CSCI 420 Computer Graphics
Lecture 9

# Polygon Meshes and Implicit Surfaces

Polygon Meshes
Implicit Surfaces
Constructive Solid Geometry
[Angel Ch. 10]

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# Modeling Complex Shapes

Source: Wikipedia

- An equation for a sphere is possible, but how about an equation for a telephone, or a face?
- Complexity is achieved using simple pieces
  - polygons, parametric surfaces, or implicit surfaces
- Goals
  - Model *anything* with arbitrary precision (in principle)
  - Easy to build and modify
  - Efficient computations (for rendering, collisions, etc.)
  - Easy to implement (a minor consideration...)

# What do we need from shapes in Computer Graphics?

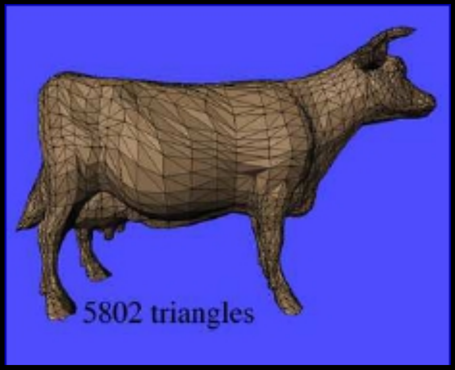
- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

No single technique solves all problems!

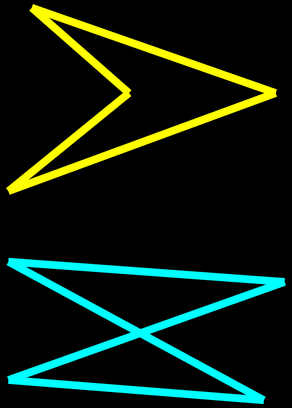
# Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces

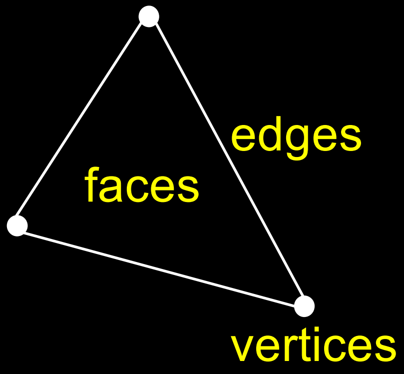
# Polygon Meshes



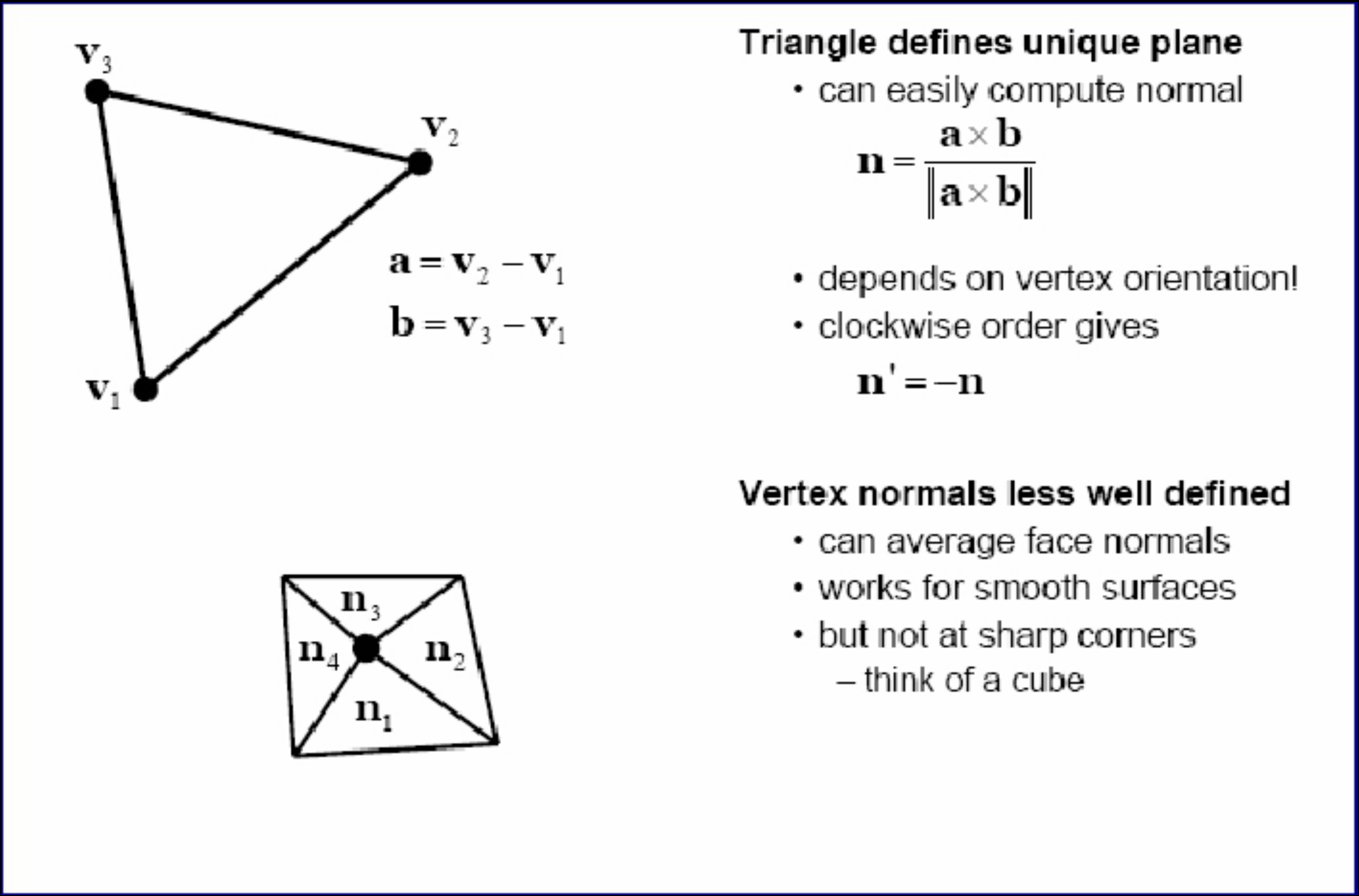
- Any shape can be modeled out of polygons
  - if you use enough of them…
- Polygons with how many sides?
  - Can use triangles, quadrilaterals, pentagons, … n-gons
  - Triangles are most common.
  - When > 3 sides are used, ambiguity about what to do when polygon nonplanar, or concave, or self-intersecting.



- Polygon meshes are built out of
  - *vertices* (points)
  - *edges* (line segments between vertices)
  - *faces* (polygons bounded by edges)



# Normals



$$\mathbf{a} = \mathbf{v}_2 - \mathbf{v}_1$$

$$\mathbf{b} = \mathbf{v}_3 - \mathbf{v}_1$$

**Triangle defines unique plane**

- can easily compute normal

$$\mathbf{n} = \frac{\mathbf{a} \times \mathbf{b}}{\|\mathbf{a} \times \mathbf{b}\|}$$

- depends on vertex orientation!
- clockwise order gives

$$\mathbf{n}' = -\mathbf{n}$$

**Vertex normals less well defined**

- can average face normals
- works for smooth surfaces
- but not at sharp corners
  - think of a cube

# Where Meshes Come From

- Specify manually
  - Write out all polygons
  - Write some code to generate them
  - Interactive editing: move vertices in space

- Acquisition from real objects
  - Laser scanners, vision systems
  - Generate set of points on the surface
  - Need to convert to polygons

# Data Structures for Polygon Meshes

- Simplest (but dumb)
  - float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
  - redundant: each vertex stored multiple times
- Vertex List, Face List
  - List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
  - List of triangles, each a triple of vertex id's (or pointers) topological (connectivity, adjacency) info only

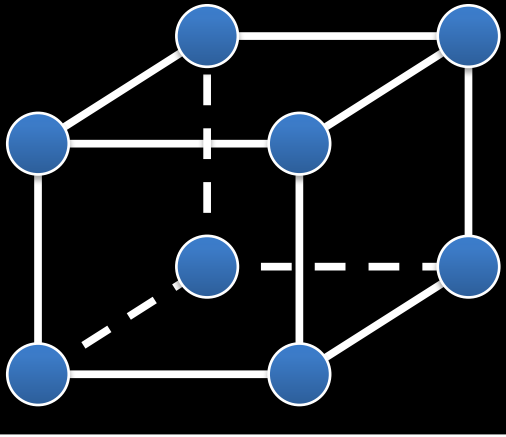
  *Fine for many purposes, but finding the faces adjacent to a vertex takes O(F) time for a model with F faces. Such queries are important for topological editing.*
- Fancier schemes:

  Store more topological info so adjacency queries can be answered in O(1) time.

  *Winged-edge data structure* – edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).

# A File Format for Polygon Models: OBJ

```
#  OBJ file for a 2x2x2  cube
v  -1.0 1.0 1.0             - vertex 1
v  -1.0 -1.0 1.0            - vertex 2
v  1.0 -1.0 1.0             - vertex 3
v  1.0 1.0 1.0              -…
v  -1.0 1.0 -1.0
v  -1.0 -1.0 -1.0
v  1.0 -1.0 -1.0
v  1.0 1.0 -1.0
f  1  2  3  4
f  8  7  6  5
f  4  3  7  8
f  5  1  4  8
f  5  6  2  1
f  2  6  7  3
```



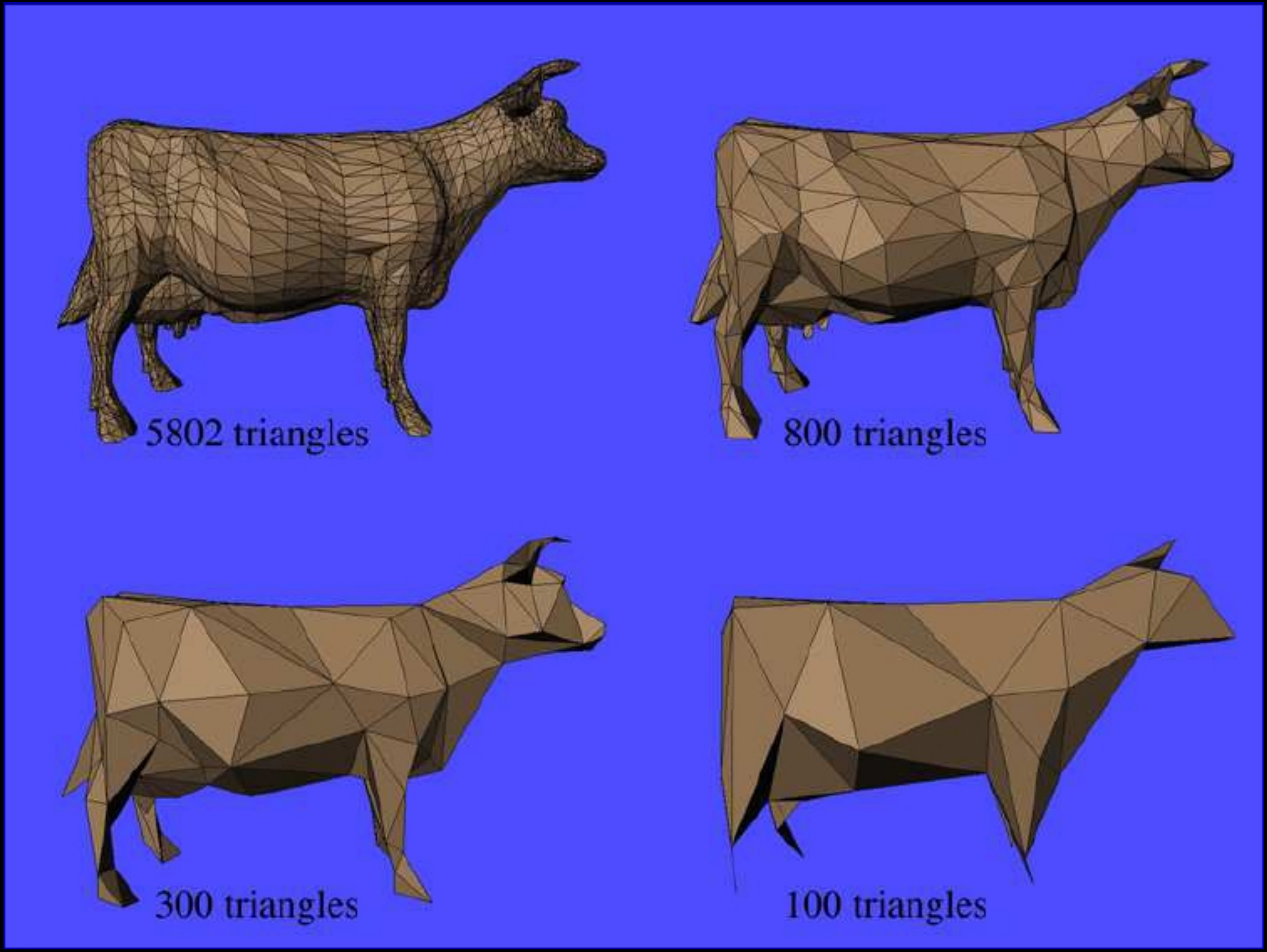
**Syntax:**

**v** $x\ y\ z$ - *a vertex at (x,y,z)*

**f** $v_1\ v_2\ \ldots\ v_n$ - *a face with vertices* $v_1, v_2, \ldots v_n$

**#** *anything* - *comment*

# How Many Polygons to Use?



# Why Level of Detail?

- Different models for near and far objects
- Different models for rendering and collision detection
- Compression of data recorded from the real world

We need automatic algorithms for reducing the polygon count without

  - losing key features
  - getting artifacts in the silhouette
  - popping

# Problems with Triangular Meshes?

- Need a lot of polygons to represent smooth shapes
- Need a lot of polygons to represent detailed shapes

- Hard to edit
- Need to move individual vertices
- Intersection test? Inside/outside test?

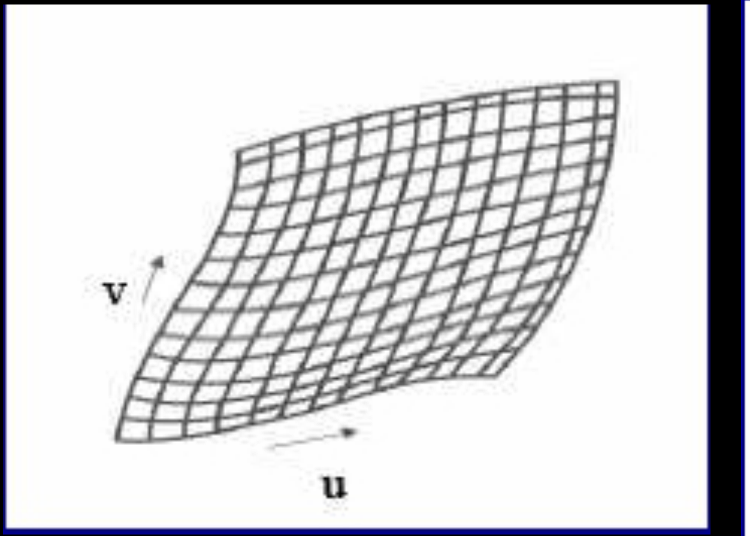
# Shape Representations

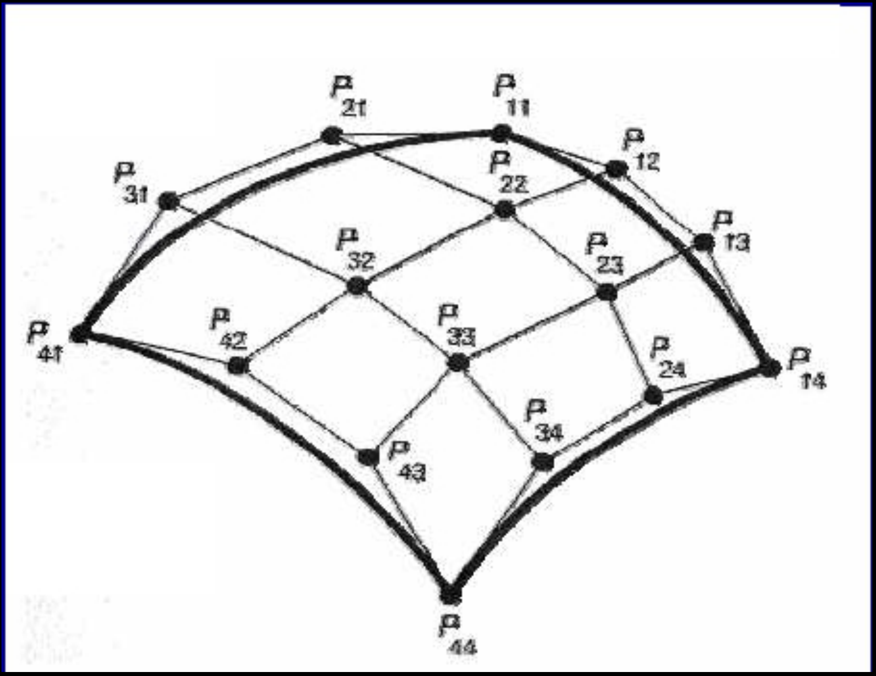
Polygon Meshes
**Parametric Surfaces**
Implicit Surfaces

# Parametric Surfaces

$$p(u,v) = [x(u,v), y(u,v), z(u,v)]$$

- e.g. plane, cylinder, bicubic surface, swept surface



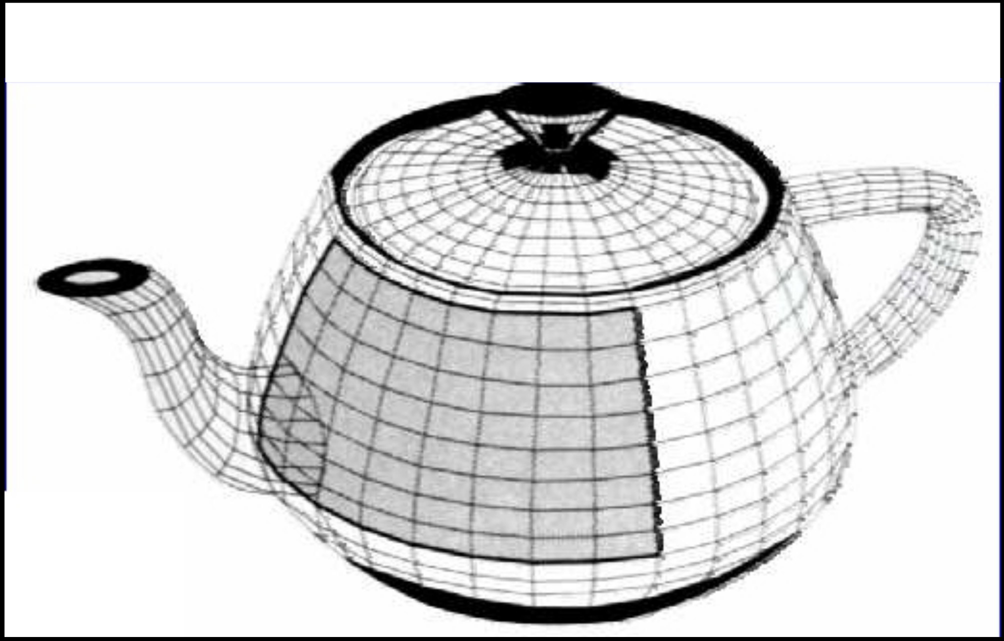


Bezier patch

# Parametric Surfaces

$$p(u,v) = [x(u,v), y(u,v), z(u,v)]$$

- e.g. plane, cylinder, bicubic surface, swept surface



the Utah teapot

# Parametric Surfaces

Why better than polygon meshes?

- Much more compact
- More convenient to control --- just edit control points
- Easy to construct from control points

What are the problems?

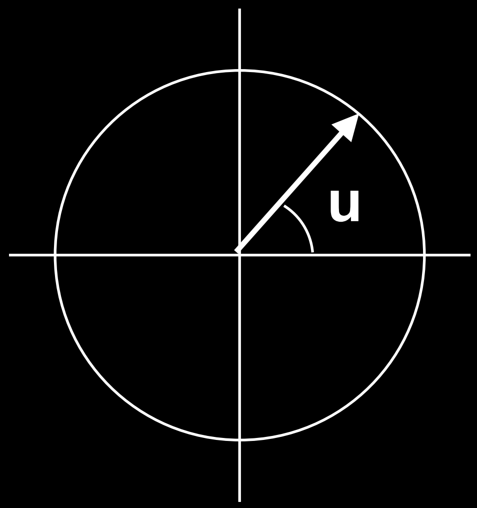
- Work well for smooth surfaces
- Must still split surfaces into discrete number of patches
- Rendering times are higher than for polygons
- Intersection test? Inside/outside test?

# Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces

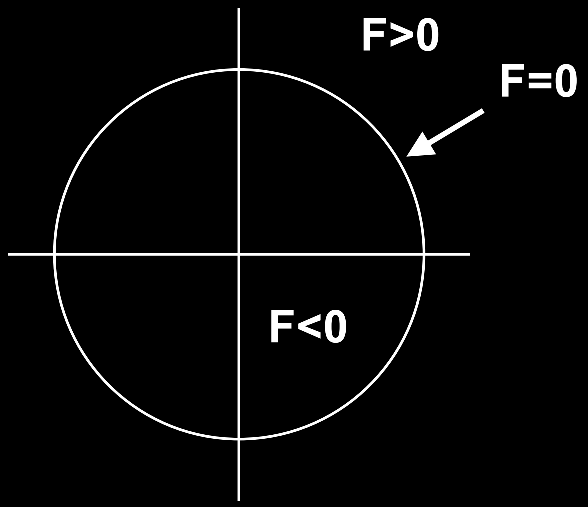
# Two Ways to Define a Circle

## Parametric



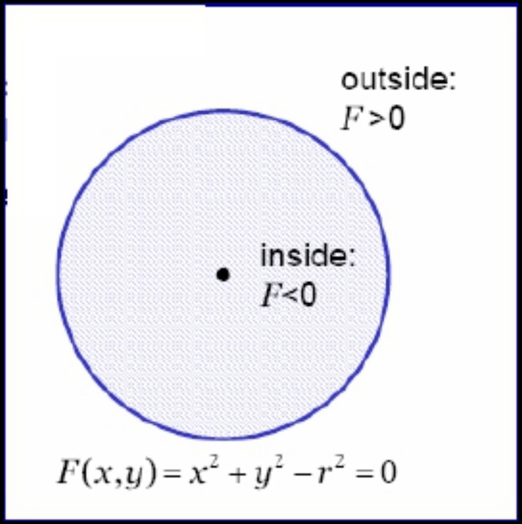
$$x = f(u) = r \cos(u)$$
$$y = g(u) = r \sin(u)$$

## Implicit



$$F(x,y) = x^2 + y^2 - r^2$$

# Implicit Surfaces



- well defined inside/outside
- polygons and parametric surfaces do not have this information

- Computing is hard:
  - implicit functions for a cube?
  - telephone?

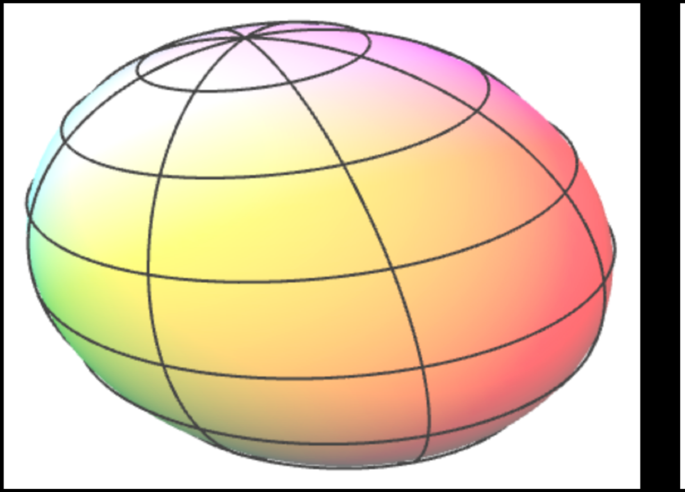
- Implicit surface: F(x,y,z) = 0
  - e.g. plane, sphere, cylinder, quadric, torus, blobby models

    sphere with radius *r* : $F(x,y,z) = x^2+y^2+z^2-r^2 = 0$

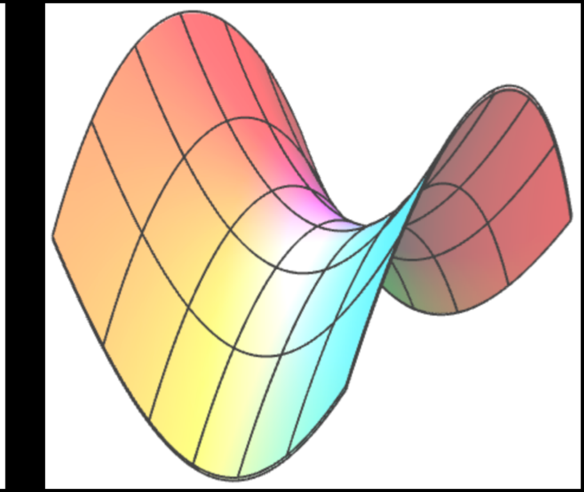
  - terrible for iterating over the surface
  - great for intersections, inside/outside test

# Quadric Surfaces

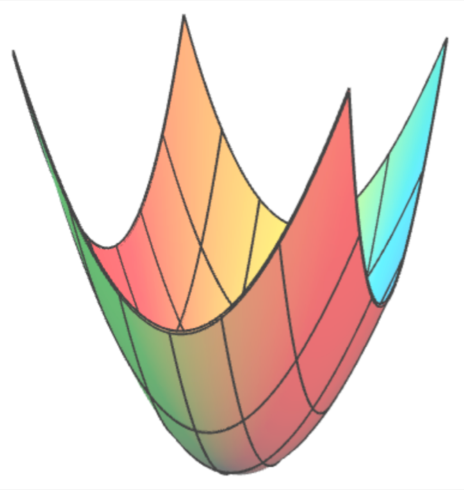
$$F(x,y,z) = ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2px+2qy+2rz+d = 0$$



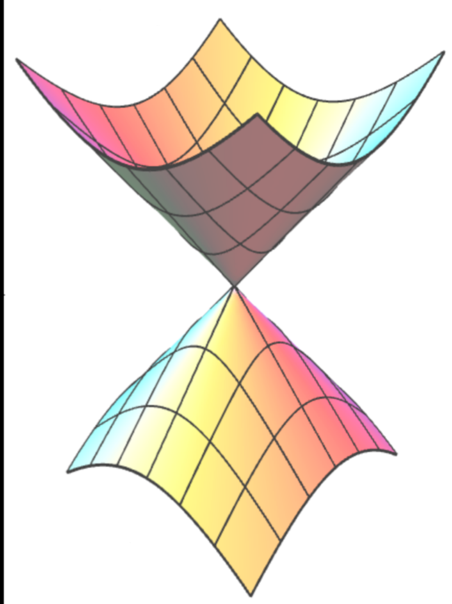
ellipsoid



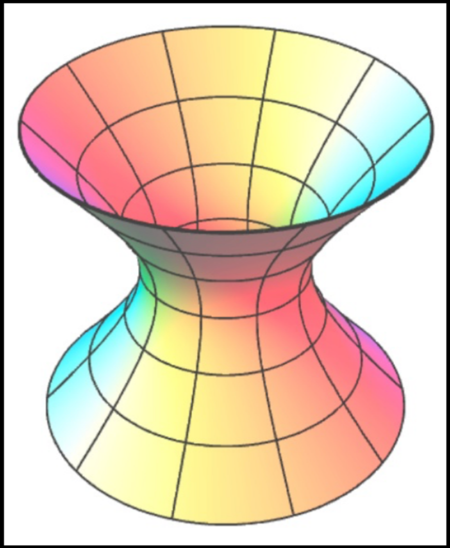
hyperbolic paraboloid



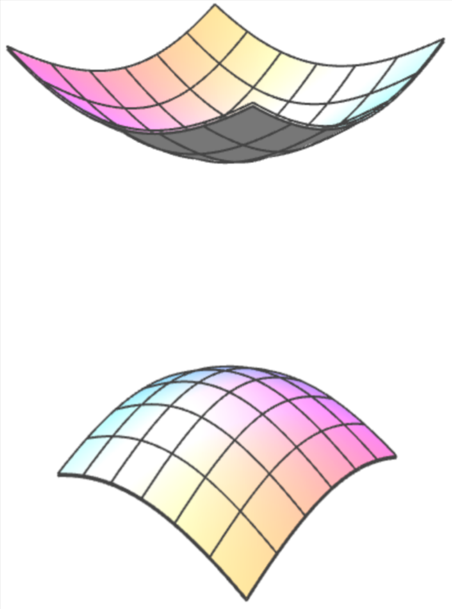
elliptic paraboloid



double cone

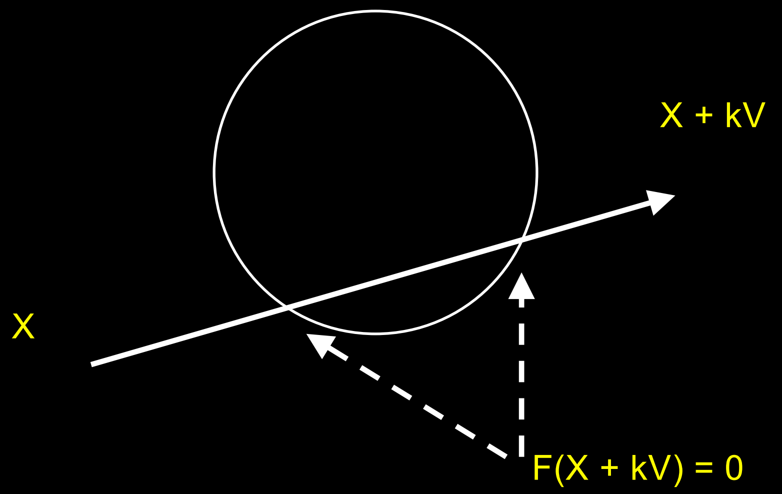


hyperboloid of one sheet

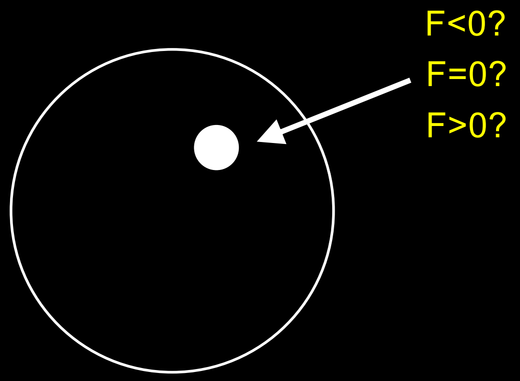


hyperboloid of two sheets

# What Implicit Functions are Good For



Ray - Surface Intersection Test



Inside/Outside Test

# Surfaces from Implicit Functions

- Constant Value Surfaces are called (depending on whom you ask):
  - constant value surfaces
  - level sets
  - isosurfaces

- Nice Feature: you can add them! (and other tricks)
  - this merges the shapes
  - When you use this with spherical exponential potentials, it's called *Blobs*, *Metaballs,* or *Soft Objects*. Great for modeling animals.

# Blobby Models

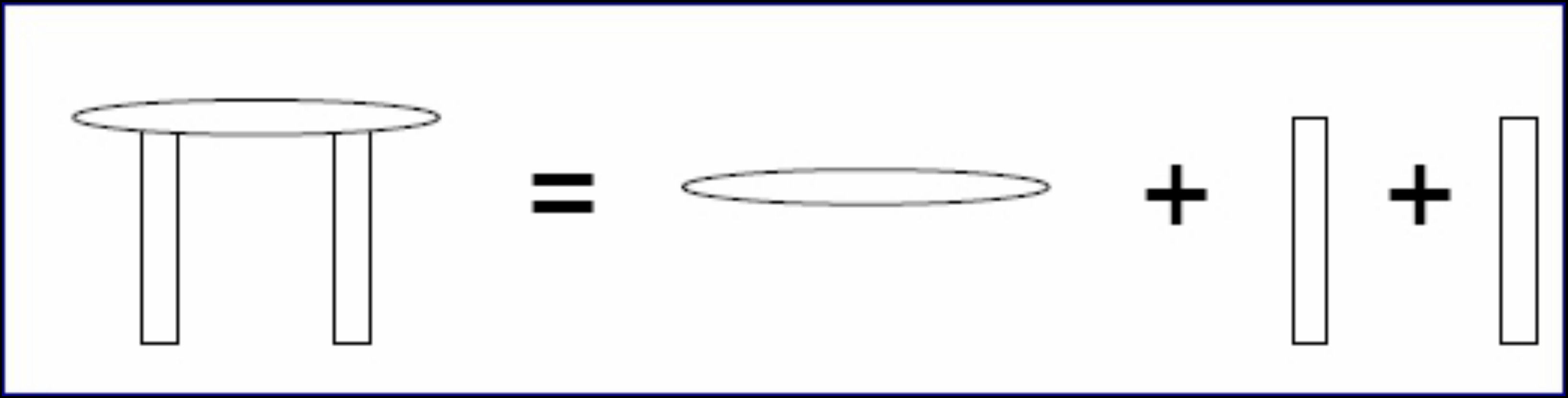
Source: blender.org (2017)

# How to draw implicit surfaces?

- It’s easy to ray trace implicit surfaces
  - because of that easy intersection test
- Volume Rendering can display them
- Convert to polygons: the Marching Cubes algorithm
  - Divide space into cubes
  - Evaluate implicit function at each cube vertex
  - Do root finding or linear interpolation along each edge
  - Polygonize on a cube-by-cube basis

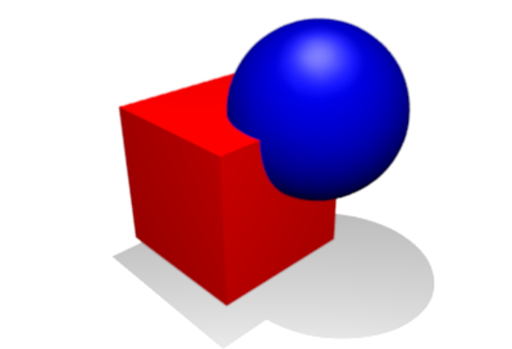
# Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together



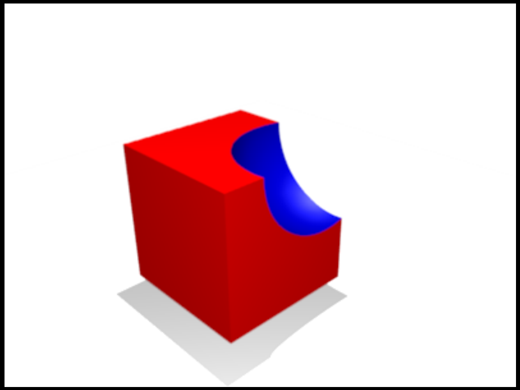
# Constructive Solid Geometry (CSG)

union



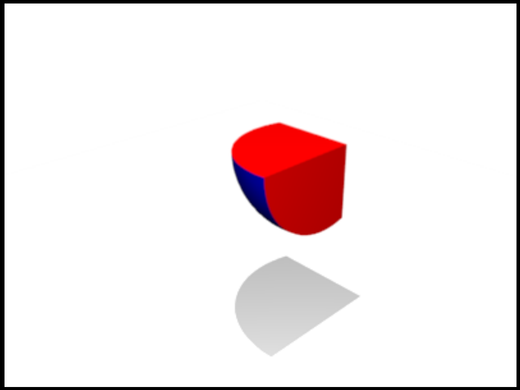
the merger
of two objects
into one

difference



the subtraction
of one object
from another

intersection



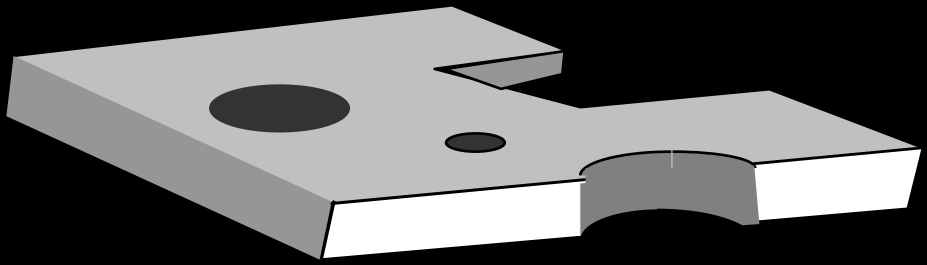
the portion
common to
both objects

Source: Wikipedia

# Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together

- This is sensible for objects that are actually made that way (human-made, particularly machined objects)



# A CSG Train

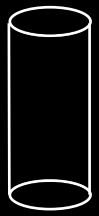
Brian Wyvill & students, Univ. of Calgary

# Negative Objects

Use point-by-point boolean functions

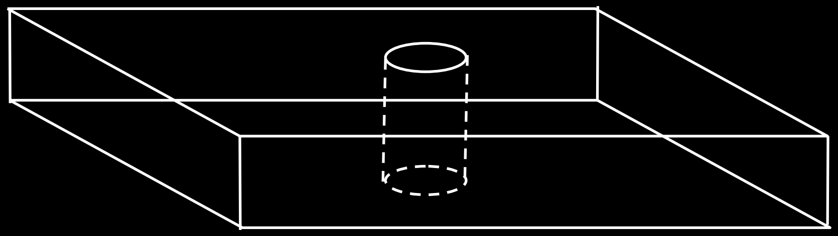
- remove a volume by using a negative object
- e.g. drill a hole by subtracting a cylinder

Subtract



From

To get



Inside(BLOCK-CYL) = Inside(BLOCK) And Not(Inside(CYL))

# Set Operations

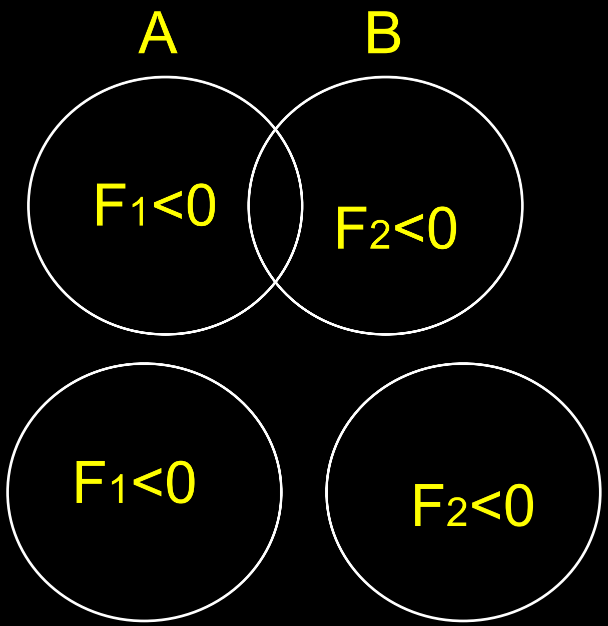
- UNION: Inside(A) || Inside(B)
  - ➢ Join A and B
- INTERSECTION: Inside(A) && Inside(B)
  - ➢ Chop off any part of A that sticks out of B
- SUBTRACTION: Inside(A) && (! Inside(B))
  - ➢ Use B to Cut A

Examples:

- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.

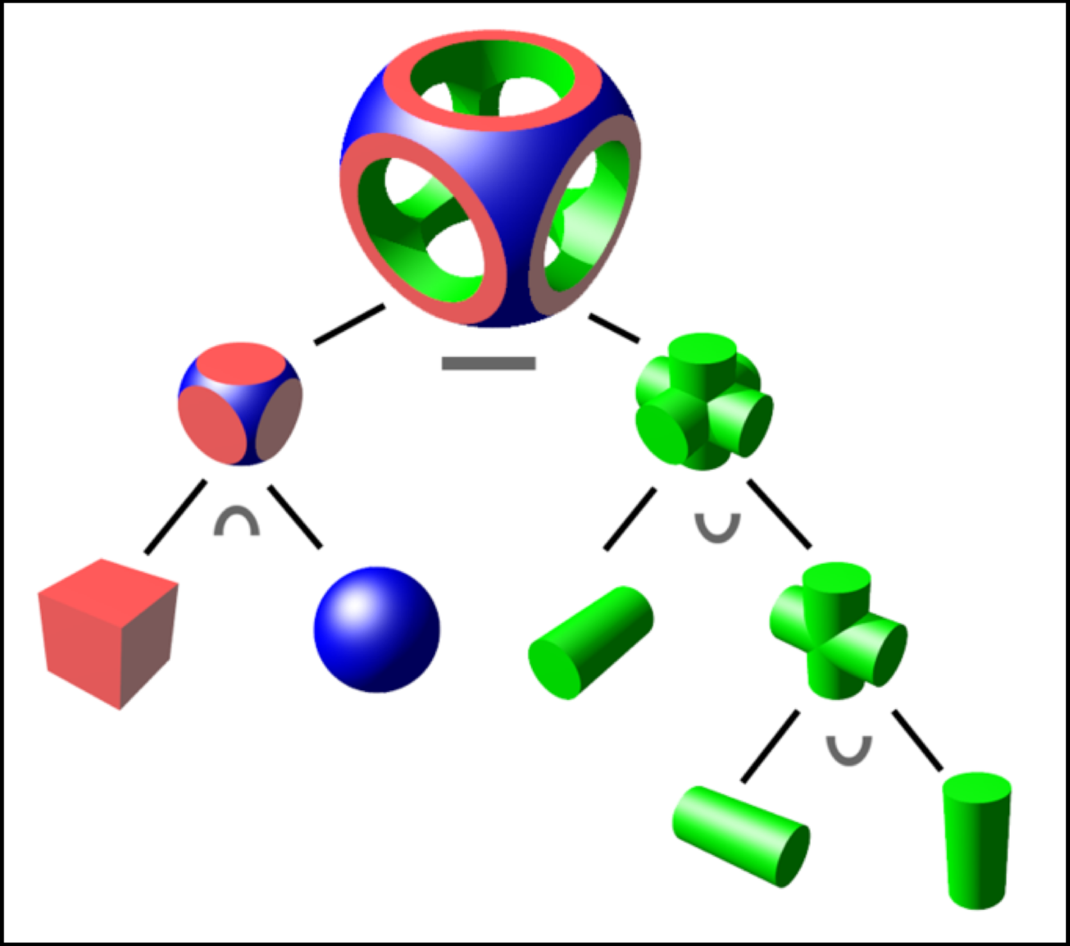
# Implicit Functions for Booleans

- Recall the implicit function for a solid: $F(x,y,z)<0$
- Boolean operations are replaced by arithmetic:
  - MAX replaces AND (intersection)
  - MIN replaces OR (union)
  - MINUS replaces NOT(unary subtraction)
- Thus
  - F(Intersect(A,B)) = MAX(F(A),F(B))
  - F(Union(A,B)) = MIN(F(A),F(B))
  - F(Subtract(A,B)) = MAX(F(A), -F(B))



# CSG Trees

- Set operations yield tree-based representation



Source: Wikipedia

# Implicit Surfaces

- Good for smoothly blending multiple components
- Clearly defined solid along with its boundary
- Intersection test and Inside/outside test are easy

- Need to polygonize to render --- expensive
- Interactive control is not easy
- Fitting to real world data is not easy
- Always smooth

# Summary

- Polygonal Meshes
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry