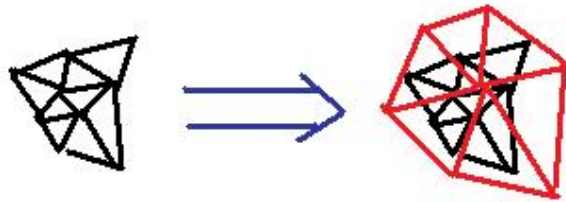


Scribe Notes of Model Reduction

Model Reduction:

For example, when the simulation is using 100K tets at 30FPS, the simulation may use a lot of computation resources. To accelerate the simulation, an intuitive and efficient method is to use coarse mesh for computation and adopt fine mesh for presentation.



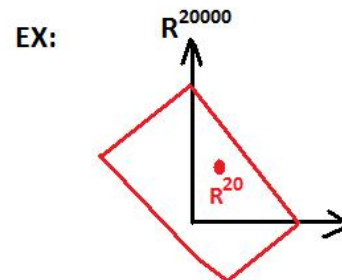
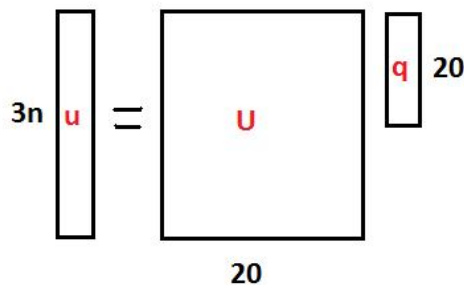
Several approaches to reduce the complexity of model:

- 1) Geometry
- 2) Material properties (Average or Weighting Average)

Projection-Based Model Reduction:

Projection-based model reduction is a popular and useful technique.

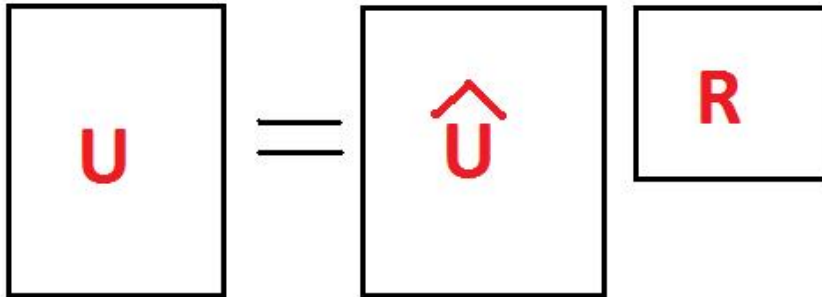
Assume: $u(t) = Uq(t)$



$$U\dot{q} = F(Uq, t) \quad , \quad \dot{q} = G(q, t)$$

$$U^T U \dot{q} = U^T F(Uq, t) \quad (\text{ODE})$$

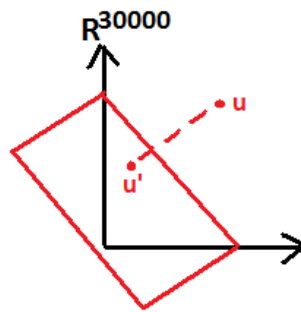
$U = \hat{U}R$, where U is a non-orthogonal matrix and \hat{U} is an orthogonal matrix.



This process is also named **QR Decomposition**

$$q = U^T u$$

$$u' = Uq$$



$$U^T U \dot{q} = U^T F(Uq, t)$$

Integration: $q_0, q_1, q_2 \dots\dots$

SIGGRAPH Paper Presentation:

Adrien Treuille et al., "Model Reduction for Real-time Fluids", SIGGRAPH 2006

This paper presents a new algorithm for the reduced-dimensional simulation of

incompressible fluids. This method is able to let fluids interact with complex boundaries for interaction.

Benefits

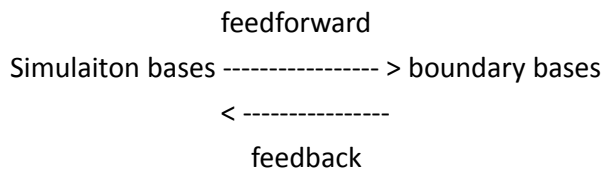
- 1) Resolution Independent
- 2) Energy Reserving
- 3) Fluid/Object interaction (Rendering with interaction)

$$\hat{F} = P \circ F \circ P^{-1}$$

$$P: u \mapsto r, P^{-1}: r \mapsto u$$

$$u \in \mathbb{R}^n, r \in \mathbb{R}^m \quad m < n$$

During the coupling process, this method takes the boundary bases into account.



this process can be done by the proposed model-reduced approach as well.

Conclusion:

Limitation	Benefits
- Accuracy	- Fast and detailed
- Generality	- Unconditional Stability
- Memory Consumption	- Fluid/Object Interaction