CSCI 420 Computer Graphics
Lecture 21

Physically Based Simulation

Examples
Particle Systems
Numerical Integration
Cloth Simulation
[Angel Ch. 9]

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Physics in Computer Graphics

- Very common
- Computer Animation, Modeling (computational mechanics)
- Rendering (computational optics)

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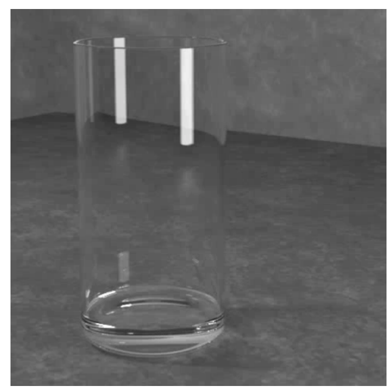
Physics in Computer Animation

- Fluids
- Smoke
- Deformable strands (rods)
- Cloth
- Solid 3D deformable objects .... and many more!

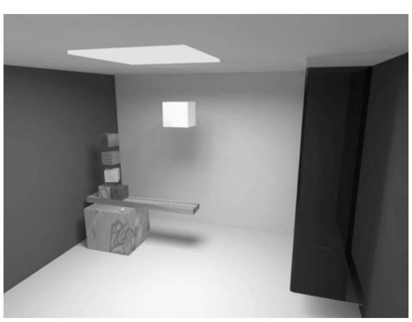
Fluids



Enright, Marschner, Fedkiw, SIGGRAPH 2002

Fluids and Rigid Bodies

[Carlson, Mucha, Turk, SIGGRAPH 2004]



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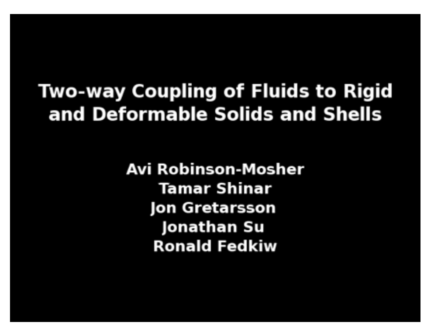
Fluids with Deformable Solid Coupling

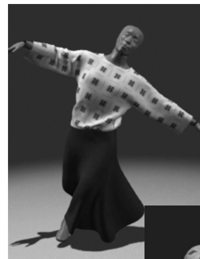
[Robinson-Mosher, Shinar, Gretarsson, Su, Fedkiw, SIGGRAPH 2008]

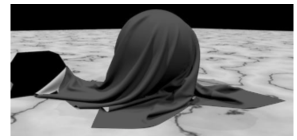


Deformations

[Barbic and James, SIGGRAPH 2005]

Cloth





Source: ACM SIGGRAPH

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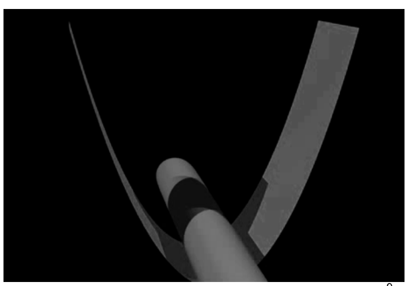
Cloth (Robustness)



[Bridson, Fedkiw, Anderson, ACM SIGGRAPH 2002

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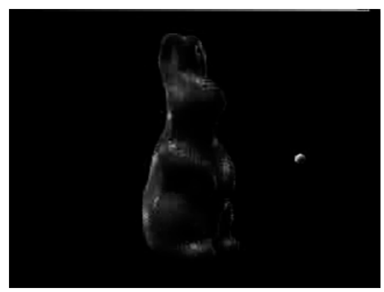
Simulating Large Models



[Doug James, PhD Thesis, UBC, 2001]

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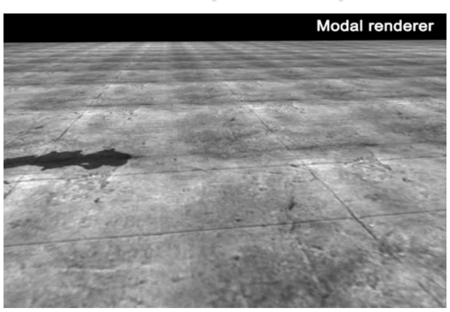
Sound Simulation (Acoustics)



[James, Barbic, Pai, SIGGRAPH 2006]

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Multibody Dynamics

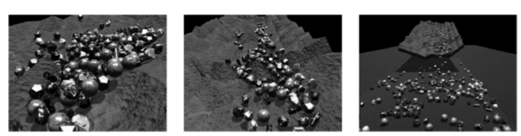

Figure 1: *Avalanche:* 300 rocks tumble down a mountainside.

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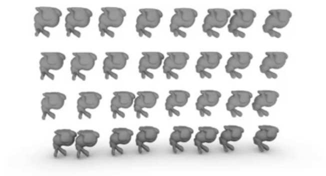
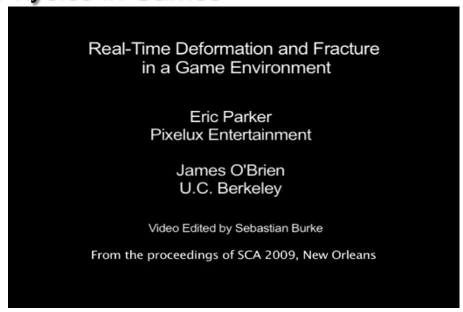
Multibody Dynamics + Self-collision Detection



[Barbic and James, SIGGRAPH 2010]

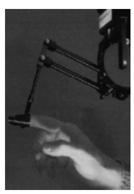
Physics in Games

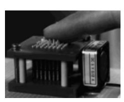


[Parker and O'Brien, Symposium on Computer Animation 2009]

Haptic Interfaces

- hap·tic ('hap-tik)
  adj.
  Of or relating to the sense of touch; tactile.





Surgical Simulation

[James and Pai, SIGGRAPH 2002]

Offline Physics

- Special effects (film, commercials)
- Large models:
  millions of particles / tetrahedra / triangles
- Use computationally expensive rendering (global illumination)
- Impressive results
- Many seconds of computation time per frame

Real-time Physics

- Interactive systems:
  computer games
  virtual medicine (surgical simulation)
- Must be fast (30 fps, preferably 60 fps for games)
  Only a small fraction of CPU time devoted to physics!
- Has to be stable, regardless of user input

Particle System

- Basic physical system in computer graphics
- We have N particles
- They interact with some forces
- Fire, Smoke, Cloth, ...
- Very popular for its simplicity

[William Reeves, SIGGRAPH 1983]

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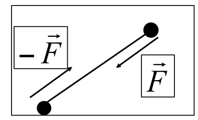
Newton's Laws

- Newton's 2nd law:

$$\vec{F} = m\vec{a}$$

- Gives acceleration, given the force and mass
- Newton's 3rd law: If object A exerts a force F on object B, then object B is at the same time exerting force -F on A.



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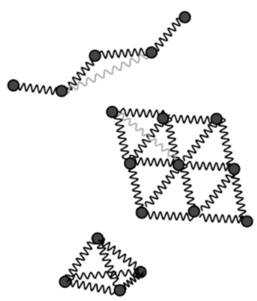
Case Study: Mass-spring Systems

- Mass particles connected by elastic springs
- One dimensional: rope, chain
- Two dimensional: cloth, shells
- Three dimensional: soft bodies



Source:Matthias Mueller, SIGGRAPH

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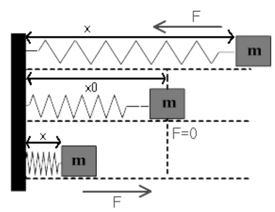
Single spring

- Obeys the *Hook's law*:
  F = k (x - x_0)
- x_0 = rest length
- k = spring elasticity (*stiffness*)
- For x<x_0, spring wants to extend
- For x>x_0, spring wants to contract



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Hook's law in 3D

- Assume A and B two mass points connected with a spring.
- Let L be the vector pointing from B to A
- Let R be the spring rest length
- Then, the elastic force exerted on A is:

$$\vec{F} = -k_{Hook}(|\vec{L}| - R)\frac{\vec{L}}{|\vec{L}|}$$

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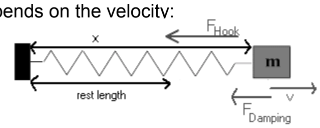
Damping

- Springs are not completely elastic
- They absorb some of the energy and tend to decrease the velocity of the mass points attached to them
- Damping force depends on the velocity:

$$\vec{F} = -k_d\vec{v}$$



- k_d = damping coefficient
- k_d different than k_{Hook} !!

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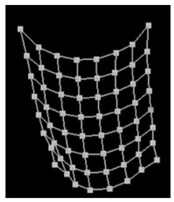
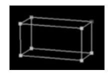
A network of springs

- Every mass point connected to some other points by springs
- Springs exert forces on mass points
  - Hook's force
  - Damping force
- Other forces
  - External force field
    - Gravity
    - Electrical or magnetic force field
  - Collision force

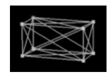


Network organization is critical

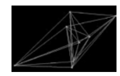
- For stability, must organize the network of springs in some clever way



Basic network

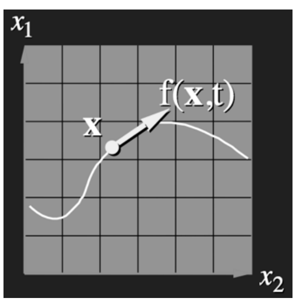


Stable network



Network out of control

Time Integration



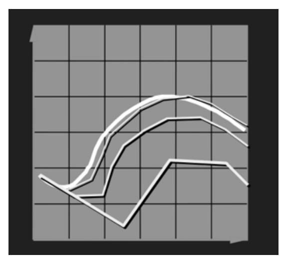
Physics equation:
$x' = f(x,t)$

$x=x(t)$ is particle trajectory

Source:Andy Witkin, SIGGRAPH

Euler Integration

$x(t + \Delta t) = x(t) + \Delta t\, f(x(t))$

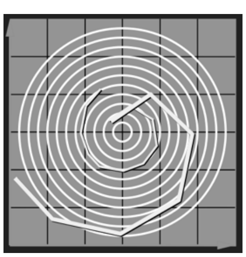


Simple,
but inaccurate.

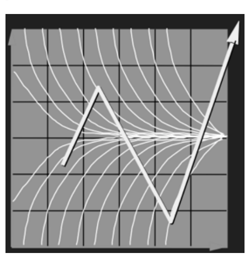
Unstable with large timesteps.

Source:Andy Witkin, SIGGRAPH

Inaccuracies with explicit Euler



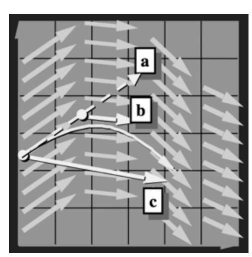
Gain energy



"Blow-up"

Source:Andy Witkin, SIGGRAPH

Midpoint Method



Source:Andy Witkin, SIGGRAPH

Improves stability

1. Compute Euler step
   $\Delta x = \Delta t\, f(x, t)$
2. Evaluate f at the midpoint
   $f_{mid} = f((x+\Delta x)/2, (t+\Delta t)/2)$
3. Take a step using the midpoint value
   $x(t+ \Delta t) = x(t) + \Delta t\, f_{mid}$

Many more methods

- Runge-Kutta (4th order and higher orders)
- Implicit methods
  - sometimes unconditionally stable
  - very popular (e.g., cloth simulations)
  - a lot of damping with large timesteps
- Symplectic methods
  - exactly preserve energy, angular momentum and/or other physical quantities
  - Symplectic Euler

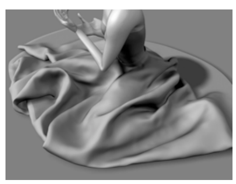
Cloth Simulation

- Cloth Forces
  - Stretch
  - Shear
  - Bend
- Many methods are a more advanced version of a mass-spring system
- Derivatives of Forces
  - necessary for stability

[Baraff and Witkin, SIGGRAPH 1998]

Challenges

- Complex Formulas
- Large Matrices
- Stability
- Collapsing triangles
- Self-collision detection

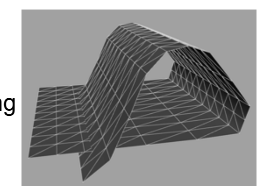


[Govindaraju et al. 2005]

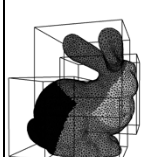
Self-collisions: definition

Deformable model is self-colliding iff

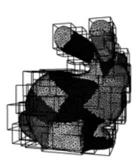
there exist non-neighboring intersecting triangles.



Bounding volume hierarchies



AABBs
Level 1



AABBs
Level 3

[Hubbard 1995]

[Gottschalk et al. 1996]

[van den Bergen 1997]

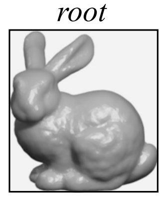
[Bridson et al. 2002]

[Teschner et al. 2002]

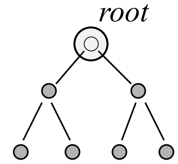
[Govindaraju et al. 2005]

Bounding volume hierarchy

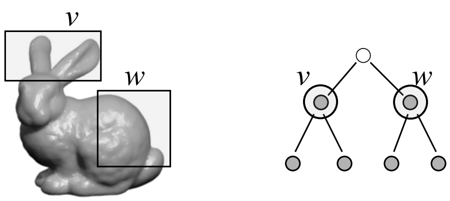
root



root



Bounding volume hierarchy



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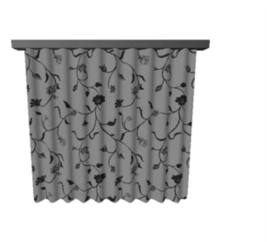
Real-time cloth simulation



Source:
Andy Pierce

Model	Triangles	FPS	% Forces + Stiffness Matrix	% Solver
Curtain	2400	25	67	33

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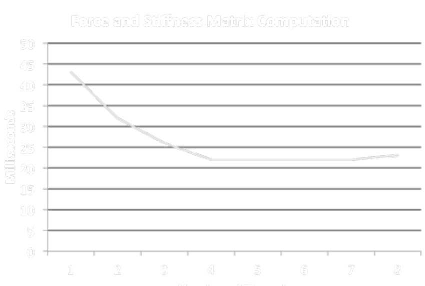
Multithreading implementation



Source:
Andy Pierce

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Summary

- Examples of physically based simulation
- Particle Systems
- Numerical Integration
- Cloth Simulation

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