

# Hierarchical Models

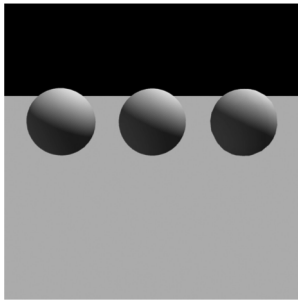
Projections and Shadows  
Hierarchical Models  
[Angel Ch. 8]

Jernej Barbic  
University of Southern California

## Roadmap

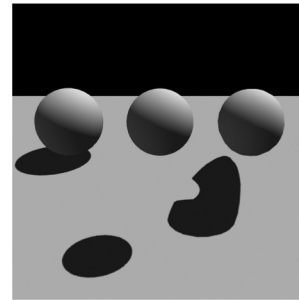
- Last lecture: Viewing and projection
- Today:
  - Shadows via projections
  - Hierarchical models
- Next: Polygonal Meshes, Curves and Surfaces
- Goal: background for Assignment 2 (next week)

## Importance of shadows

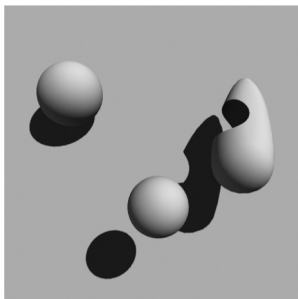


Source: UNC

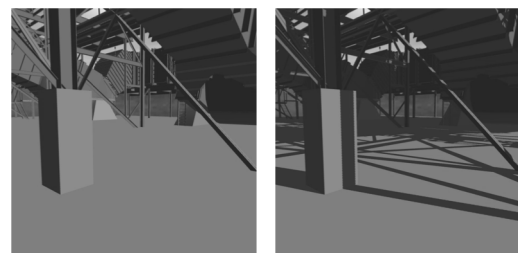
## Importance of shadows



## Importance of shadows



## Importance of shadows



Without shadows

With shadows

Source: UNC

## Doom III

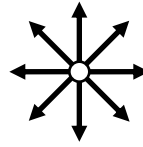


Source: Wikipedia

Reported to spend 50% of time rendering shadows!

7

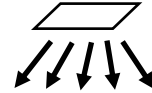
## Light sources



point light source



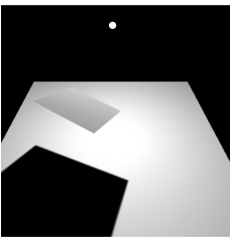
directional light source



area light source

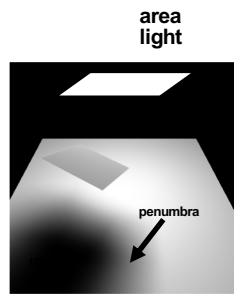
8

## Hard and soft shadows



Source: UNC

Hard shadow



Soft shadow

9

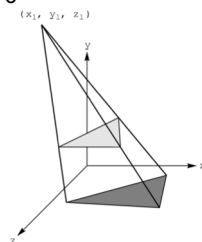
## Shadow Algorithms

- With visibility tests
  - Accurate yet expensive
  - Example: ray casting or ray tracing
  - Example: 2-pass z-buffer [Foley, Ch. 16.4.4] [RTR 6.12]
- Without visibility tests (“fake” shadows)
  - Approximate and inexpensive
  - Using a model-view matrix “trick”

10

## Projection-based Shadows

- Assume light source at  $[x_l \ y_l \ z_l]^T$
- Assume shadow on plane  $y = 0$
- Viewing = shadow projection
  - Center of projection = light
  - Viewing plane = shadow plane
- Construct a modelview matrix to flatten the geometry onto the shadow plane



11

## Shadow Projection Strategy

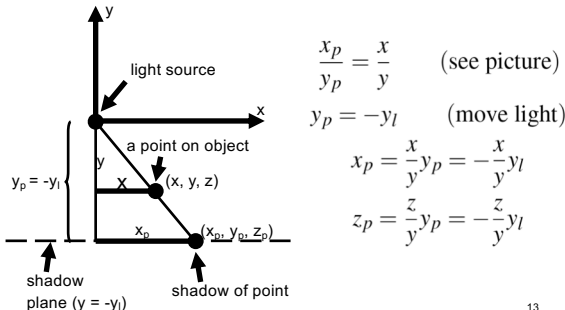
- Move light source to origin
- Apply appropriate projection matrix
- Move light source back
- Instance of general strategy: compose complex transformation from simpler ones!

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_l \\ 0 & 1 & 0 & -y_l \\ 0 & 0 & 1 & -z_l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12

## Derive Equation

- Now, light source at origin



13

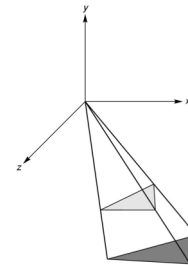
## Light Source at Origin

- After translation, solve

$$M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = w \begin{bmatrix} -\frac{xy_l}{y} \\ -y_l \\ -\frac{zy_l}{y} \\ 1 \end{bmatrix}$$

- w can be chosen freely
- Use w = -y / y<sub>l</sub>

$$M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -\frac{y}{y_l} \end{bmatrix}$$



14

## Shadow Projection Matrix

- Solution of previous equation

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{y_l} & 0 & 0 \end{bmatrix}$$

- Total shadow projection matrix

$$S = T^{-1}MT = \dots$$

15

## Implementation

- Recall column-major form

```
GLfloat m[16] =
{1.0, 0.0, 0.0, 0.0,
 0.0, 1.0, 0.0, -1.0 / y_l,
 0.0, 0.0, 1.0, 0.0,
 0.0, 0.0, 0.0, 0.0};
```

- y<sub>l</sub> is light source height
- Assume drawPolygon(); draws object

16

## Saving the ModelView Matrix State

- Assume xl, yl, zl hold light coordinates
- Core OpenGL code (compatibility code is similar)

```
openGLMatrix.MatrixMode(openGLMatrix::ModelView);
// here, set the model view matrix, in the usual way
// ...
```

```
drawPolygon(); // draw normally
openGLMatrix.PushMatrix(); // save current matrix
openGLMatrix.Translate(xl, yl, zl); // translate back
openGLMatrix.MultMatrix(m); // project
openGLMatrix.Translate(-xl, -yl, -zl); // move light to origin
```

```
float ms[16];
openGLMatrix.GetMatrix(ms); // read the shadow matrix
```

17

## Saving the ModelView Matrix State (cont.)

```
// upload the shadow matrix to the GPU
glUniformMatrix4fv(h_modelViewMatrix, 1, GL_FALSE, ms);
```

```
drawPolygon(); // draw polygon again for shadow
```

```
// restore original modelview matrix
openGLMatrix.PopMatrix();
openGLMatrix.GetMatrix(ms);
glUniformMatrix4fv(h_modelViewMatrix, 1, GL_FALSE, ms);
```

```
// continue rendering more objects, as usual ...
```

18

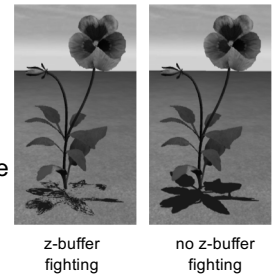
## The Matrix and Attribute Stacks

- Mechanism to save and restore state
  - {OpenGLMatrix::, gl}PushMatrix();
  - {OpenGLMatrix::, gl}PopMatrix();
- Apply to current matrix
- In compatibility profile, can also save current attribute values
  - Examples: color, lighting
  - glPushAttrib(GLbitfield mask);
  - glPopAttrib();
  - Mask determines which attributes are saved
  - This feature has been removed in the core profile

19

## Drawing on a Surface

- Shimmering (“z-buffer fighting”) when drawing shadow on surface
- Due to limited precision of depth buffer
- Solution: slightly displace either the surface or the shadow (glPolygonOffset in OpenGL)

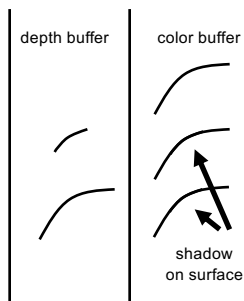


20

## Drawing on a Surface

Or use general technique

1. Set depth buffer to read-only, draw surface
2. Set depth buffer to read-write, draw shadow
3. Set color buffer to read-only, draw surface again
4. Set color buffer to read-write



21

## Outline

- Projections and Shadows
- Hierarchical Models

22

## Hierarchical Models

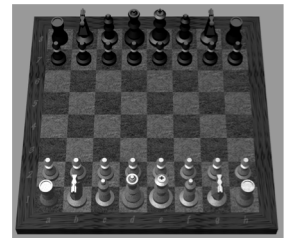
- Many graphical objects are structured
- Exploit structure for
  - Efficient rendering
  - Example: tree leaves
  - Concise specification of model parameters
  - Example: joint angles
  - Physical realism
- Structure often naturally hierarchical



23

## Instance Transformation

- Often we need several instances of an object
  - Wheels of a car
  - Arms or legs of a figure
  - Chess pieces



24

## Instance Transformation

- Instances can be shared across space or time
- Write a function that renders the object in “standard” configuration
- Apply transformations to different instances
- Typical order: scaling, rotation, translation

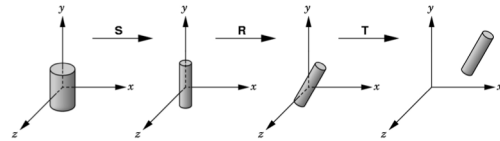


25

## Sample Instance Transformation

```

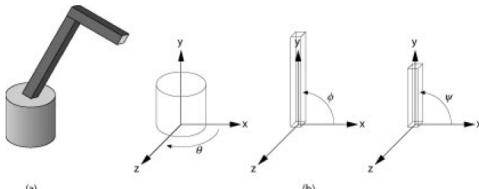
OpenGLMatrix.MatrixMode(OpenGLMatrix::ModelView);
OpenGLMatrix.LoadIdentity();
OpenGLMatrix.Translate(...);
OpenGLMatrix.Rotate(...);
OpenGLMatrix.Scale(...);
// ... upload modelview matrix to GPU, as usual ...
renderCylinder(...);
    
```



26

## Drawing a Compound Object

- Example: simple “robot arm”

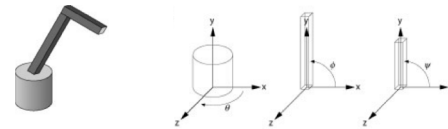


Base rotation  $\theta$ , arm angle  $\phi$ , joint angle  $\psi$

27

## Hierarchical Objects and Animation

- Drawing functions are time-invariant and draw the object in a canonical position:  
drawBase(); drawLowerArm(); drawUpperArm();
- Can be easily stored in a VBO
- Change parameters of model with time



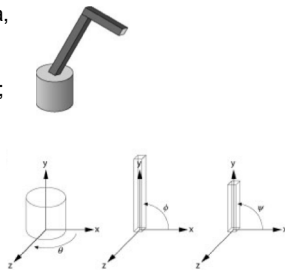
28

## Interleave Drawing & Transformation

- $h1$  = height of base,  $h2$  = length of lower arm
- This is pseudocode (must upload matrix to GPU)

```

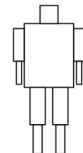
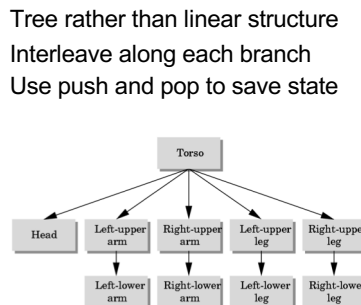
void drawRobot(GLfloat theta,
              GLfloat phi, GLfloat psi)
{
    Rotate(theta, 0.0, 1.0, 0.0);
    drawBase();
    Translate(0.0, h1, 0.0);
    Rotate(phi, 0.0, 0.0, 1.0);
    drawLowerArm();
    Translate(0.0, h2, 0.0);
    Rotate(psi, 0.0, 0.0, 1.0);
    drawUpperArm();
}
    
```



29

## More Complex Objects

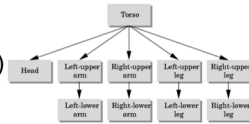
- Tree rather than linear structure
- Interleave along each branch
- Use push and pop to save state



30

## Hierarchical Tree Traversal

- Order not necessarily fixed (breadth-first, depth-first, etc.)



- Example:

```
void drawFigure()
{
  PushMatrix(); // save           Translate(...);
  drawTorso();                 Rotate(...);
  Translate(...); // move head drawLeftUpperArm();
  Rotate(...); // rotate head Translate(...);
  drawHead();                 Rotate(...);
  PopMatrix(); // restore       drawLeftLowerArm();
  PushMatrix();               PopMatrix();
  ... }                        ... }
```

31

## Summary

- Projections and Shadows
- Hierarchical Models

35