

## Lighting and Shading

Light Sources  
Phong Illumination Model  
Normal Vectors  
[Angel Ch. 5]

Jernej Barbic  
University of Southern California

1

## Outline

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

2

## Global Illumination

- Ray tracing
- Radiosity
- Photon Mapping
- Follow light rays through a scene
- Accurate, but expensive (off-line)



Tobias R. Metoc

3

## Raytracing Example



Martin Moeck,  
Siemens Lighting

4

## Radiosity Example

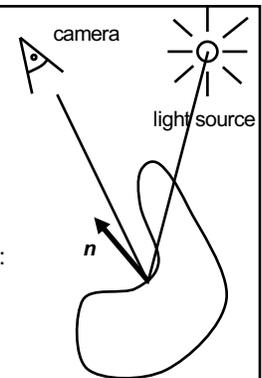


Restaurant Interior. Guillermo Leal, Evolucion Visual

5

## Local Illumination

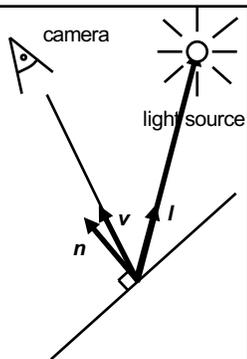
- Approximate model
- Local interaction between light, surface, viewer
- Phong model (this lecture): fast, supported in OpenGL
- GPU shaders
- Pixar Renderman (offline)



6

## Local Illumination

- Approximate model
- Local interaction between light, surface, viewer
- Color determined only based on surface normal, relative camera position and relative light position
- What effects does this ignore?



7

## Outline

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

8

## Normal Vectors

- Must calculate and specify the normal vector
  - Even in OpenGL!
- Two examples: plane and sphere

9

## Normals of a Plane, Method I

- Method I: given by  $ax + by + cz + d = 0$
- Let  $p_0$  be a known point on the plane
- Let  $p$  be an arbitrary point on the plane
- Recall:  $u \cdot v = 0$  if and only if  $u$  orthogonal to  $v$
- $n \cdot (p - p_0) = n \cdot p - n \cdot p_0 = 0$
- Consequently  $n_0 = [a \ b \ c]^T$
- Normalize to  $n = n_0/|n_0|$

10

## Normals of a Plane, Method II

- Method II: plane given by  $p_0, p_1, p_2$
- Points must not be collinear
- Recall:  $u \times v$  orthogonal to  $u$  and  $v$
- $n_0 = (p_1 - p_0) \times (p_2 - p_0)$
- Order of cross product determines orientation
- Normalize to  $n = n_0/|n_0|$

11

## Normals of Sphere

- Implicit Equation  $f(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$
- Vector form:  $f(p) = p \cdot p - 1 = 0$
- Normal given by gradient vector

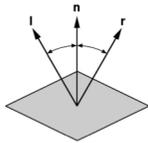
$$n_0 = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix} = 2p$$

- Normalize  $n_0/|n_0| = 2p/2 = p$

12

## Reflected Vector

- Perfect reflection: angle of incident equals angle of reflection
- Also:  $l$ ,  $n$ , and  $r$  lie in the same plane
- Assume  $|l| = |n| = 1$ , guarantee  $|r| = 1$



$$l \cdot n = \cos(\theta) = n \cdot r$$

$$r = \alpha l + \beta n$$

$$\text{Solution: } \alpha = -1 \text{ and } \beta = 2(l \cdot n)$$

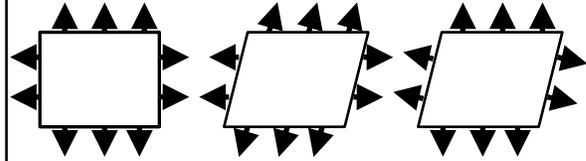
$$r = 2(l \cdot n)n - l$$

13

## Normals Transformed by Modelview Matrix

Modelview matrix  $M$  (shear in this example)

Only keep linear transform in  $M$  (discard any translation).



Undeformed

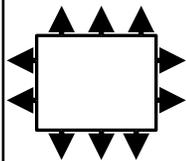
Transformed with  $M$   
(incorrect)

Transformed with  $(M^{-1})^T$   
(correct)

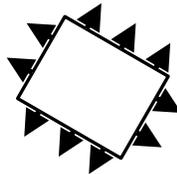
14

## Normals Transformed by Modelview Matrix

When  $M$  is rotation,  $M = (M^{-1})^T$



Undeformed



Transformed with  $M = (M^{-1})^T$   
(correct)

15

## Normals Transformed by Modelview Matrix (proof of $(M^{-1})^T$ transform)

Point  $(x,y,z,w)$  is on a plane in 3D (homogeneous coordinates) if and only if

$$a x + b y + c z + d w = 0, \text{ or } [a \ b \ c \ d] [x \ y \ z \ w]^T = 0.$$

Now, let's transform the plane by  $M$ .

Point  $(x,y,z,w)$  is on the transformed plane if and only if

$M^{-1} [x \ y \ z \ w]^T$  is on the original plane:

$$[a \ b \ c \ d] M^{-1} [x \ y \ z \ w]^T = 0.$$

So, equation of transformed plane is

$$[a' \ b' \ c' \ d'] [x \ y \ z \ w]^T = 0, \text{ for } [a' \ b' \ c' \ d']^T = (M^{-1})^T [a \ b \ c \ d]^T.$$

16

## Outline

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

17

## Light Sources and Material Properties

- Appearance depends on
  - Light sources, their locations and properties
  - Material (surface) properties:



– Viewer position

18

## Types of Light Sources

- Ambient light: no identifiable source or direction
- Point source: given only by point
- Distant light: given only by direction
- Spotlight: from source in direction
  - Cut-off angle defines a cone of light
  - Attenuation function (brighter in center)



19

## Point Source

- Given by a point  $p_0$
- Light emitted equally in all directions
- Intensity decreases with square of distance

$$I \propto \frac{1}{|p - p_0|^2}$$

20

## Limitations of Point Sources

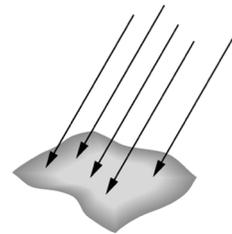
- Shading and shadows inaccurate
- Example: penumbra (partial “soft” shadow)
- Similar problems with highlights
- Compensate with attenuation
 
$$\frac{1}{a + bq + cq^2} \quad \begin{array}{l} q = \text{distance } |p - p_0| \\ a, b, c \text{ constants} \end{array}$$
- Softens lighting
- Better with ray tracing
- Better with radiosity



21

## Distant Light Source

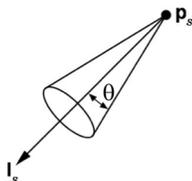
- Given by a direction vector  $[x \ y \ z]$



22

## Spotlight

- Light still emanates from point
- Cut-off by cone determined by angle  $\theta$



23

## Global Ambient Light

- Independent of light source
- Lights entire scene
- Computationally inexpensive
- Simply add  $[G_R \ G_G \ G_B]$  to every pixel on every object
- Not very interesting on its own.  
A cheap hack to make the scene brighter.

24

## Outline

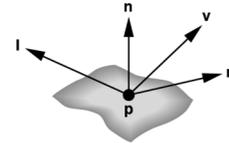
- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

25

## Phong Illumination Model

- Calculate color for arbitrary point on surface
- Compromise between realism and efficiency
- Local computation (no visibility calculations)
- Basic inputs are material properties and  $\mathbf{l}$ ,  $\mathbf{n}$ ,  $\mathbf{v}$ :

$\mathbf{l}$  = unit vector to light source  
 $\mathbf{n}$  = surface normal  
 $\mathbf{v}$  = unit vector to viewer  
 $\mathbf{r}$  = reflection of  $\mathbf{l}$  at  $\mathbf{p}$   
 (determined by  $\mathbf{l}$  and  $\mathbf{n}$ )



26

## Phong Illumination Overview

1. Start with global ambient light [ $G_R$   $G_G$   $G_B$ ]
  2. Add contributions from each light source
  3. Clamp the final result to  $[0, 1]$
- Calculate each color channel (R,G,B) **separately**
  - Light source contributions decomposed into
    - Ambient reflection
    - Diffuse reflection
    - Specular reflection
  - Based on ambient, diffuse, and specular lighting and material properties

27

## Ambient Reflection

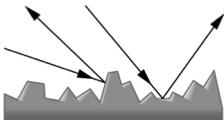
$$I_a = k_a L_a$$

- Intensity of ambient light is uniform at every point
- Ambient reflection coefficient  $k_a \geq 0$
- May be different for every surface and r,g,b
- Determines reflected fraction of ambient light
- $L_a$  = ambient component of light source (can be set to different value for each light source)
- Note:  $L_a$  is not a physically meaningful quantity

28

## Diffuse Reflection

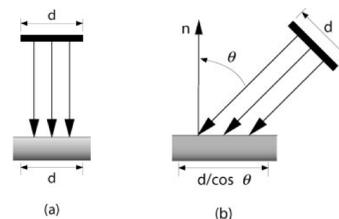
- Diffuse reflector scatters light
- Assume equally all direction
- Called Lambertian surface
- Diffuse reflection coefficient  $k_d \geq 0$
- Angle of incoming light is important



29

## Lambert's Law

Intensity depends on angle of incoming light.



30

## Diffuse Light Intensity Depends On Angle Of Incoming Light

- Recall

$l$  = unit vector to light  
 $n$  = unit surface normal  
 $\theta$  = angle to normal

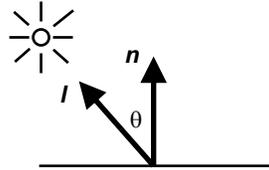
- $\cos \theta = l \cdot n$

- $I_d = k_d L_d (l \cdot n)$

- With attenuation:

$$I_d = \frac{k_d L_d}{a + bq + cq^2} (l \cdot n)$$

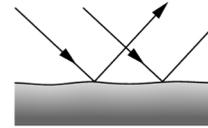
$q$  = distance to light source,  
 $L_d$  = diffuse component of light



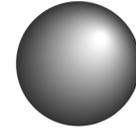
31

## Specular Reflection

- Specular reflection coefficient  $k_s \geq 0$
- Shiny surfaces have high specular coefficient
- Used to model specular highlights
- Does not give the mirror effect (need other techniques)



specular reflection



specular highlights

32

## Specular Reflection

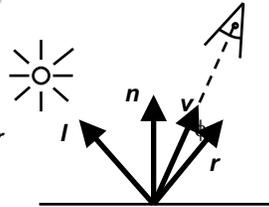
- Recall

$v$  = unit vector to camera  
 $r$  = unit reflected vector  
 $\phi$  = angle between  $v$  and  $r$

- $\cos \phi = v \cdot r$

- $I_s = k_s L_s (\cos \phi)^\alpha$

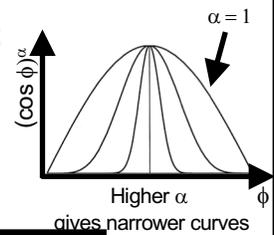
- $L_s$  is specular component of light
- $\alpha$  is shininess coefficient
- Can add distance term as well



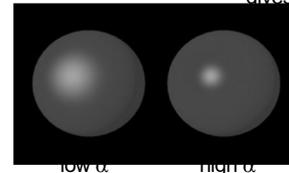
33

## Shininess Coefficient

- $I_s = k_s L_s (\cos \phi)^\alpha$
- $\alpha$  is the shininess coefficient



Higher  $\alpha$  gives narrower curves



Source: Univ. of Calgary

34

## Summary of Phong Model

- Light components for each color:
  - Ambient ( $L_a$ ), diffuse ( $L_d$ ), specular ( $L_s$ )
- Material coefficients for each color:
  - Ambient ( $k_a$ ), diffuse ( $k_d$ ), specular ( $k_s$ )
- Distance  $q$  for surface point from light source

$$I = \frac{1}{a + bq + cq^2} (k_d L_d (l \cdot n) + k_s L_s (r \cdot v)^\alpha) + k_a L_a$$

$l$  = unit vector to light       $r$  = reflected about  $n$   
 $n$  = surface normal       $v$  = vector to viewer

35

## BRDF

- Bidirectional Reflection Distribution Function
- Must measure for real materials
- Isotropic vs. anisotropic
- Mathematically complex
- Implement in a fragment shader



Lighting properties of a human face were captured and face re-rendered; Institute for Creative Technologies

36

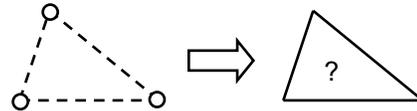
## Outline

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

37

## Polygonal Shading

- Now we know vertex colors
  - either via OpenGL lighting,
  - or by setting directly via `glColor3f` if lighting disabled
- How do we shade the interior of the triangle ?



38

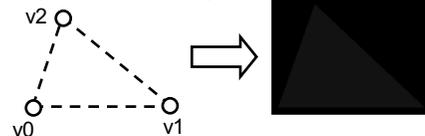
## Polygonal Shading

- Curved surfaces are approximated by polygons
- How do we shade?
  - Flat shading
  - Interpolative shading
  - Gouraud shading
  - Phong shading (different from Phong illumination!)

39

## Flat Shading

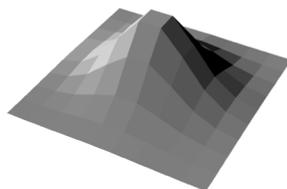
- Shading constant across polygon
- Core profile: Use interpolation qualifiers in the fragment shader
- Compatibility profile: Enable with `glShadeModel(GL_FLAT);`
- Color of last vertex determines interior color
- Only suitable for *very* small polygons



40

## Flat Shading Assessment

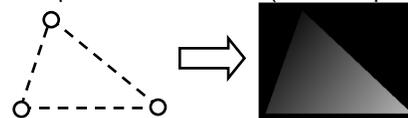
- Inexpensive to compute
- Appropriate for objects with flat faces
- Less pleasant for smooth surfaces



41

## Interpolative Shading

- Interpolate color in interior
- Computed during scan conversion (rasterization)
- Core profile: enabled by default
- Compatibility profile: enable with `glShadeModel(GL_SMOOTH);`
- Much better than flat shading
- More expensive to calculate (but not a problem)



42

## Gouraud Shading

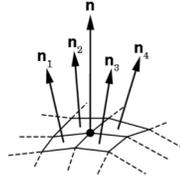
Invented by Henri Gouraud, Univ. of Utah, 1971

- Special case of interpolative shading
- How do we calculate vertex normals for a polygonal surface? Gouraud:
  1. average all adjacent face normals

$$n = \frac{n_1 + n_2 + n_3 + n_4}{|n_1 + n_2 + n_3 + n_4|}$$

2. use  $n$  for Phong lighting
3. interpolate vertex colors into the interior

- Requires knowledge about which faces share a vertex



43

## Data Structures for Gouraud Shading

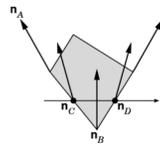
- Sometimes vertex normals can be computed directly (e.g. height field with uniform mesh)
- More generally, need data structure for mesh
- Key: which polygons meet at each vertex

44

## Phong Shading (“per-pixel lighting”)

Invented by Bui Tuong Phong, Univ. of Utah, 1973

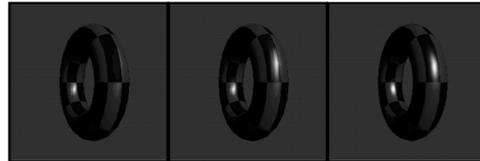
- *At each pixel* (as opposed to at each vertex) :
  1. Interpolate *normals* (rather than colors)
  2. Apply Phong lighting to the interpolated normal
- Significantly more expensive
- Done off-line or in GPU shaders (not supported in OpenGL directly)



45

## Phong Shading Results

Michael Gold, Nvidia



Single light  
Phong Lighting  
Gouraud Shading

Two lights  
Phong Lighting  
Gouraud Shading

Two lights  
Phong Lighting  
Phong Shading

46

## Outline

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

47

## Phong Shader: Vertex Program

```
#version 150
```

```
in vec3 position; } input vertex position and normal,
in vec3 normal; } in world-space
```

```
out vec3 viewPosition; } vertex position and
out vec3 viewNormal; } normal, in view-space } these will be
passsed to
fragment
program
(interpolated by
hardware)
```

```
uniform mat4 modelViewMatrix; }
uniform mat4 normalMatrix; } transformation matrices
uniform mat4 projectionMatrix; }
```

48



## Upload the light direction vector to GPU

```
float lightDirection[3] = { 0, 1, 0 }; // the "Sun" at noon
float viewLightDirection[3]; // light direction in the view space
// the following line is pseudo-code:
viewLightDirection = (view * float4(lightDirection, 0.0)).xyz;

// upload viewLightDirection to the GPU
glUniform3fv(h_viewLightDirection, 1, viewLightDirection);

// continue with model transformations
openGLMatrix->Translate(x, y, z);
...

renderBunny(); // render, via VAO
glutSwapBuffers();
}
```

55

## Upload the normal matrix to GPU

```
// in the display function:

// get a handle to the program
GLuint program = pipelineProgram->GetProgramHandle();
// get a handle to the normalMatrix shader variable
GLint h_normalMatrix =
    glGetUniformLocation(program, "normalMatrix");

float n[16];
matrix->SetMatrixMode(OpenGLMatrix::ModelView);
matrix->GetNormalMatrix(n); // get normal matrix

// upload n to the GPU
GLboolean isRowMajor = GL_FALSE;
glUniformMatrix4fv(h_normalMatrix, 1, isRowMajor, n);
```

56

## Summary

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

57