Physically Based Modeling for Interactive Simulation and Games

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Scribe notes – April 1, 2010

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Acoustics – Deals with the study of sound. This field is dealt by the physicists but is not much explored in the field of Computer Science.

In graphics, sound is dealt with much more complex objects.

Related Research Papers:

Synthesizing Sounds from Physically Based Motion - James F. O'Brien, Perry R. Cook, Georg Essl

ACM SIGGRAPH 2001

http://graphics.eecs.berkeley.edu/site_root/papers/Obrien-SSR-2002-07/

Precomputed Acoustic Transfer: Output-sensitive, accurate sound generation for geometrically complex vibration sources - Doug L. James, Jernej Barbič, Dinesh K. Pai

ACM SIGGRAPH 2006

http://graphics.cs.cmu.edu/projects/pat/

Harmonic Shells: A Practical Nonlinear Sound Model for Near-Rigid Thin Shells - J. Chadwick, S. An, and D. L. James

ACM SIGGRAPH 2009

http://www.cs.cornell.edu/projects/HarmonicShells/

Linear Dynamics + Monopole	Non-Linear Dynamics + Monopole
Linear Dynamics + Transfer	Non-Linear Dynamics + Transfer

The method that we are going to study is <u>Linear Dynamics + Monopole.</u> This method is the simplest method to simulate sound.

Let us take a simple perfectly rigid cubical bar and let it fall on the ground. When this bar falls on the ground, forces exerted on the object are calculated and stored. These forces are then applied to a similar model which is not perfectly rigid and undergoes some transformation. This transformation of the model sends out pressure waves, thus simulating sound. Figure 1 shows the simulation when an object falls on the ground and the sound that it simulates in various time-stamps.

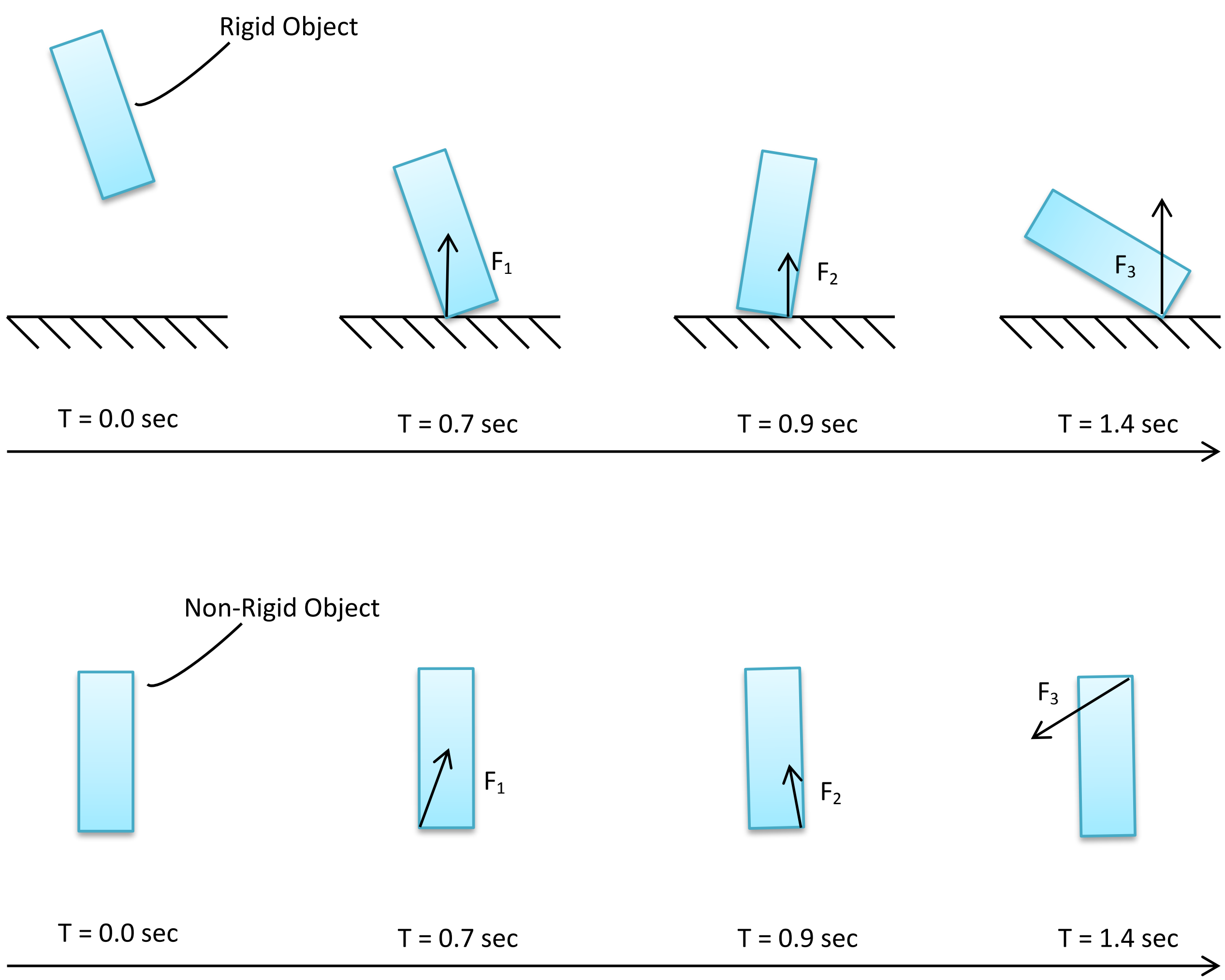


Figure 1

We have Rayleigh's damping equation

$$M \ddot{u} + (\alpha M + \beta K) \dot{u} + K u = F \qquad - (1)$$

This represents generalized Eigen Value problem,

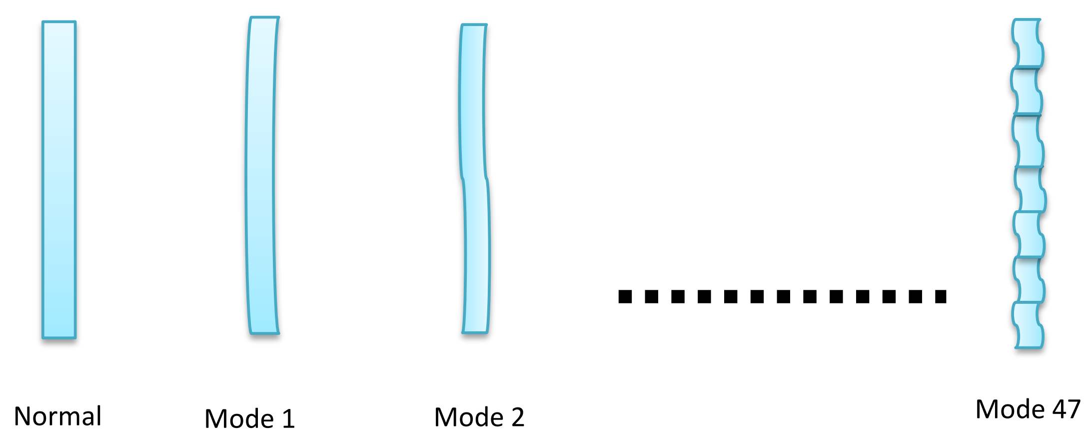
$$K x = \lambda M x$$

We have various modes Ψ_i for the model. On solving above equation, we get different Eigen values for every Ψ_i, which is λ_i. This λ_i is always equal to the frequency of the sound.

$$\lambda_i = \omega_i^2$$

Out of these, very few frequencies are audible to human, thus we ignore rest of the Eigen values. In practice, we take only around 50 Eigen values.

$$U = [\Psi_1 \quad \Psi_2 \quad \Psi_3 \quad \Psi_4 \; . \; . \; . \; . \; . \; . \; . \; . \; \Psi_{50}]$$



$$u = U q$$

Replacing $u = U q$ in equation (1) and pre-multiplying by U^T, we get

$$(U^T M U) \ddot{q} + (\alpha U^T M U + \beta U^T K U) \dot{q} + U^T K U q = U^T F \qquad - (2)$$

Now, $U^T M U = I$ and $U^T K U$ can be written as diagonal matrix Λ, where Λ equals

$$\Lambda = \begin{bmatrix} \lambda_1 & & & & \\ & \lambda_2 & & & \\ & & \lambda_3 & & \\ & & & \ddots & \\ & & & & \lambda_{50} \end{bmatrix}$$

Therefore, equation (2) becomes

$$\ddot{q} + (\alpha I + \beta \Lambda)\, \dot{q} + \Lambda\, q = \tilde{F} \qquad (3)$$

where $\tilde{F} = U^T F$

Solve this equation for q by replacing the values of $\tilde{F}$. This is a system of ODEs. So there are 50 equations for 50 different unknowns. Hence sound at any time t can be calculated as

$$\text{Sound}\ (t) = \sum_{i=1}^{50} a_i\, q_i\, (t) \qquad -(4)$$

Since all values of q are independent of each other, equation (3) can be solved independently for each value of q.Thus, equation (3) becomes,

$$\ddot{q}_i + (\alpha + \beta\, \lambda_i)\, \dot{q}_i + \lambda_i\, q_i = \tilde{F}_i$$

This can be written as

$$\ddot{q}_i + C_i\, \dot{q}_i + \lambda_i\, q_i = \tilde{F}_i \qquad -(5)$$

Equation (5) represents equation of a harmonic oscillator and C_i is the damping coefficient.

In equation (4), a_i can be calculated in different ways.

1) $a_i = 1$ or $a_i = 1 / R$ (i.e. Some constant for all a*s*)
   If depends on time, it would become $a_i(t)$. e.g. If a microphone changes its position with time in respect to the object.

2) $a_i = \int_S \Psi_i\, \vec{n}\ dS$

   where, $\Psi_i\ \vec{n}$ is the volume velocity.

   This would tell us how much each point changes its position.

3) $a_i = a_i\ (x, t)$
   Here, a_i is a function of space and time.

For sounds, $C_i << 1$ in equation (5)

Therefore some possible solutions for equation (5) are

$\sin (\omega_i t)$

$\cos (\omega_i t)$

$\exp (- \beta t)$

Solution is usually $e^{- \beta t} \sin (\omega_i t)$