Fluids (Navier-Stokes)

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Introduction

- Most common method
- Not blow up with large timestamps
- If timestamp is very large -> large error but doesn't blow up (Fluid slows down)
- N-S equations -> pde (Partial Diffrential Equations)
- Nabla Operator $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

The Navier-Stokes Equation

- $\partial u/\partial t = - (u \cdot \nabla)u + \nu \nabla^2 u - 1/\rho \nabla p + f$

  $\partial u/\partial t$ -> Tells how velocities change over time

- Mass Conservation Condition : $\nabla \cdot u = 0$

Explanation of the Four Terms

- 1st term -> Advection
- 2nd term -> Laplacian/diffuse term
  - ν = viscosity coefficient ($\nu >= 0$)
  - $\nu = 0$ -> Euler equation -> no loss of energy

  -> Add this term to slow down the velocity.
  -> If velocity differs from neighbors, want to dissipate the difference.
  -> If you had only this term -> difference in velocities between neighbors will become zero.
  -> [We don't solve it exactly so even if this term is not present(or set to zero), the fluid will slow down. Error causes slowdown (also called numerical viscosity)]
  -> If this term is set to zero -> corresponds to no real material. Still commonly set to zero because of slowdown by error. Therefore low levels of viscosity difficult to obtain.
- 3rd term
  $- 1/\rho \nabla p$

  -> pressure term.
- 4th term
  f
  ->external forces

Mass Conservation Condition : $\nabla \cdot u = 0$

–> $\nabla \cdot u = 0$ vector field with respect to (x,y,z)

-> Divergence

-> Incompressibility

<u>Helmholtz – Hodge Decomposition</u>

$W = u + \nabla p$

We must have some method to ensure that the field is always divergence free.

$$u = Pw = W - \nabla p$$

<u>Solving The Equation</u>

Many ways to solve. One method is :

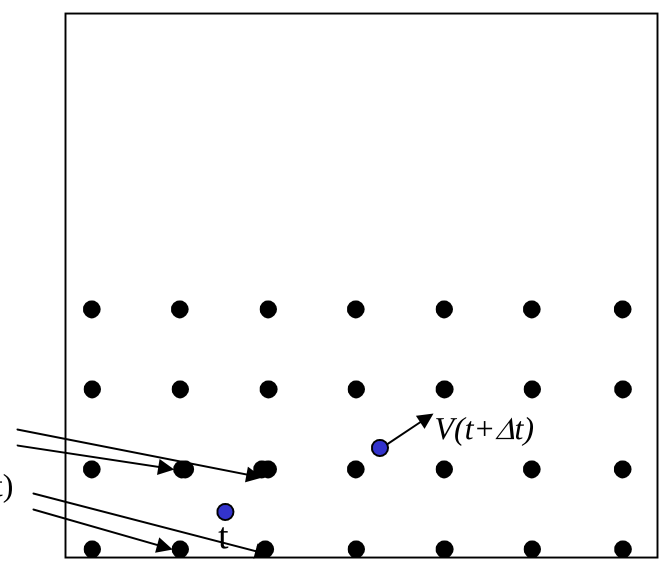
- Split the equation and solve the terms individually
- That means pretending other terms are not there
- Last step is projection, to make the field divergence free.

This method does not give very accurate solution, but for small time steps it is reasonably accurate.

- Add force fields.

<u>Advection</u>

Calculating velocity at the next time step:
- We have fixed locations on the grid
- Which particle will drive into the location
- Set the velocity at $t + \Delta t$ equal to the velocity of the particle at time t.



- Even if velocity so high that the particle flies over a grid cell this method will not blow up.

Projection

- This is the most expensive step
- Conjugate gradients is a common method to solve it.

Weakness of the method

- A fundamental weekness of the method is that the system loses energy very fast.

Rendering Aspect

- We cannot show velocities, hence we can immerse some smoke i.e. introduce particles. However this is not commonly used, as it would need a lot of particles.
- A common method is to render densities. At every gird location we have velocity and a scalar density. Densities get conveyed by velocities.
- Density/smoke will be dissipated if it differs from neighbors.
- We can render the density as the intensity of pixels in that grid cell.

- For 3D we may combine density with particles. We can render each particle as a sprite or radially decreasing intensity.