CS-520 Assignment #3: Constrained Particle System

Instructor: Jernej Barbic

Models of 1D deformable chain-like structures are important tools for computer animation [3, 6, 5]. In this assignment, we will use particle systems to model a simple chain, with an emphasis on "hard" constraints.

Consider an open chain composed of $N+1$ point masses, $m = 1/(N+1)[kg]$, at positions $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_N$ (in meters), with adjacent masses connected by rigid (inextensible) edges of length $h = 1/N$ [m]. We will use the Lagrangian dynamics approach of [2] ("Constrained Dynamics" chapter) to apply suitable constraints to the particle system using Lagrange multipliers (see also [1]). The "rigid edge constraint" can be modeled with the scalar constraint function,

$$C^{RIGID}(\mathbf{x}_i, \mathbf{x}_{i+1}; h) = \|\mathbf{x}_i - \mathbf{x}_{i+1}\| - h = 0, \quad i = 0, \ldots, N\texttt{-1} \tag{1}$$

To keep the chain from falling away under gravity, use the "pin constraint"

$$C^{PIN}(\mathbf{x}_i; \mathbf{p}) = \|\mathbf{x}_i - \mathbf{p}\| = 0 \tag{2}$$

to pin mass #0 at the origin, $\mathbf{p} = 0$. To make things a little more interesting, similar to [2] we'll constrain mass # N to lie on a ring of unit diameter using

$$C^{RING}(\mathbf{x}_N; \mathbf{p}^{RING}) = \|\mathbf{x}_N - \mathbf{p}^{RING}\| - 0.5 = 0 \tag{3}$$

where $\mathbf{p}^{RING} = (0, -0.5, 0)^T$. Also, make the simulation run entirely in the xy plane, i.e., z=0 for all particles.

Implement a simple interactive environment to observe the dynamics of the chain, e.g., for $N = 11$, with the masses drawn as small spheres, and edges as cylinders. In addition to downward gravitational acceleration, $\vec{\mathbf{g}} = (0, -1, 0)$ (in $[ms^{-2}]$), use the cursor keys to interact with the model by applying an additional nonzero translational acceleration $\vec{\mathbf{a}}$ of your choosing. Use the simple forward Euler method (or midpoint if you choose) to integrate the resulting equations of motion.

Make sure to implement Baumgarte stabilization [4] to solve the constrained equations using linear feedback control, i.e., replace each constraint equation $C = 0$ by

$$0 = \ddot{C} + 2b\dot{C} + b^2 C. \tag{4}$$

What b value works best? What happens when the damping parameter b is set too high, or too low? Can you determine b automatically? Momentarily disable C^{RING} and drop the chain from a horizontal position: (a) compare the stabilized system ($b > 0$) to the unstabilized one ($b = 0$); (b) plot the error in the constraints as a function of time.

Next, try adding some simple velocity damping, $\mathbf{f_i} = -\alpha\dot{\mathbf{x}}_i$ to give the chain an "underwater effect." How much damping can you add before stability becomes a problem?

In addition to submitting your code, briefly write-up your derived equations, and describe your results and answers to the previous questions. Record brief animations of your results in any convenient way.

References

[1] David Baraff. Linear-Time Dynamics using Lagrange Multipliers. In *Proceedings of SIGGRAPH 96*, Computer Graphics Proceedings, Annual Conference Series, pages 137–146, August 1996.

[2] David Baraff and Andrew Witkin. Physically Based Modeling: Principles and Practice. In *SIGGRAPH 2001 Course Notes*. ACM SIGGRAPH, 2001.

[3] Ronen Barzel. Faking Dynamics of Ropes and Springs. *IEEE Computer Graphics & Applications*, 17(3):31–39, May - June 1997.

[4] J. Baumgarte. Stabilization of constraints and integrals of motion in dynamical systems. *Comp. Meth. in Appl. Mech. and Eng.*, 1:1–16, 1972.

[5] Johnny T. Chang, Jingyi Jin, and Yizhou Yu. A Practical Model for Hair Mutual Interactions. In *ACM SIGGRAPH Symposium on Computer Animation*, pages 73–80, July 2002.

[6] D. K. Pai. STRANDS: Interactive Simulation of Thin Solids using Cosserat Models. *Computer Graphics Forum*, 21(3):347–352, 2002.