

CSCI 420 Computer Graphics  
Lecture 21

# Physically Based Simulation

Examples

Particle Systems

Numerical Integration

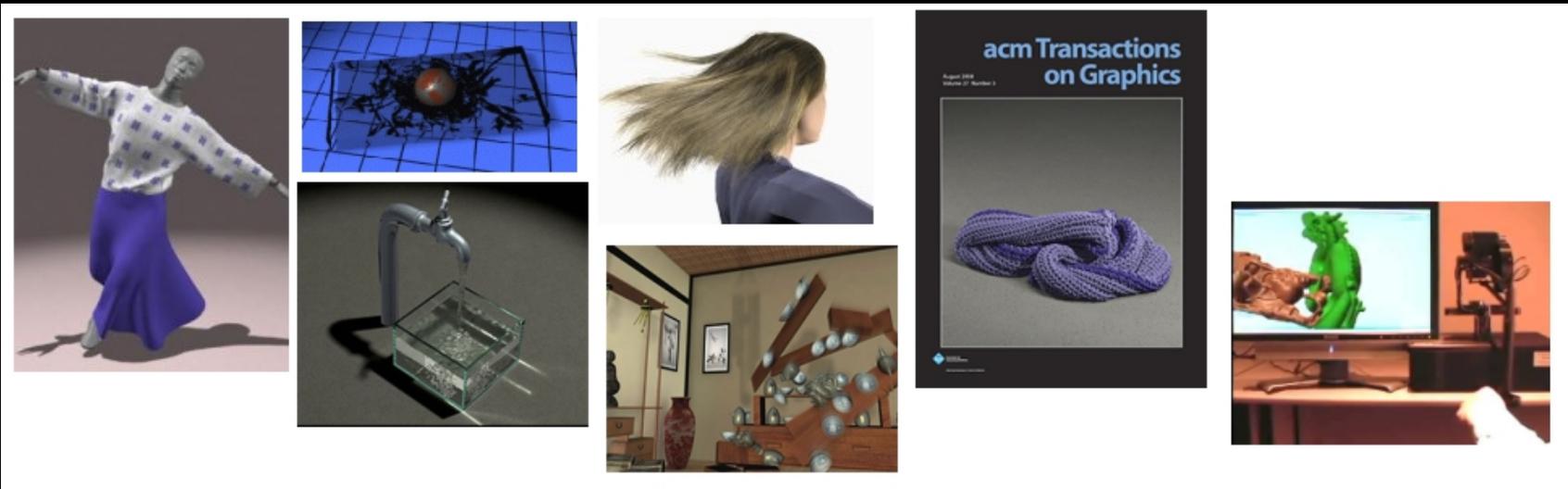
Cloth Simulation

[Angel Ch. 9]

Jernej Barbic  
University of Southern California

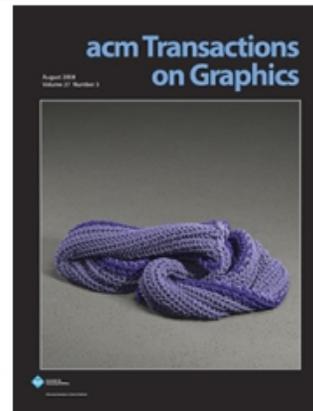
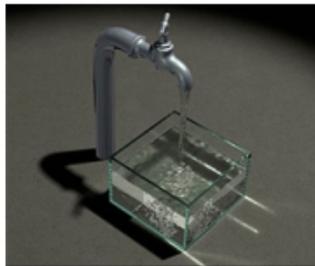
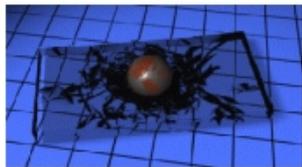
# Physics in Computer Graphics

- Very common
- Computer Animation, Modeling (computational mechanics)
- Rendering (computational optics)

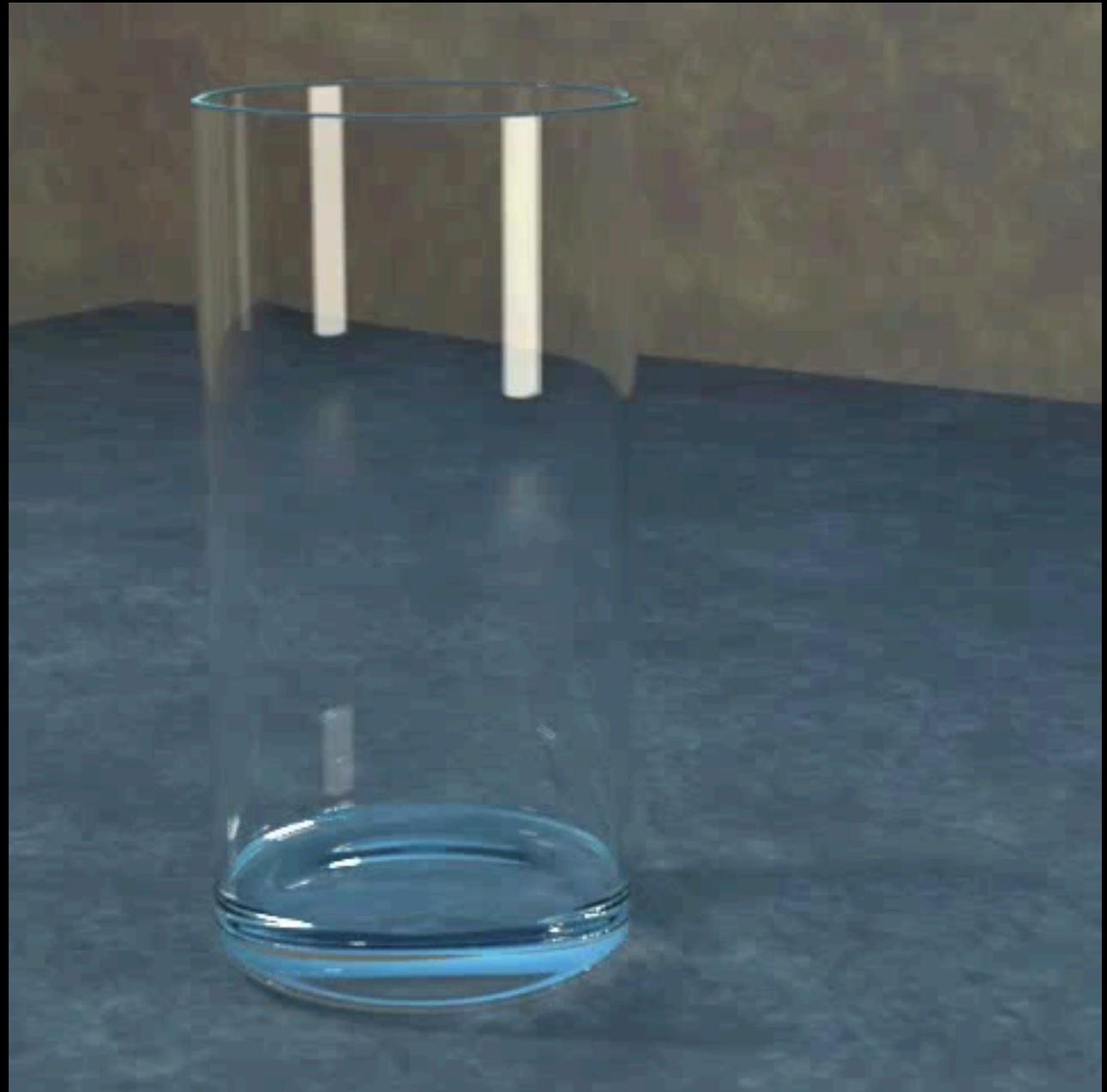


# Physics in Computer Animation

- Fluids
- Smoke
- Deformable strands (rods)
- Cloth
- Solid 3D deformable objects .... and many more!



# Fluids



Enright, Marschner,  
Fedkiw,  
SIGGRAPH 2002

# Fluids and Rigid Bodies

[Carlson, Mucha, Turk,  
SIGGRAPH 2004]



# Fluids with Deformable Solid Coupling

[Robinson-Mosher,  
Shinar,  
Gretarsson,  
Su, Fedkiw,  
SIGGRAPH 2008]

## **Two-way Coupling of Fluids to Rigid and Deformable Solids and Shells**

**Avi Robinson-Mosher  
Tamar Shinar  
Jon Gretarsson  
Jonathan Su  
Ronald Fedkiw**

# Deformations

Vertices: 45882  
Triangles: 105788



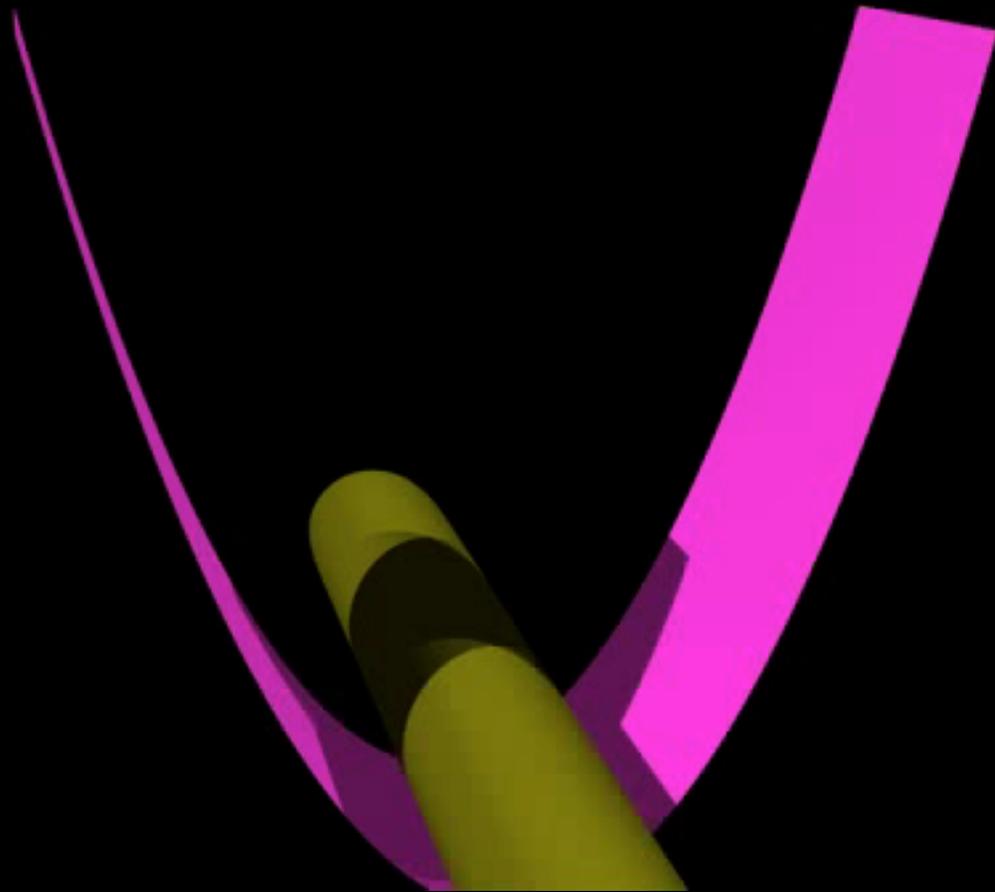
[Barbic and James,  
SIGGRAPH 2005]

# Cloth



Source:  
ACM SIGGRAPH

# Cloth (Robustness)



[Bridson, Fedkiw,  
Anderson, ACM  
SIGGRAPH 2002

# Simulating Large Models



[Doug James,  
PhD Thesis, UBC, 2001]

# Sound Simulation (Acoustics)

Modal renderer



[James, Barbic, Pai,  
SIGGRAPH 2006]

# Multibody Dynamics

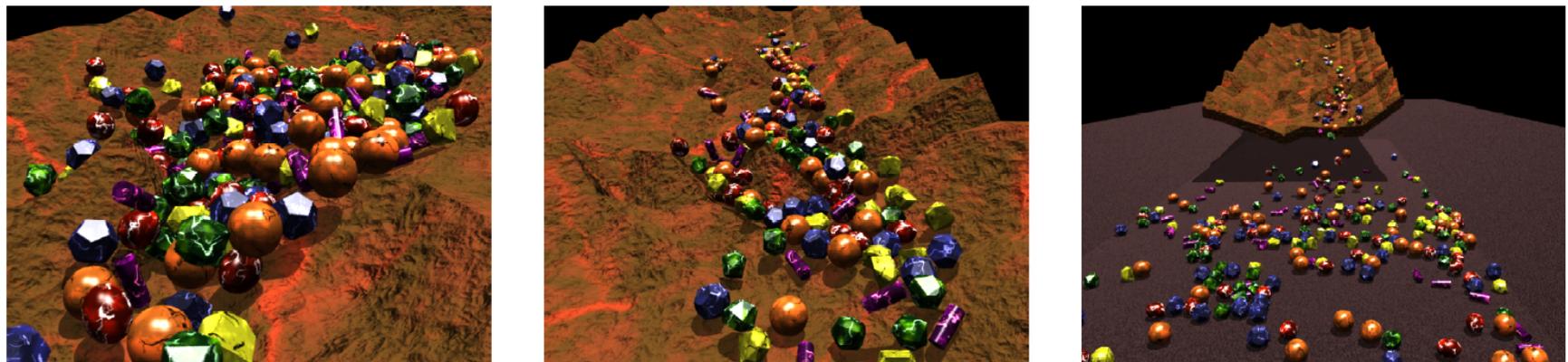
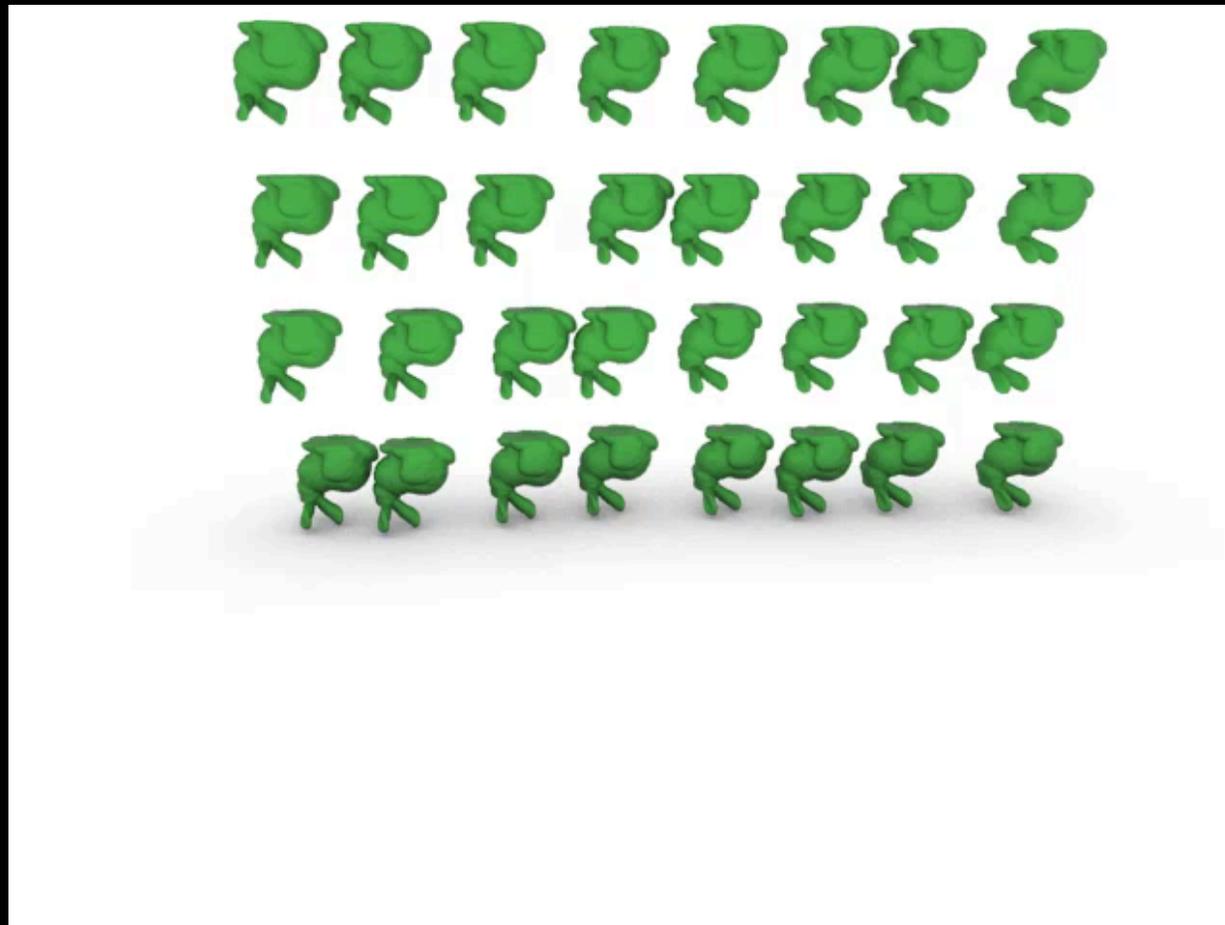


Figure 1: *Avalanche*: 300 rocks tumble down a mountainside.

# Multibody Dynamics + Self-collision Detection



# Physics in Games

Real-Time Deformation and Fracture  
in a Game Environment

Eric Parker  
Pixelux Entertainment

James O'Brien  
U.C. Berkeley

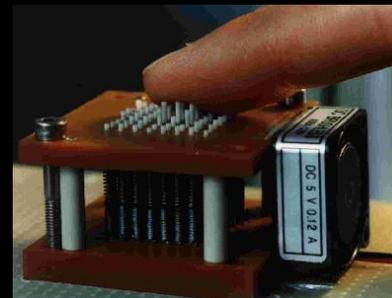
Video Edited by Sebastian Burke

From the proceedings of SCA 2009, New Orleans

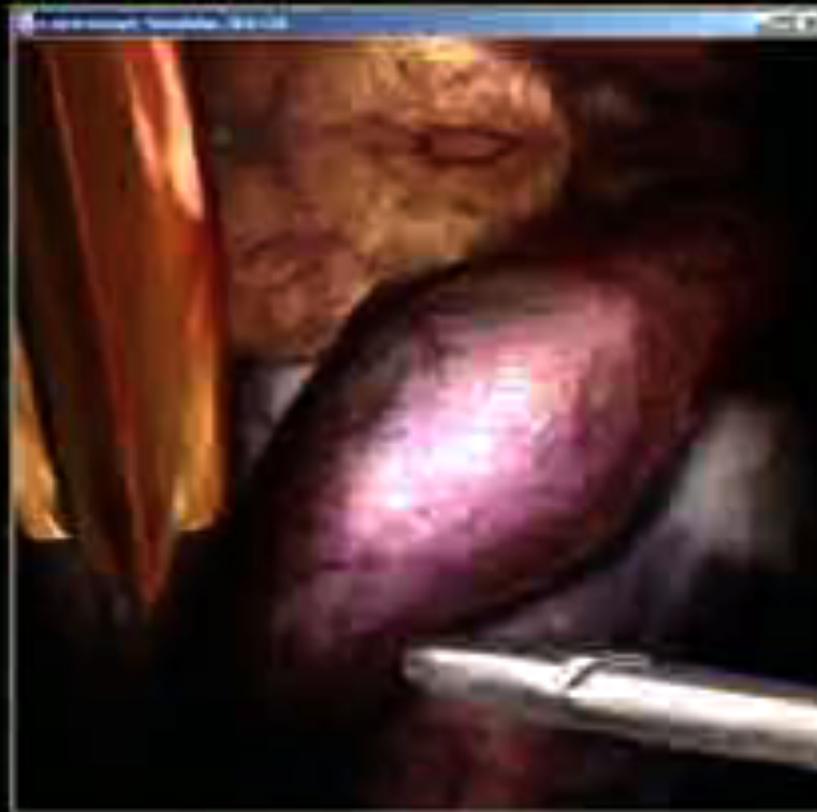
[Parker and O'Brien,  
Symposium on Computer Animation 2009]

# Haptic Interfaces

- hap·tic ('hap-tik)  
*adj.*  
Of or relating to the sense of touch; tactile.



# Surgical Simulation



[James and Pai,  
SIGGRAPH 2002]

# Offline Physics

- Special effects (film, commercials)
- Large models:  
millions of particles / tetrahedra / triangles
- Use computationally expensive rendering  
(global illumination)
- Impressive results
- Many seconds of computation time per frame

# Real-time Physics

- Interactive systems:  
computer games  
virtual medicine (surgical simulation)
- Must be fast (30 fps, preferably 60 fps for games)  
Only a small fraction of CPU time devoted to physics!
- Has to be stable, regardless of user input

# Particle System

- Basic physical system in computer graphics
- We have  $N$  particles
- They interact with some forces
- Fire, Smoke, Cloth, ...
- Very popular for its simplicity

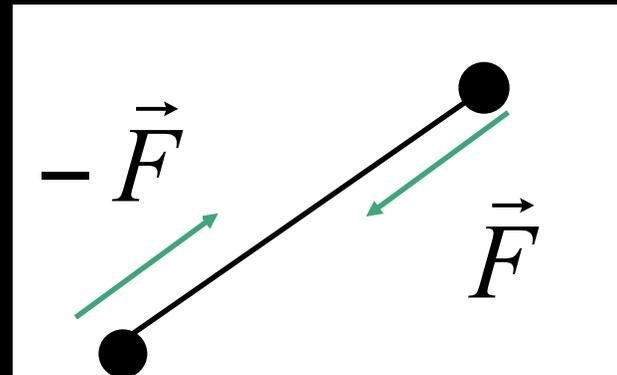


# Newton's Laws

- Newton's 2nd law:

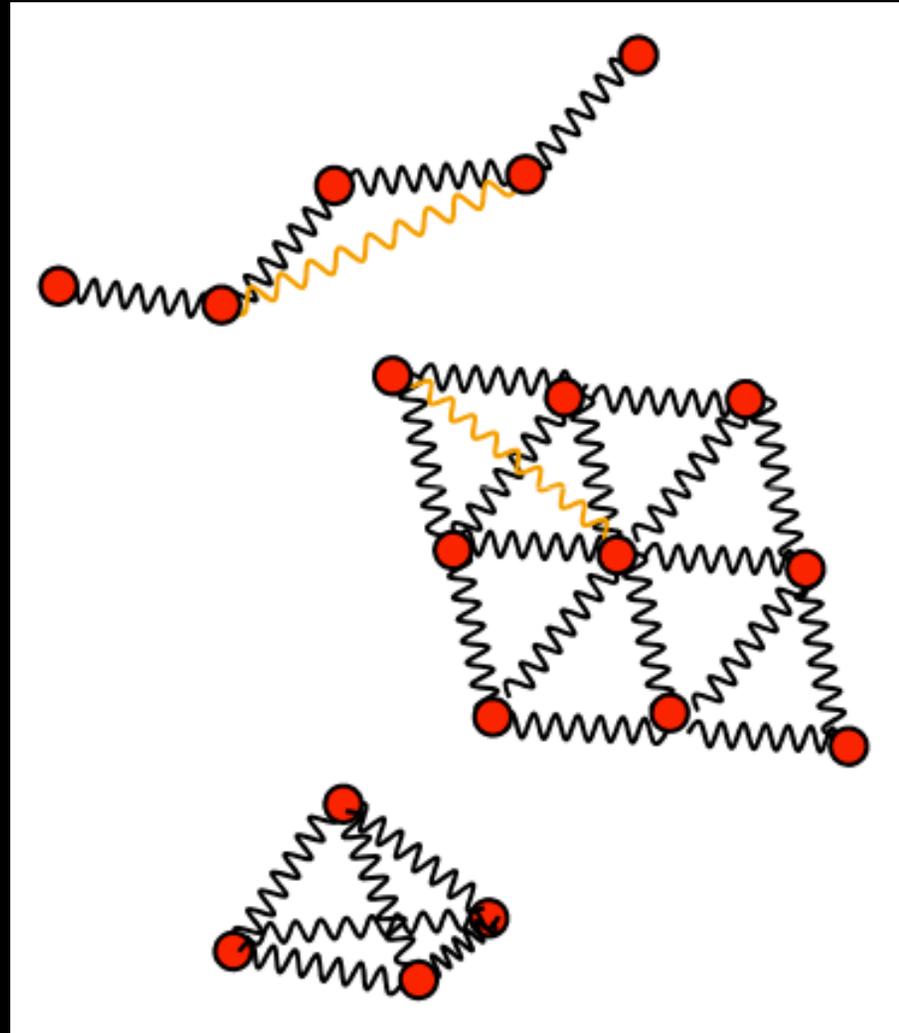
$$\vec{F} = m\vec{a}$$

- Gives acceleration, given the force and mass
- Newton's 3rd law: If object A exerts a force  $F$  on object B, then object B is at the same time exerting force  $-F$  on A.



# Case Study: Mass-spring Systems

- Mass particles connected by elastic springs
- One dimensional: rope, chain
- Two dimensional: cloth, shells
- Three dimensional: soft bodies



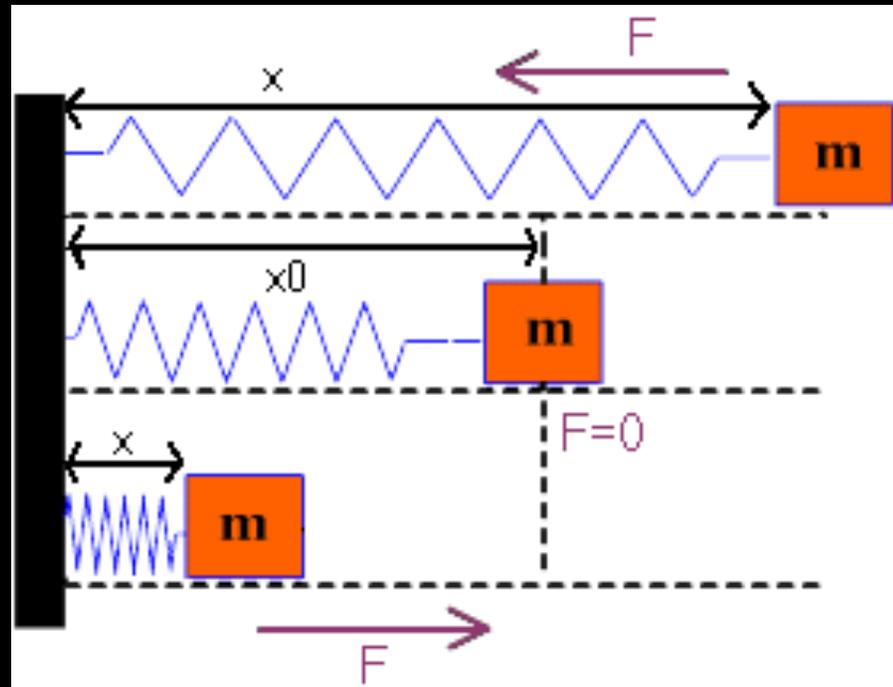
Source:Matthias Mueller, SIGGRAPH

# Single spring

- Obeys the *Hook's law*:

$$F = k (x - x_0)$$

- $x_0$  = rest length
- $k$  = spring elasticity (*stiffness*)
- For  $x < x_0$ , spring wants to extend
- For  $x > x_0$ , spring wants to contract



## Hook' s law in 3D

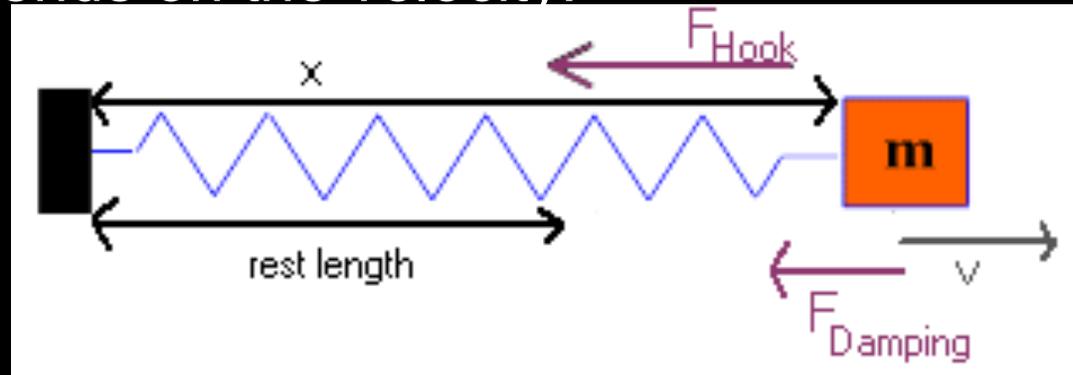
- Assume A and B two mass points connected with a spring.
- Let  $L$  be the vector pointing from B to A
- Let  $R$  be the spring rest length
- Then, the elastic force exerted on A is:

$$\vec{F} = -k_{Hook} (|\vec{L}| - R) \frac{\vec{L}}{|\vec{L}|}$$

# Damping

- Springs are not completely elastic
- They absorb some of the energy and tend to decrease the velocity of the mass points attached to them
- Damping force depends on the velocity:

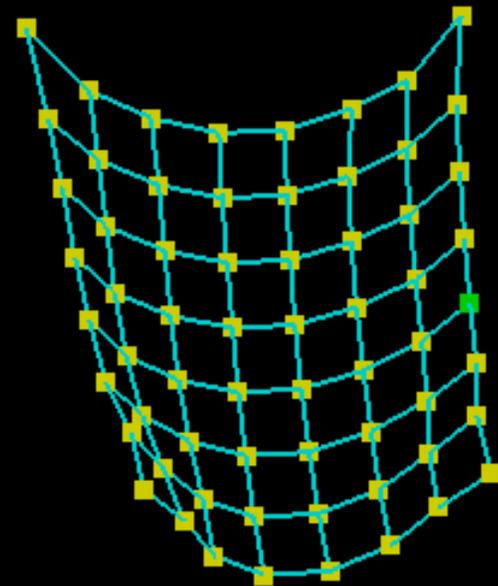
$$\vec{F} = -k_d \vec{v}$$



- $k_d$  = damping coefficient
- $k_d$  different than  $k_{\text{Hook}}$  !!

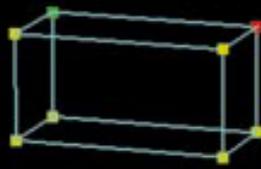
# A network of springs

- Every mass point connected to some other points by springs
- Springs exert forces on mass points
  - Hook's force
  - Damping force
- Other forces
  - External force field
    - Gravity
    - Electrical or magnetic force field
  - Collision force

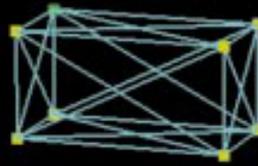


# Network organization is critical

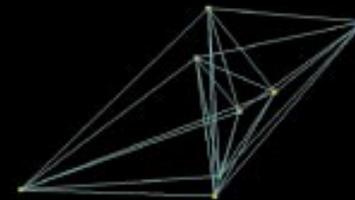
- For stability, must organize the network of springs in some clever way



Basic network

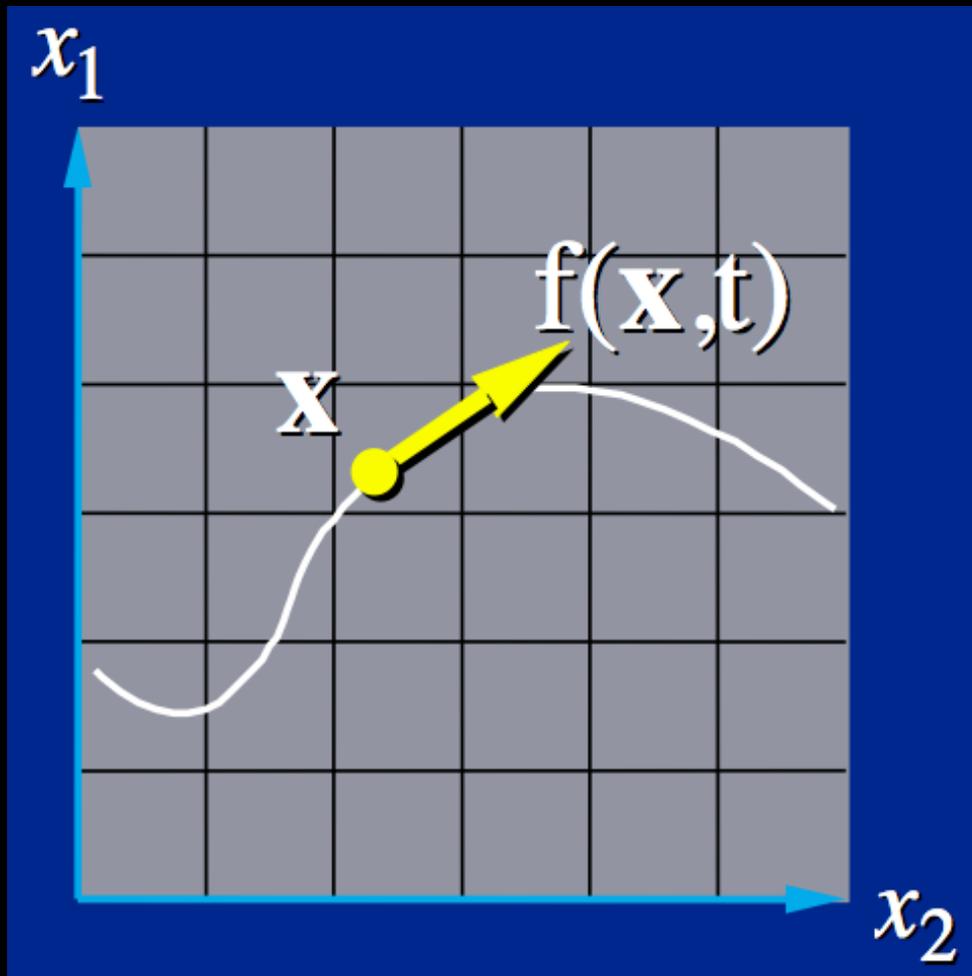


Stable network



Network out  
of control

# Time Integration

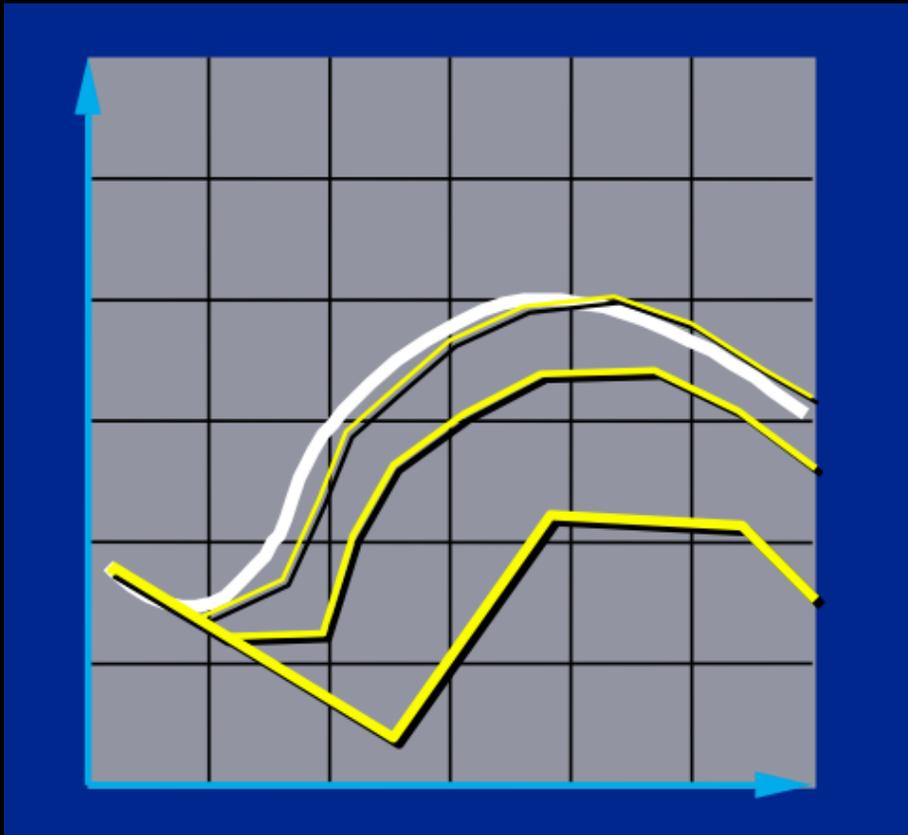


Physics equation:  
 $\mathbf{x}' = f(\mathbf{x},t)$

$\mathbf{x}=\mathbf{x}(t)$  is particle trajectory

# Euler Integration

$$x(t + \Delta t) = x(t) + \Delta t f(x(t))$$



Simple,  
but inaccurate.

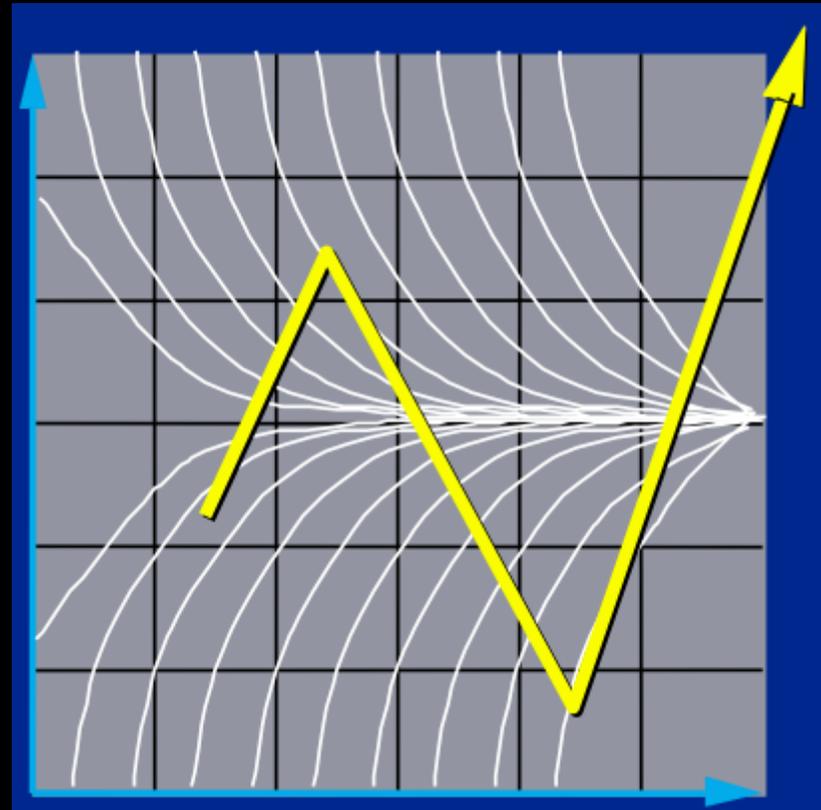
Unstable with  
large timesteps.

Source: Andy Witkin, SIGGRAPH

# Inaccuracies with explicit Euler



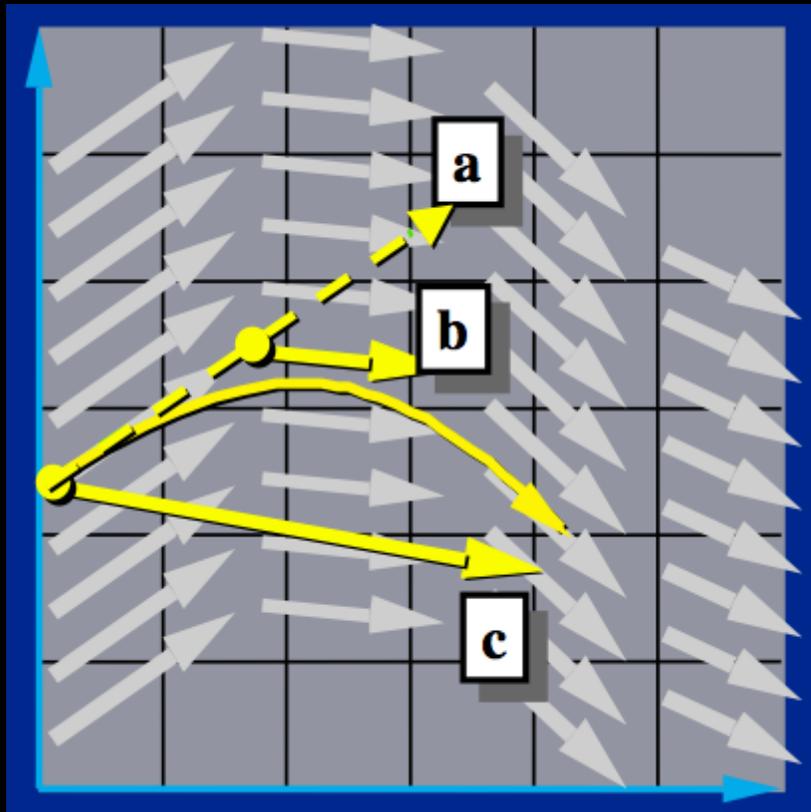
Gain energy



“Blow-up”

Source: Andy Witkin, SIGGRAPH

# Midpoint Method



Source: Andy Witkin, SIGGRAPH

Improves stability

1. Compute Euler step  
 $\Delta x = \Delta t f(x, t)$
2. Evaluate  $f$  at the midpoint  
 $f_{\text{mid}} = f((x+\Delta x)/2, (t+\Delta t)/2)$
3. Take a step using the midpoint value  
 $x(t + \Delta t) = x(t) + \Delta t f_{\text{mid}}$

# Many more methods

- Runge-Kutta (4th order and higher orders)
- Implicit methods
  - sometimes unconditionally stable
  - very popular (e.g., cloth simulations)
  - a lot of damping with large timesteps
- Symplectic methods
  - exactly preserve energy, angular momentum and/or other physical quantities
  - Symplectic Euler

# Cloth Simulation

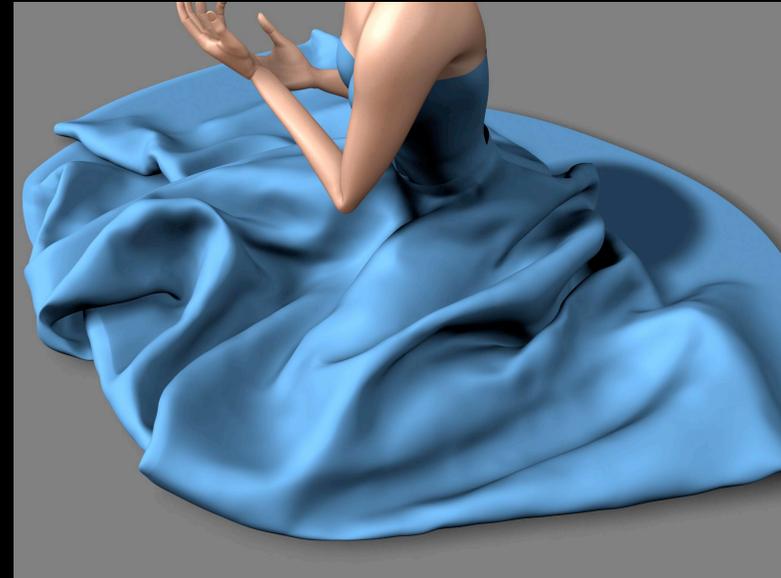
- Cloth Forces
  - Stretch 
  - Shear 
  - Bend 
- Many methods are a more advanced version of a mass-spring system
- Derivatives of Forces
  - necessary for stability



[Baraff and Witkin,  
SIGGRAPH 1998]

# Challenges

- Complex Formulas
- Large Matrices
- Stability
- Collapsing triangles
- Self-collision detection

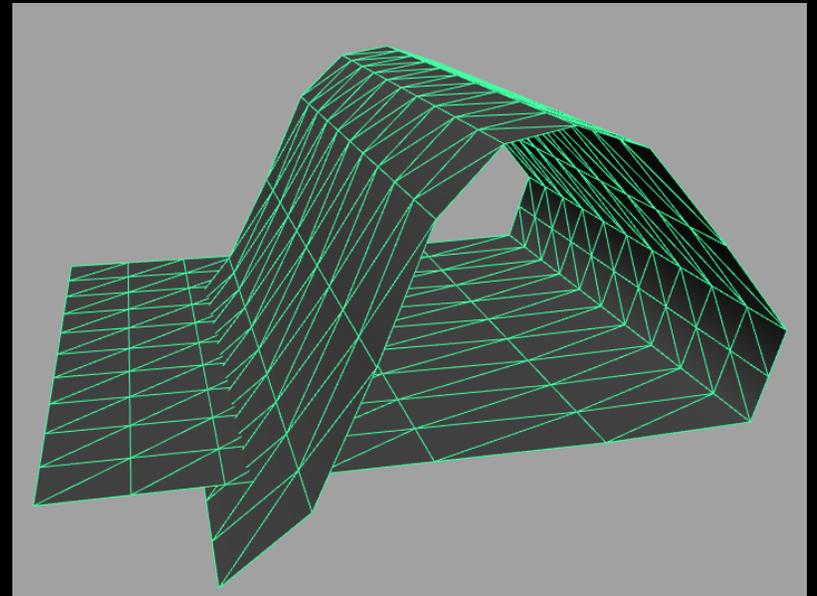


[Govindaraju et al. 2005]

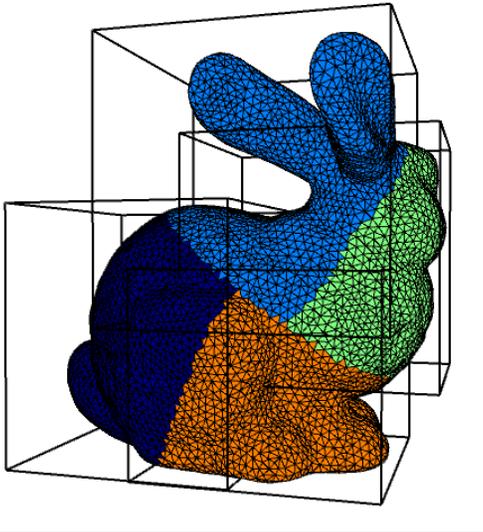
# Self-collisions: definition

Deformable model is  
self-colliding iff

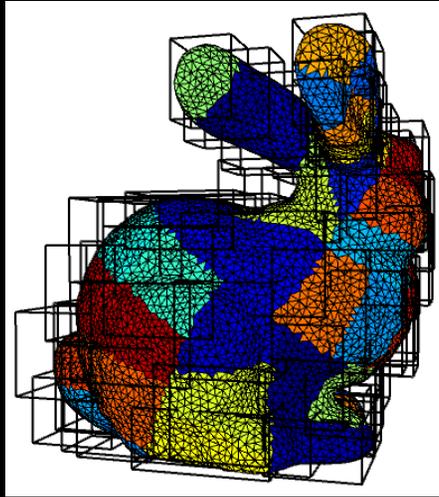
there exist non-neighboring  
intersecting triangles.



# Bounding volume hierarchies



AABBs  
Level 1



AABBs  
Level 3

[Hubbard 1995]

[Gottschalk et al. 1996]

[van den Bergen 1997]

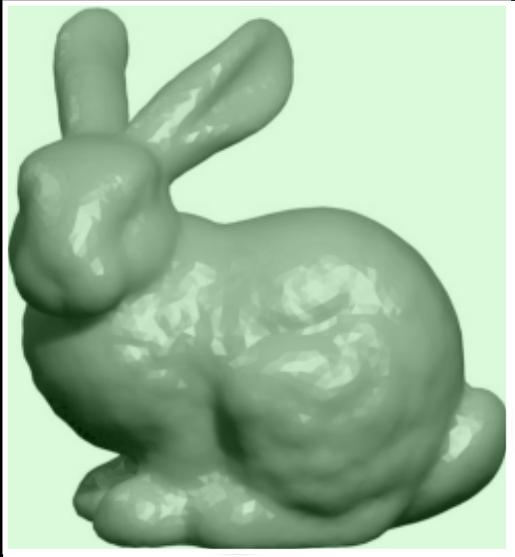
[Bridson et al. 2002]

[Teschner et al. 2002]

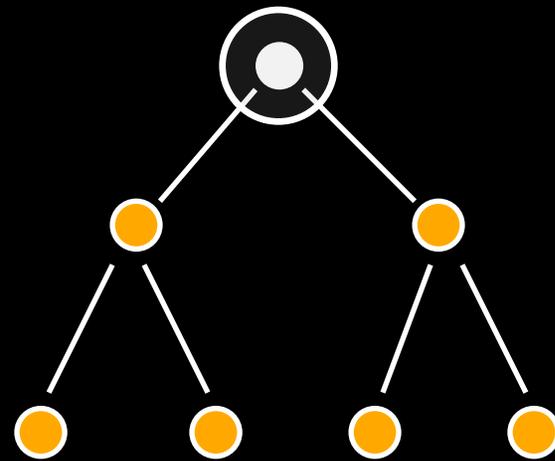
[Govindaraju et al. 2005]

# Bounding volume hierarchy

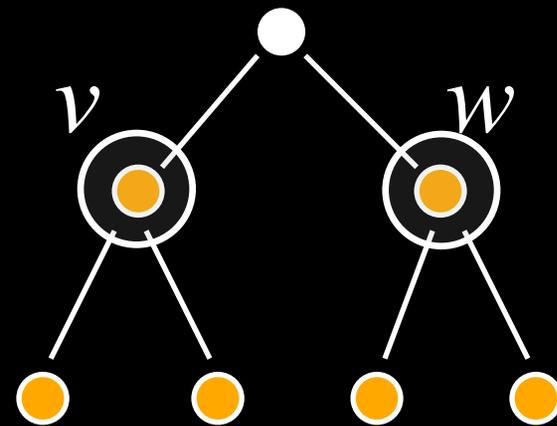
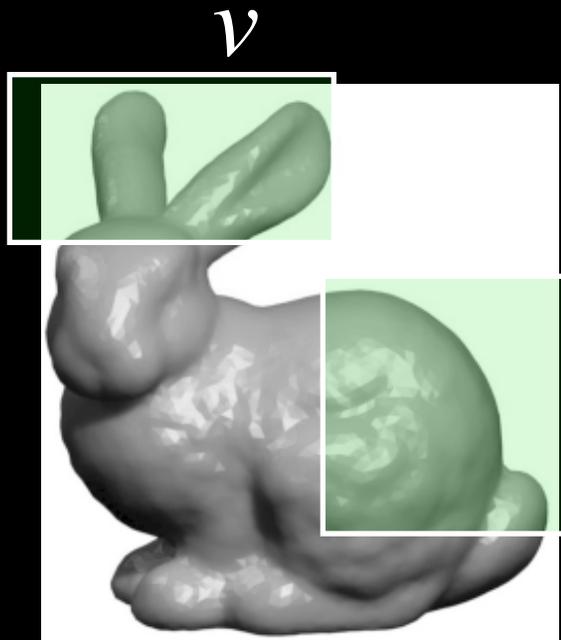
*root*



*root*



# Bounding volume hierarchy



# Real-time cloth simulation

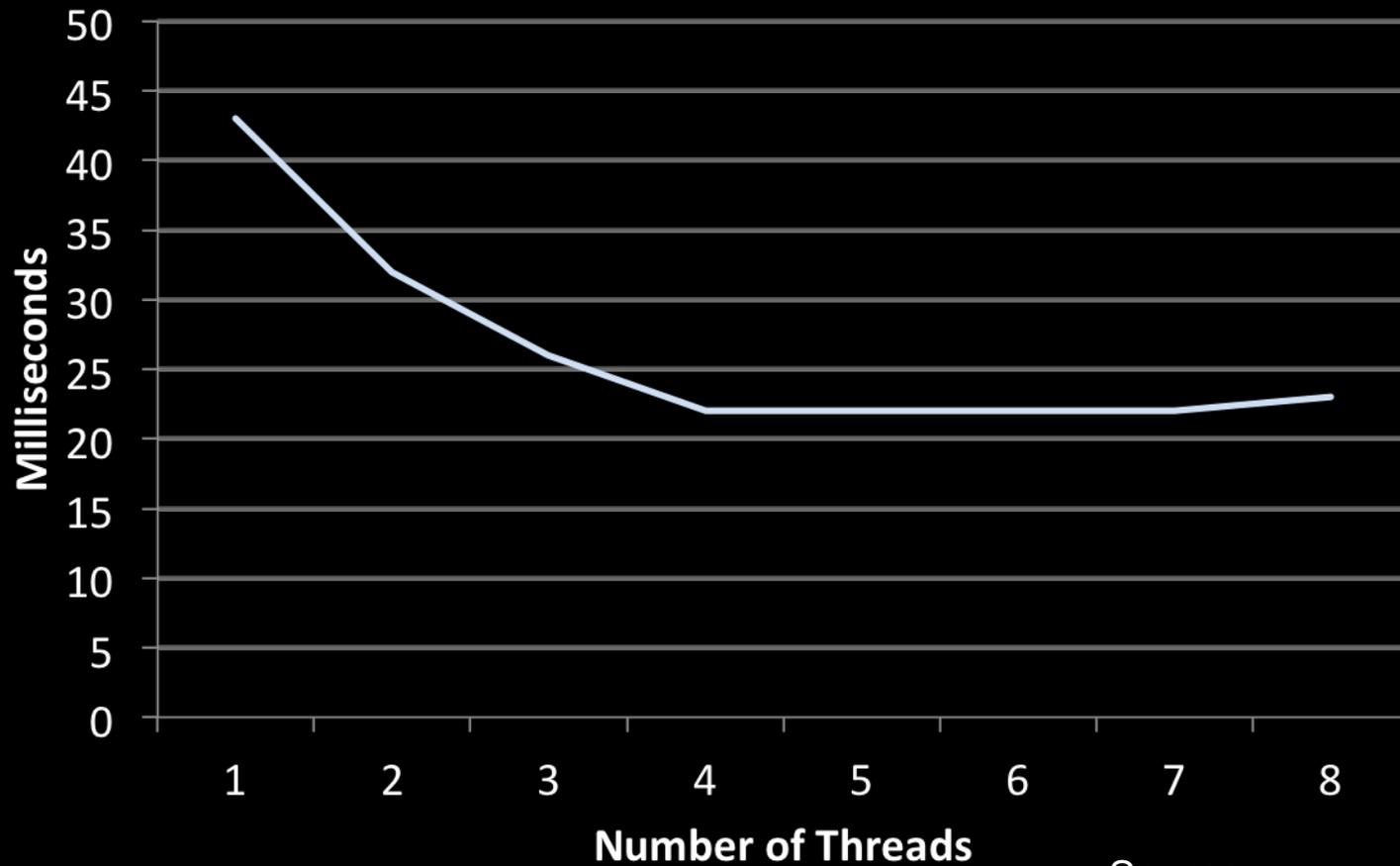


Source:  
Andy Pierce

Model	Triangles	FPS	% Forces + Stiffness Matrix	% Solver
Curtain	2400	25	67	33

# Multithreading implementation

Force and Stiffness Matrix Computation



Source:  
Andy Pierce

# Summary

- Examples of physically based simulation
- Particle Systems
- Numerical Integration
- Cloth Simulation