

CSCI 420 Computer Graphics  
Lecture 8

# Hierarchical Models

Projections and Shadows  
 Hierarchical Models  
 [Angel Ch. 8]

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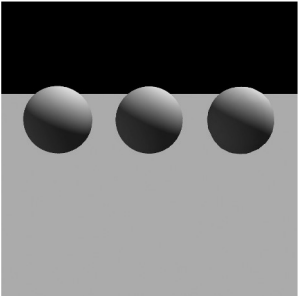
## Roadmap

- Last lecture: Viewing and projection
- Today:
  - Shadows via projections
  - Hierarchical models
- Next: Polygonal Meshes, Curves and Surfaces
- Goal: background for Assignment 2 (next week)

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### Importance of shadows

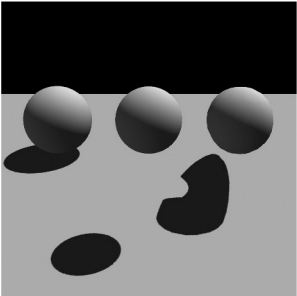


Source: UNC

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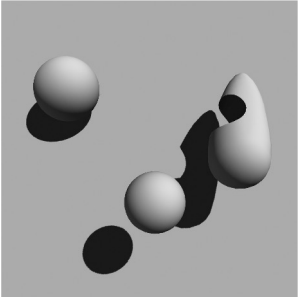
### Importance of shadows



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### Importance of shadows



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### Importance of shadows

Without shadows


With shadows

Source: UNC

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### Doom III

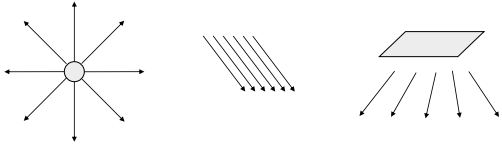


Reported to spend 50% of time rendering shadows!

Source: Wikipedia

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### Light sources



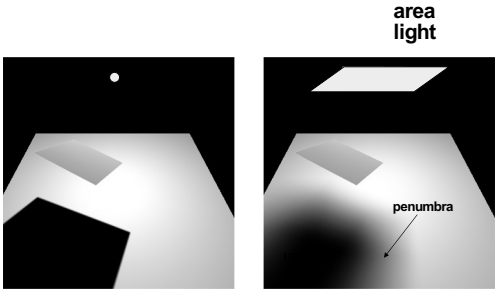
point light source      directional light source      area light source

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### Hard and soft shadows



area light

penumbra

Hard shadow      Soft shadow

Source: UNC

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### Shadow Algorithms

- With visibility tests
  - Accurate yet expensive
  - Example: ray casting or ray tracing
  - Example: 2-pass z-buffer [Foley, Ch. 16.4.4] [RTR 6.12]
- Without visibility tests (“fake” shadows)
  - Approximate and inexpensive
  - Using a model-view matrix “trick”

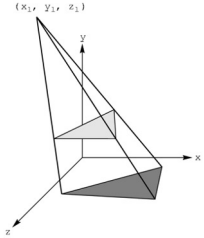
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### Projection-based Shadows

- Assume light source at  $[x_l \ y_l \ z_l]^T$
- Assume shadow on plane  $y = 0$
- Construct a modelview matrix to flatten the geometry onto the shadow plane



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### Shadow Projection Strategy

- Move light source to origin
- Apply appropriate projection matrix
- Move light source back
- Instance of general strategy: compose complex transformation from simpler ones!

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_l \\ 0 & 1 & 0 & -y_l \\ 0 & 0 & 1 & -z_l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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### Derive Equation

- Now, light source at origin

$$\frac{x_p}{y_p} = \frac{x}{y} \quad (\text{see picture})$$

$$y_p = -y_l \quad (\text{move light})$$

$$x_p = \frac{x}{y} y_p = -\frac{x}{y} y_l$$

$$z_p = \frac{z}{y} y_p = -\frac{z}{y} y_l$$

shadow plane ( $y = -y_l$ ) shadow of point

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### Light Source at Origin

- After translation, solve

$$M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = w \begin{bmatrix} -\frac{xy_l}{y} \\ -y_l \\ -\frac{zy_l}{y} \\ 1 \end{bmatrix}$$

- w can be chosen freely
- Use  $w = -y / y_l$

$$M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -\frac{y}{y_l} \end{bmatrix}$$

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### Shadow Projection Matrix

- Solution of previous equation

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{y_l} & 0 & 0 \end{bmatrix}$$

- Total shadow projection matrix

$$S = T^{-1} M T = \dots$$

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### Implementation

- Recall column-major form

```
GLfloat m[16] =
{1.0, 0.0, 0.0, 0.0,
0.0, 1.0, 0.0, -1.0 / y_l,
0.0, 0.0, 1.0, 0.0,
0.0, 0.0, 0.0, 0.0};
```

- $y_l$  is light source height
- Assume drawPolygon(); draws object

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### Saving the ModelView Matrix State

- Assume  $x_l, y_l, z_l$  hold light coordinates
- Core OpenGL code (compatibility code is similar)

```
openGLMatrix.MatrixMode(openGLMatrix::ModelView);
// here, set the model view matrix, in the usual way
// ...

drawPolygon(); // draw normally
openGLMatrix.PushMatrix(); // save current matrix
openGLMatrix.Translate(x_l, y_l, z_l); // translate back
openGLMatrix.MultMatrix(m); // project
openGLMatrix.Translate(-x_l, -y_l, -z_l); // move light to origin

float ms[16];
openGLMatrix.GetMatrix(ms); // read the shadow matrix
```

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### Saving the ModelView Matrix State (cont.)

```
// upload the shadow matrix to the GPU
pipelineProgram->SetUniformVariableMatrix4fv(
    "modelViewMatrix", GL_FALSE, ms);

drawPolygon(); // draw polygon again for shadow

// restore original modelview matrix
openGLMatrix.PopMatrix();
openGLMatrix.GetMatrix(ms);
pipelineProgram->SetUniformVariableMatrix4fv(
    "modelViewMatrix", GL_FALSE, ms);

// continue rendering more objects, as usual ...
```

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## The Matrix and Attribute Stacks

- Mechanism to save and restore state
  - {OpenGLMatrix::, gl}PushMatrix();
  - {OpenGLMatrix::, gl}PopMatrix();
- Apply to current matrix
- In compatibility profile, can also save current attribute values
  - Examples: color, lighting
  - glPushAttrib(GLbitfield mask);
  - glPopAttrib();
  - Mask determines which attributes are saved
  - This feature has been removed in the core profile

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## Drawing on a Surface

- Shimmering (“z-buffer fighting”) when drawing shadow on surface
- Due to limited precision of depth buffer
- Solution: slightly displace either the surface or the shadow (glPolygonOffset in OpenGL)



z-buffer fighting

no z-buffer fighting

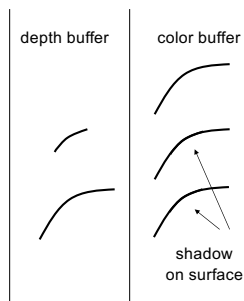
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## Drawing on a Surface

Or use general technique

1. Set depth buffer to read-only, draw surface
2. Set depth buffer to read-write, draw shadow
3. Set color buffer to read-only, draw surface again
4. Set color buffer to read-write



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## Outline

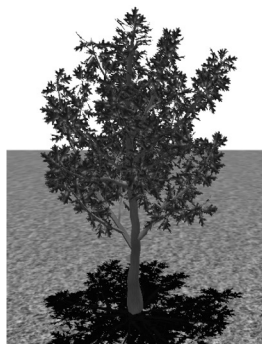
- Projections and Shadows
- Hierarchical Models

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## Hierarchical Models

- Many graphical objects are structured
- Exploit structure for
  - Efficient rendering
  - Example: tree leaves
  - Concise specification of model parameters
  - Example: joint angles
  - Physical realism
- Structure often naturally hierarchical

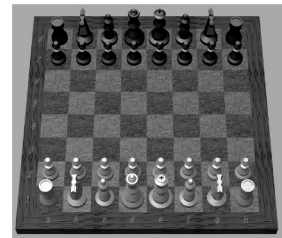


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## Instance Transformation

- Often we need several instances of an object
  - Wheels of a car
  - Arms or legs of a figure
  - Chess pieces



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## Instance Transformation

- Instances can be shared across space or time
- Write a function that renders the object in “standard” configuration
- Apply transformations to different instances
- Typical order: scaling, rotation, translation



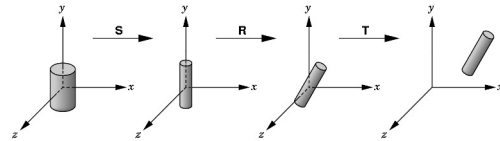
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## Sample Instance Transformation

```

OpenGLMatrix.MatrixMode(OpenGLMatrix::ModelView);
OpenGLMatrix.LoadIdentity();
OpenGLMatrix.Translate(...);
OpenGLMatrix.Rotate(...);
OpenGLMatrix.Scale(...);
// ... upload modelview matrix to GPU, as usual ...
renderCylinder(...);
    
```

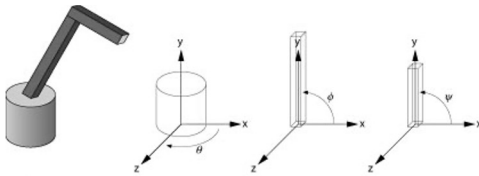


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## Drawing a Compound Object

- Example: simple “robot arm”



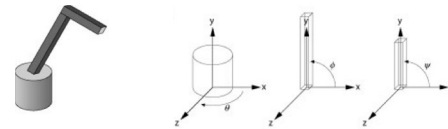
Base rotation  $\theta$ , arm angle  $\phi$ , joint angle  $\psi$

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## Hierarchical Objects and Animation

- Drawing functions are time-invariant and draw the object in a canonical position:  
drawBase(); drawLowerArm(); drawUpperArm();
- Can be easily stored in a VBO
- Change parameters of model with time



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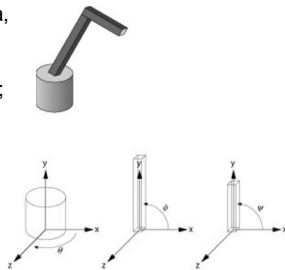
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## Interleave Drawing & Transformation

- $h1$  = height of base,  $h2$  = length of lower arm
- This is pseudocode (must upload matrix to GPU)

```

void drawRobot(GLfloat theta,
              GLfloat phi, GLfloat psi)
{
    Rotate(theta, 0.0, 1.0, 0.0);
    drawBase();
    Translate(0.0, h1, 0.0);
    Rotate(phi, 0.0, 0.0, 1.0);
    drawLowerArm();
    Translate(0.0, h2, 0.0);
    Rotate(psi, 0.0, 0.0, 1.0);
    drawUpperArm();
}
    
```

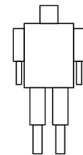
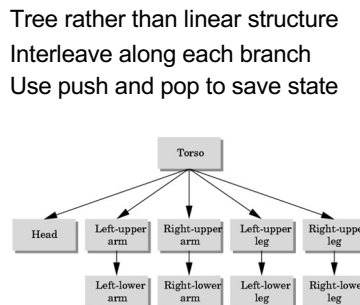


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## More Complex Objects

- Tree rather than linear structure
- Interleave along each branch
- Use push and pop to save state

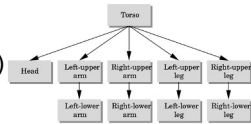


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## Hierarchical Tree Traversal

- Order not necessarily fixed (breadth-first, depth-first, etc.)



- Example:

```
void drawFigure()
{
  PushMatrix(); // save           Translate(...);
  drawTorso();                   Rotate(...);
  Translate(...); // move head   drawLeftUpperArm();
  Rotate(...); // rotate head   Translate(...)
  drawHead();                    Rotate(...)
  PopMatrix(); // restore        drawLeftLowerArm();
  PushMatrix();                  PopMatrix();
  ... }                           ... }
```

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## Summary

- Projections and Shadows
- Hierarchical Models

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