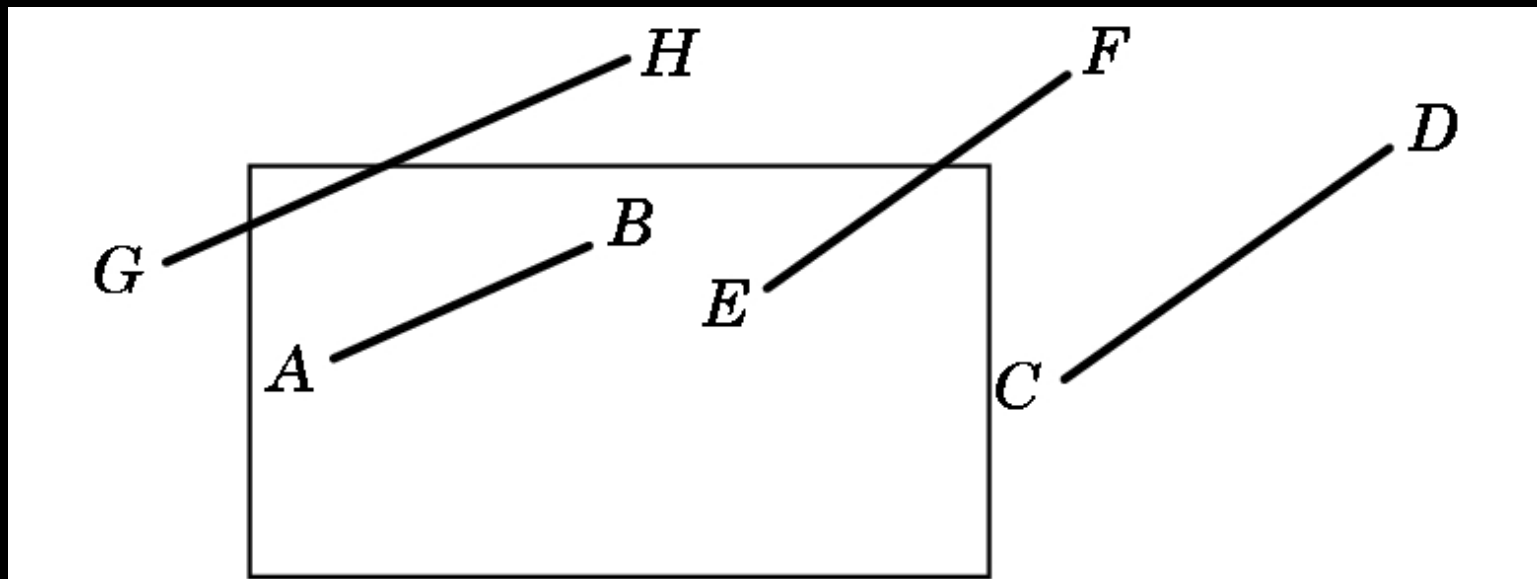
# Clipping

- General: 3D object against cube
- Simpler case:
  - In 2D: line against square or rectangle
  - Later: polygon clipping



# Clipping Against Rectangle in 2D

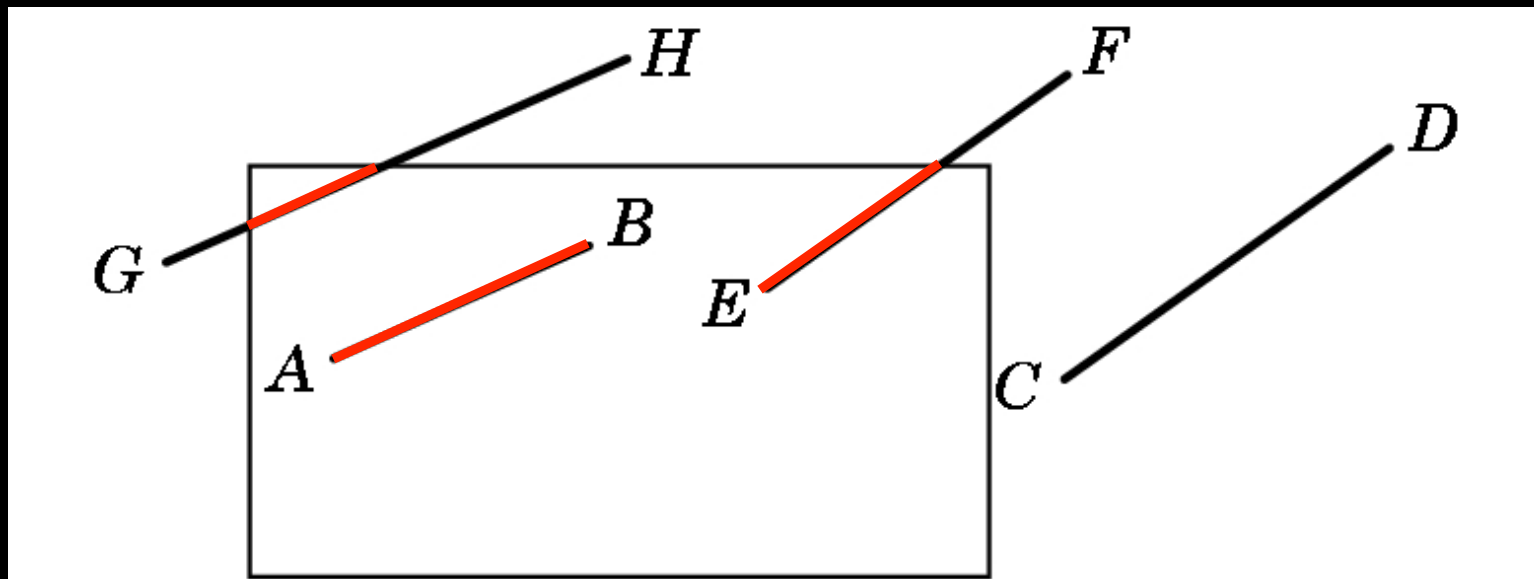
- **Line-segment clipping:** modify endpoints of lines to lie within clipping rectangle





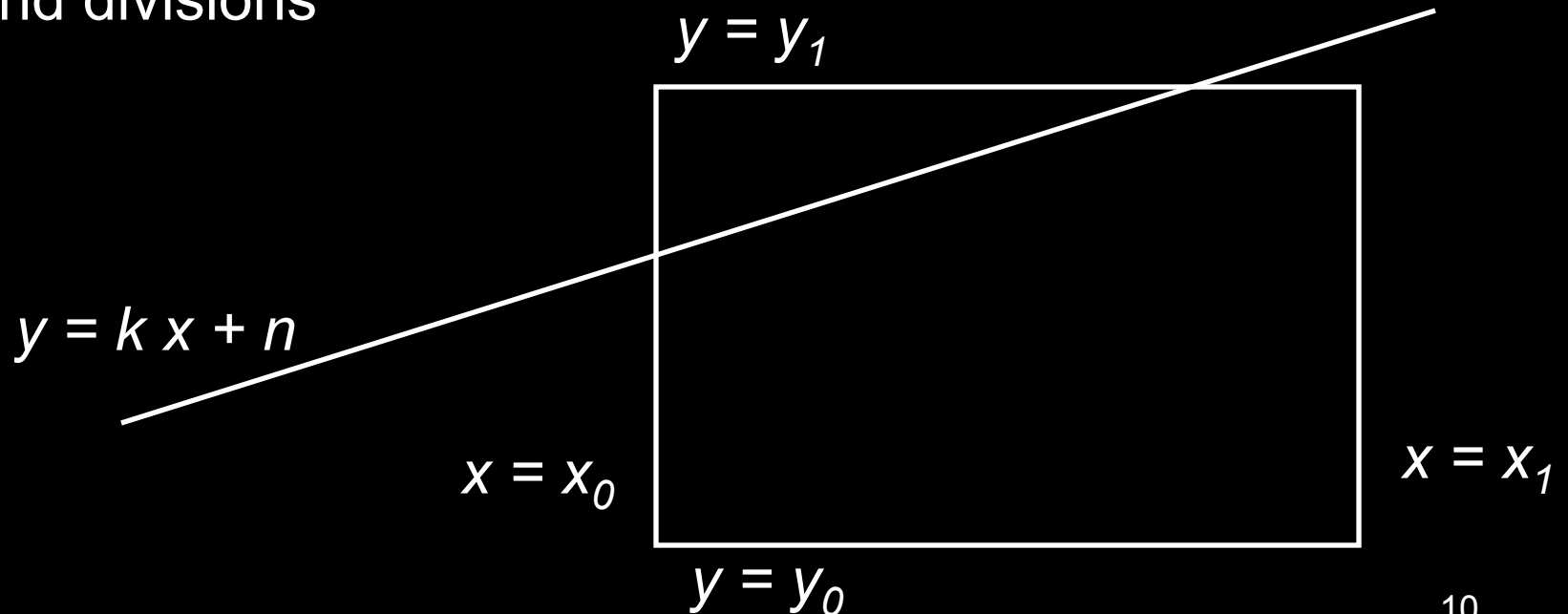
# Clipping Against Rectangle in 2D

- The result (in red)



# Clipping Against Rectangle in 2D

- Could calculate intersections of line segments with clipping rectangle
  - expensive, due to floating point multiplications and divisions
- Want to minimize the number of multiplications and divisions



# Several practical algorithms for clipping

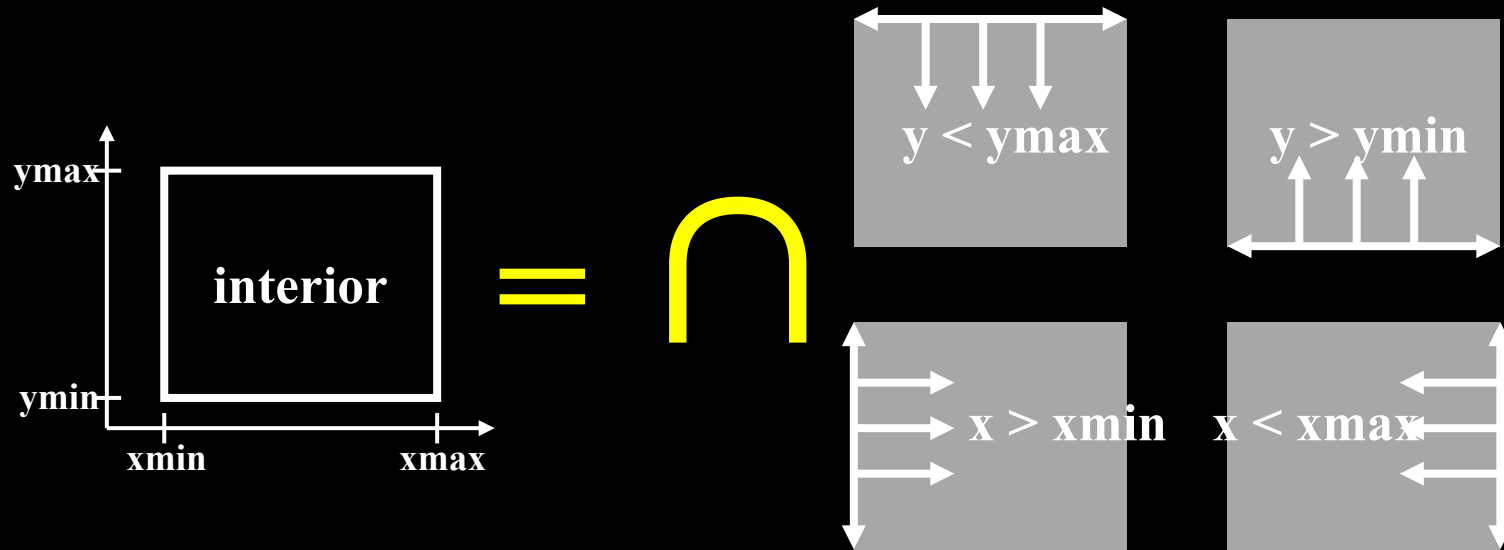
- Main motivation:

Avoid expensive line-rectangle intersections  
(which require floating point divisions)

- Cohen-Sutherland Clipping
- Liang-Barsky Clipping
- There are many more  
(but many only work in 2D)

# Cohen-Sutherland Clipping

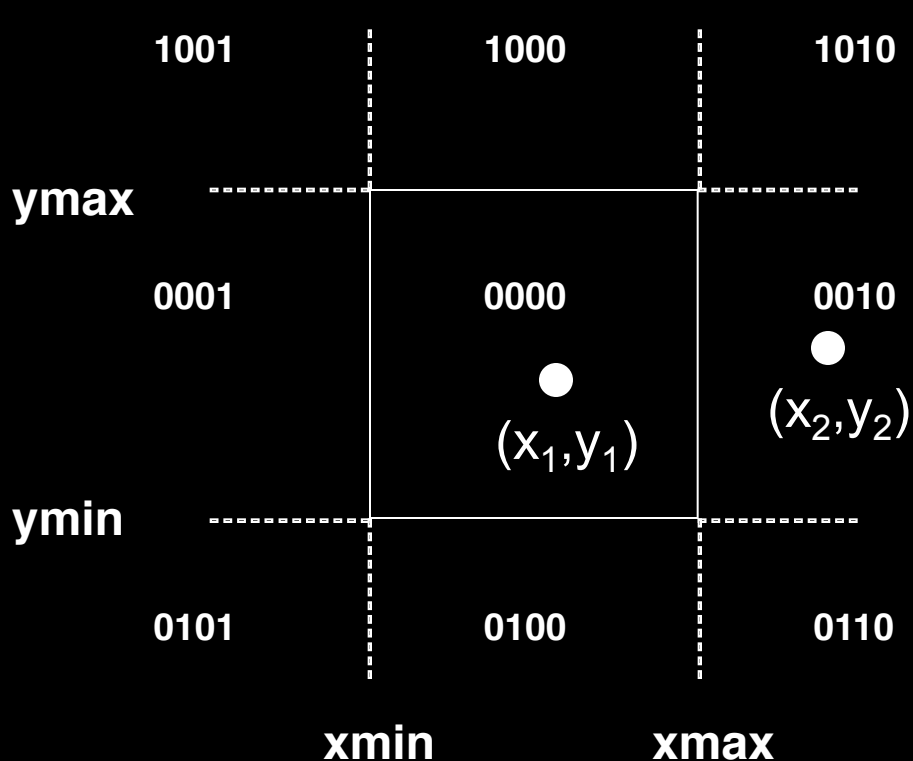
- Clipping rectangle is an intersection of 4 half-planes



- Encode results of four half-plane tests
- Generalizes to 3 dimensions (6 half-planes)

# Outcodes (Cohen-Sutherland)

- Divide space into 9 regions
- 4-bit **outcode** determined by comparisons



$b_0: y > y_{max}$

$b_1: y < y_{min}$

$b_2: x > x_{max}$

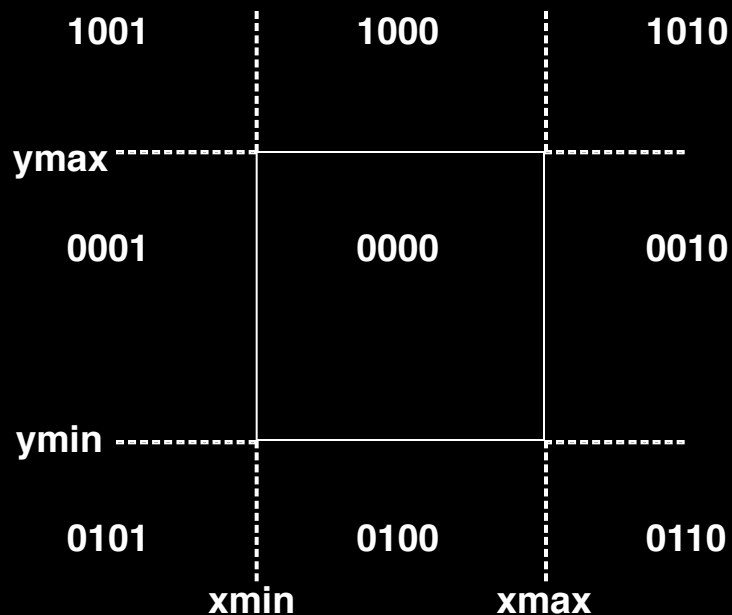
$b_3: x < x_{min}$

$o_1 = \text{outcode}(x_1, y_1)$

$o_2 = \text{outcode}(x_2, y_2)$

# Cases for Outcodes

- Outcomes: accept, reject, subdivide



$o_1 = o_2 = 0000$ : accept entire segment

$o_1 \& o_2 \neq 0000$ : reject entire segment

$o_1 = 0000, o_2 \neq 0000$ : subdivide

$o_1 \neq 0000, o_2 = 0000$ : subdivide

bitwise AND

$o_1 \& o_2 = 0000$ : subdivide

# Cohen-Sutherland Subdivision

- Pick outside endpoint ( $o \neq 0000$ )
- Pick a crossed edge ( $o = b_0b_1b_2b_3$  and  $b_k \neq 0$ )
- Compute intersection of this line and this edge
- Replace endpoint with intersection point
- Restart with new line segment
  - Outcodes of second point are unchanged
- This algorithm converges

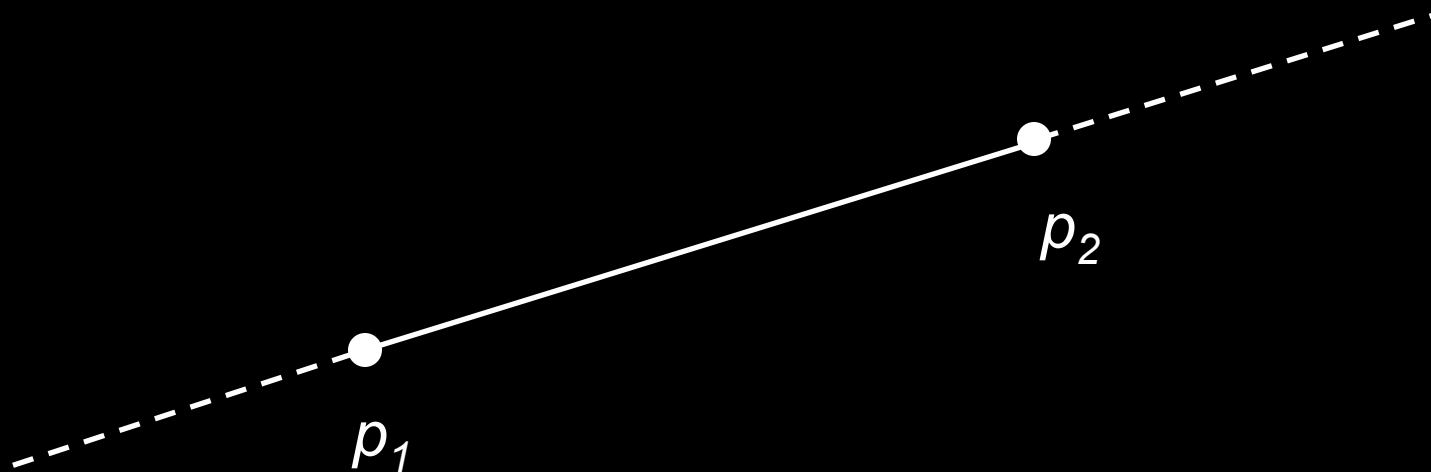
# Liang-Barsky Clipping

- Start with parametric form for a line

$$p(\alpha) = (1 - \alpha)p_1 + \alpha p_2, \quad 0 \leq \alpha \leq 1$$

$$x(\alpha) = (1 - \alpha)x_1 + \alpha x_2$$

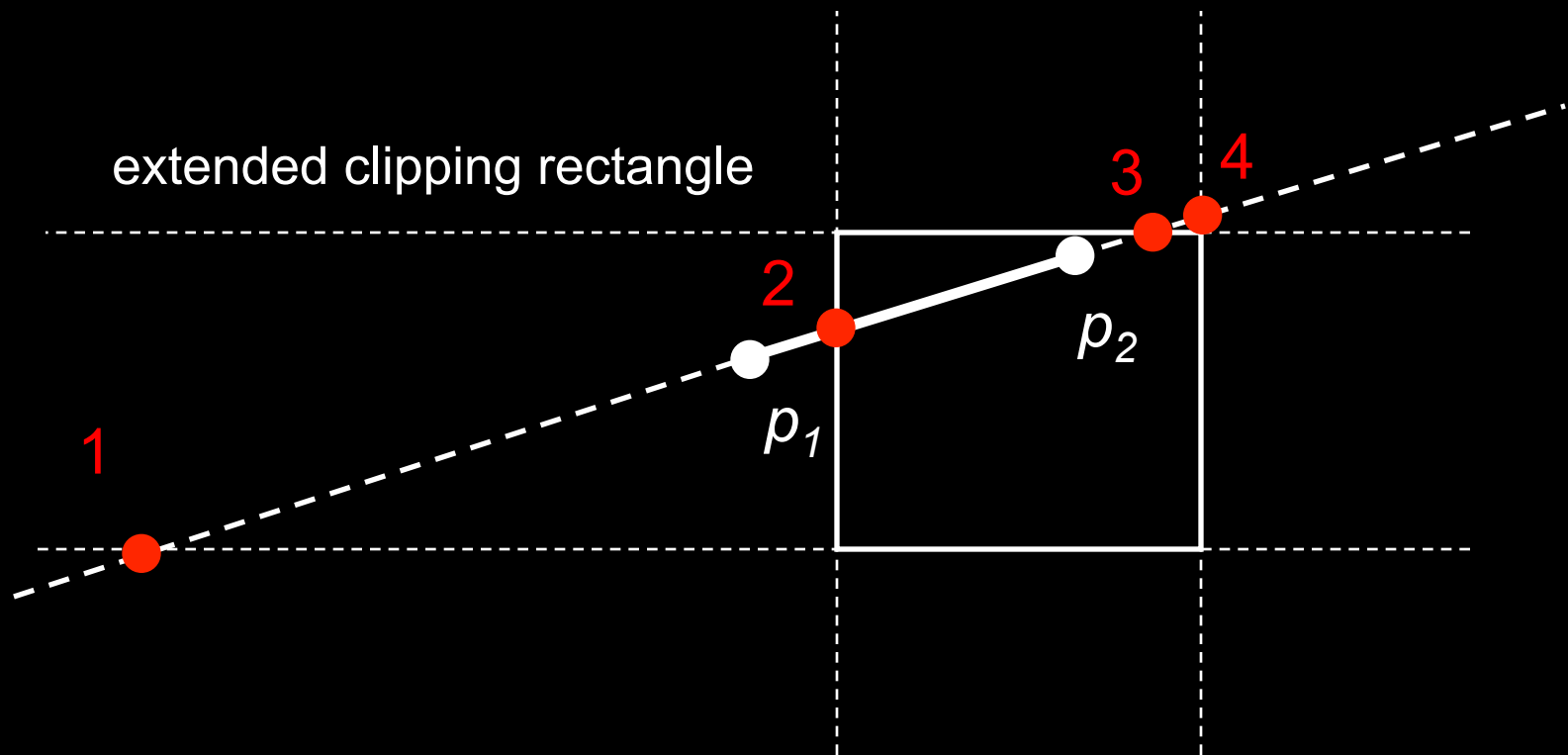
$$y(\alpha) = (1 - \alpha)y_1 + \alpha y_2$$



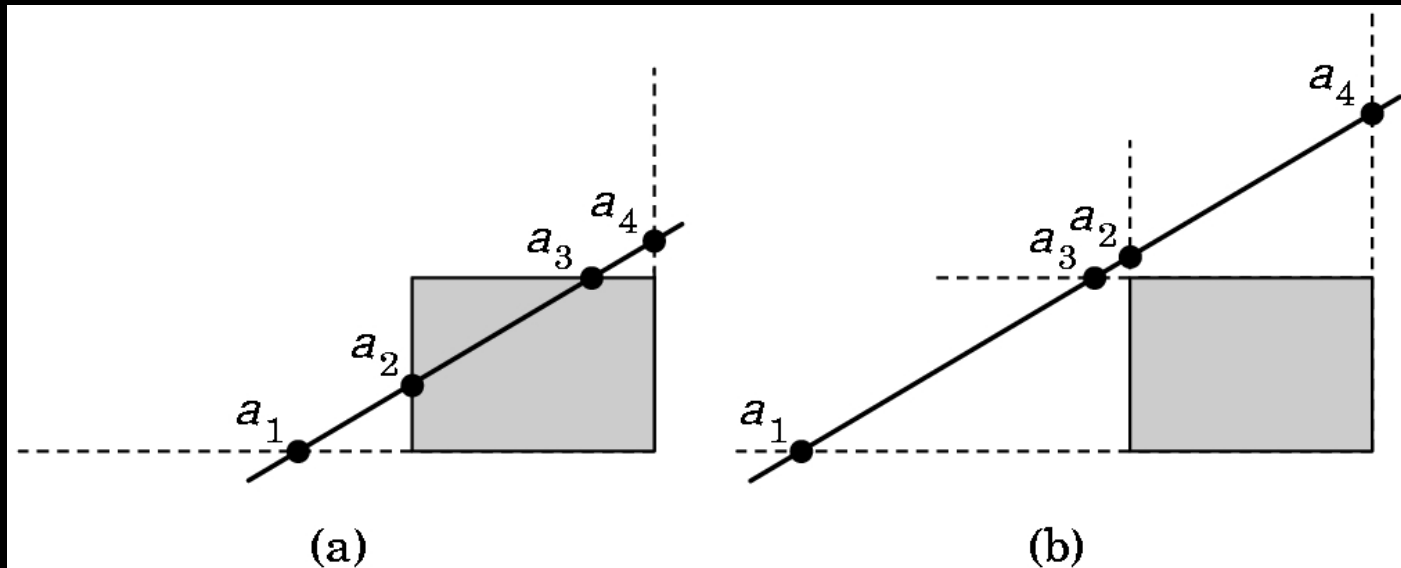


# Liang-Barsky Clipping

- Compute all four intersections **1,2,3,4** with **extended clipping rectangle**
- Often, no need to compute all four intersections

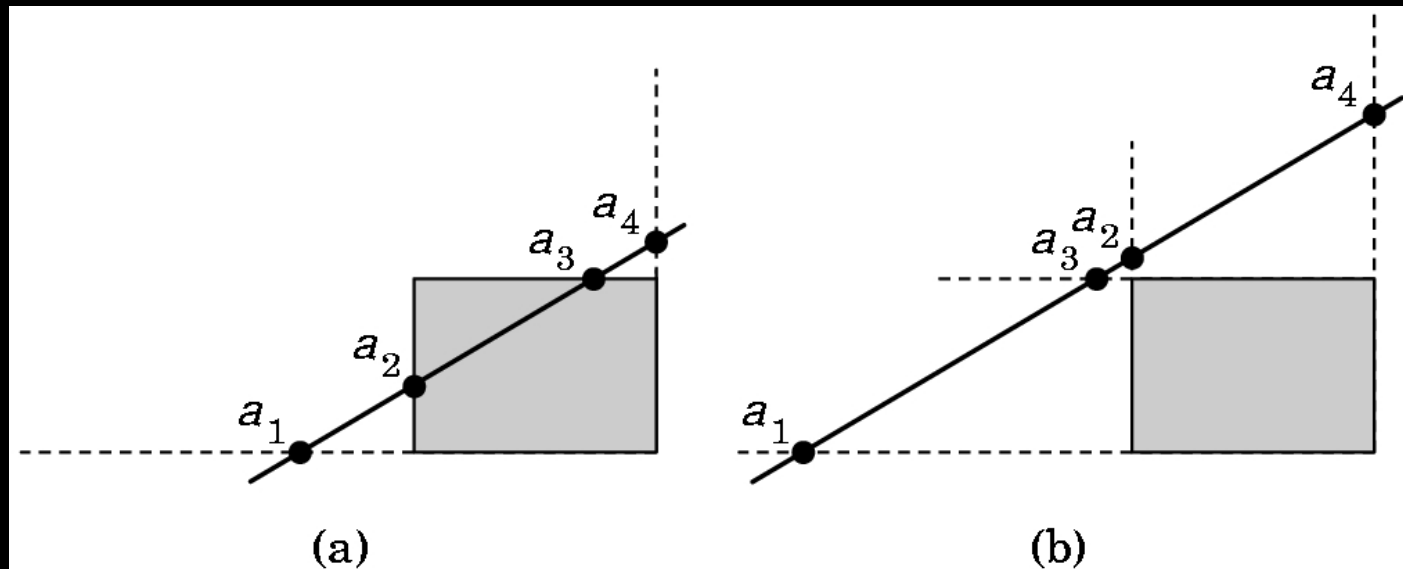


# Ordering of intersection points



- Order the intersection points
- Figure (a):  $1 > \alpha_4 > \alpha_3 > \alpha_2 > \alpha_1 > 0$
- Figure (b):  $1 > \alpha_4 > \alpha_2 > \alpha_3 > \alpha_1 > 0$

# Liang-Barsky Idea



- It is possible to clip already if one knows the order of the four intersection points !
- Even if the actual intersections were not computed !
- Can enumerate all ordering cases

# Liang-Barsky efficiency improvements

- Efficiency improvement 1:
  - Compute intersections one by one
  - Often can reject before all four are computed
- Efficiency improvement 2:
  - Equations for  $\alpha_3, \alpha_2$

$$y_{\max} = (1 - \alpha_3)y_1 + \alpha_3y_2$$

$$x_{\min} = (1 - \alpha_2)x_1 + \alpha_2x_2$$

$$\alpha_3 = \frac{y_{\max} - y_1}{y_2 - y_1} \quad \alpha_2 = \frac{x_{\min} - x_1}{x_2 - x_1}$$

- Compare  $\alpha_3, \alpha_2$  without floating-point division

# Line-Segment Clipping Assessment

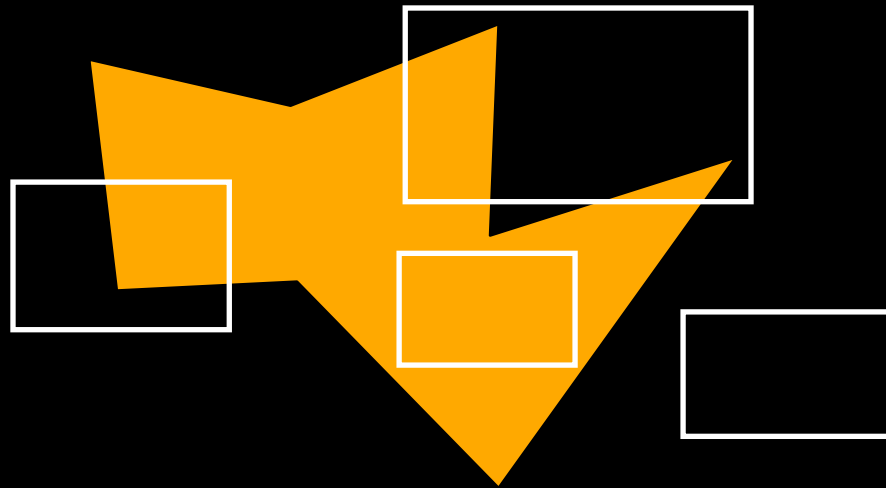
- Cohen-Sutherland
  - Works well if many lines can be rejected early
  - Recursive structure (multiple subdivisions) is a drawback
- Liang-Barsky
  - Avoids recursive calls
  - Many cases to consider (tedious, but not expensive)

# Outline

- Line-Segment Clipping
  - Cohen-Sutherland
  - Liang-Barsky
- Polygon Clipping
  - Sutherland-Hodgeman
- Clipping in Three Dimensions

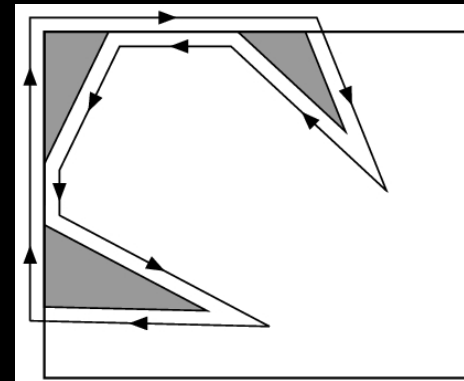
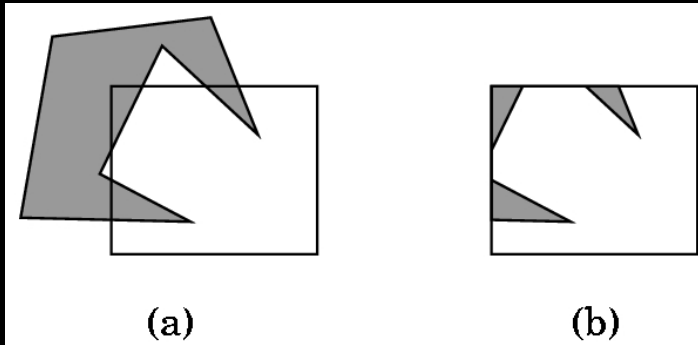
# Polygon Clipping

- Polygon is clipped into **one or more** polygons
- Their union is intersection with clip window
- Alternatively, we can first tessellate concave polygons (OpenGL supported)

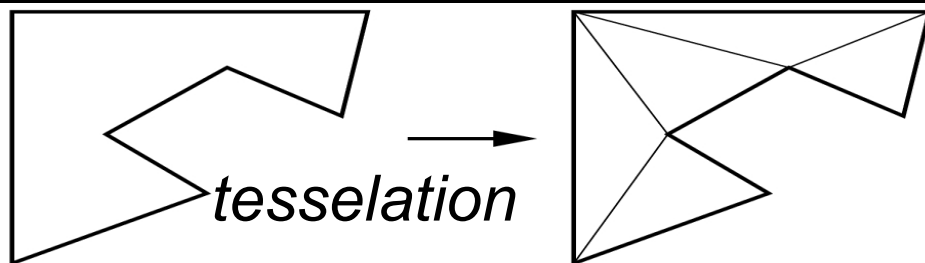


# Concave Polygons

- Approach 1: clip, and then join pieces to a single polygon
  - often difficult to manage



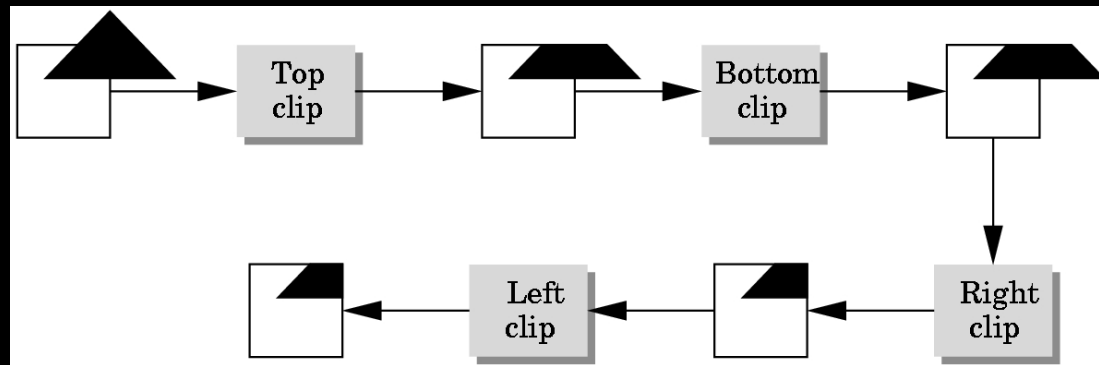
- Approach 2: tessellate and clip triangles
  - this is the common solution





# Sutherland-Hodgeman (part 1)

- Subproblem:
  - Input: polygon (vertex list) and single clip plane
  - Output: new (clipped) polygon (vertex list)
- Apply once for each clip plane
  - 4 in two dimensions
  - 6 in three dimensions
  - Can arrange in pipeline

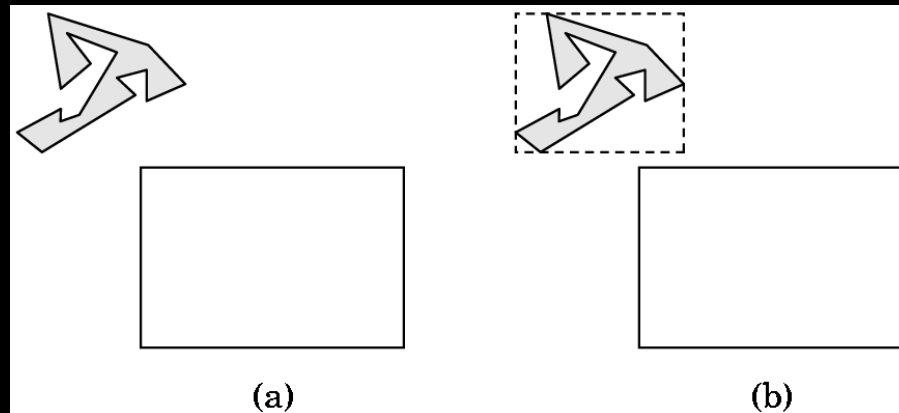


# Sutherland-Hodgeman (part 2)

- To clip vertex list (polygon) against a **half-plane**:
  - Test first vertex. Output if inside, otherwise skip.
  - Then loop through list, testing transitions
    - In-to-in: output vertex
    - In-to-out: output intersection
    - out-to-in: output intersection and vertex
    - out-to-out: no output
  - Will output clipped polygon as vertex list
- May need some cleanup in concave case
- Can combine with Liang-Barsky idea

# Other Cases and Optimizations

- Curves and surfaces
  - Do it analytically if possible
  - Otherwise, approximate curves / surfaces by lines and polygons
- Bounding boxes
  - Easy to calculate and maintain
  - Sometimes big savings

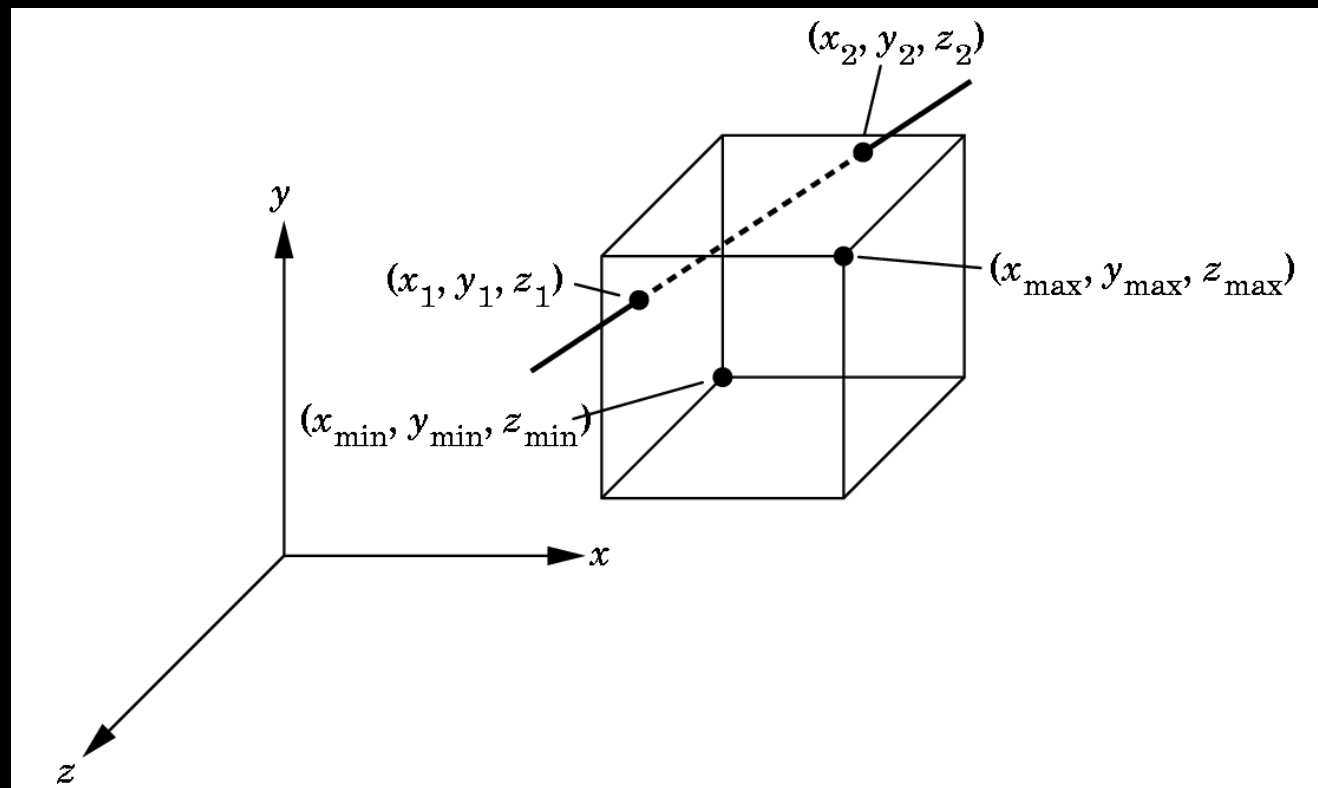


# Outline

- Line-Segment Clipping
  - Cohen-Sutherland
  - Liang-Barsky
- Polygon Clipping
  - Sutherland-Hodgeman
- Clipping in Three Dimensions

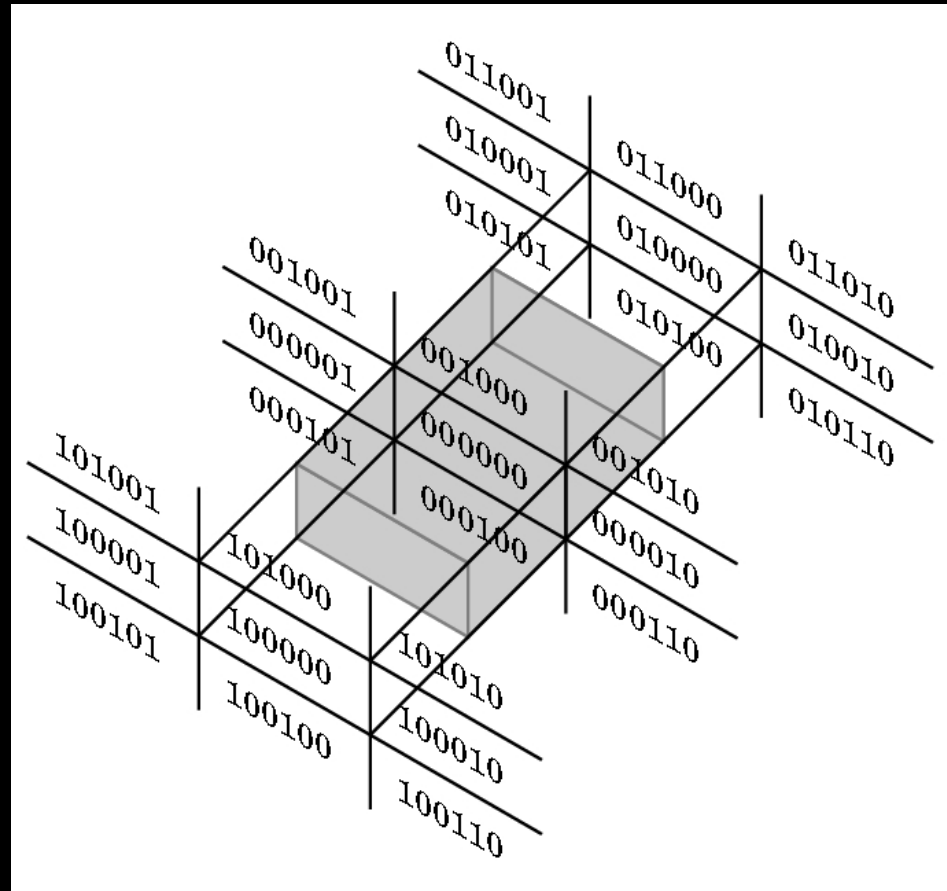
# Clipping Against Cube

- Derived from earlier algorithms
- Can allow right parallelepiped



# Cohen-Sutherland in 3D

- Use 6 bits in outcode
  - $b_4: z > z_{\max}$
  - $b_5: z < z_{\min}$
- Other calculations as before



# Liang-Barsky in 3D

- Add equation  $z(\alpha) = (1 - \alpha) z_1 + \alpha z_2$
- Solve, for  $p_0$  in plane and normal  $n$ :

$$p(\alpha) = (1 - \alpha)p_1 + \alpha p_2$$
$$n \cdot (p(\alpha) - p_0) = 0$$

- Yields

$$\alpha = \frac{n \cdot (p_0 - p_1)}{n \cdot (p_2 - p_1)}$$

- Optimizations as for Liang-Barsky in 2D

# Summary: Clipping

- Clipping line segments to rectangle or cube
  - Avoid expensive multiplications and divisions
  - Cohen-Sutherland or Liang-Barsky
- Polygon clipping
  - Sutherland-Hodgeman pipeline
- Clipping in 3D
  - essentially extensions of 2D algorithms



# Preview and Announcements

- Scan conversion
- Anti-aliasing
- Other pixel-level operations
- **Assignment 2 due a week from today!**