CS599 Class Notes 2010-03-02 Yuyu Xu

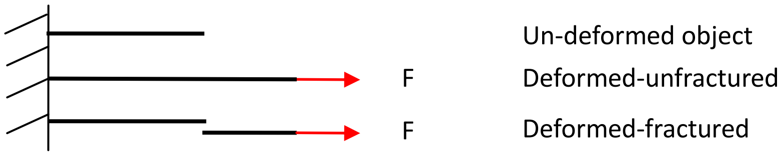
Fracture and Cutting

(Reference) J. F. O'Brien and J. K. Hodgins: *Graphical Modeling and Animation of Brittle Fracture*, SIGGRAPH 99

There are two kinds of fracture, one happens before large deformation and the other one happens after large deformation. The reference paper is about the previous situation which is called brittle fracture. Typical example of brittle fracture is glass.

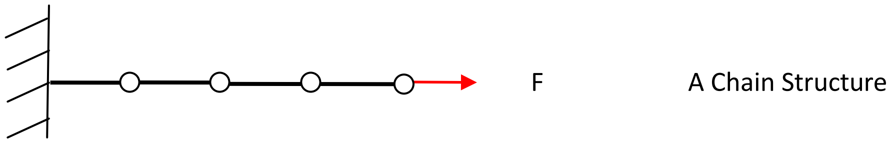
Overall Introduction

When objects deformed, there would be stress inside the material. If the stress force is larger than some threshold, the material cannot afford it, fracture happens. Consider a 1D example like the following.



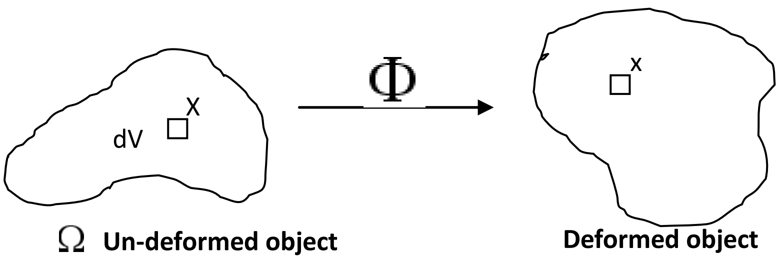
The force exerted on the object stretched the object, if F is larger than the threshold F0 (F0 > 0), object breaks. The two parts are treated as two objects and the process repeats.

Now considering a 1.5D example like the following:



In this situation, there exists a lot of cross section; larger force would be needed to make it depart. This is the same principle that is inside 3D objects fracture.

3D Object Fracture Model – Terminology and Formula



Deformation Configuration

Assume there is an infinite small sub object Ω inside the object, its volume is dV, location is X, after an deformation function mapping, its location becomes x.

$$\Phi(X,t) = x$$

Define deformation gradient:

$$F = \frac{\partial \Phi}{\partial X} \quad \text{(3×3 Jacobian matrix)}$$

Define Strain Tensor:

$$E = \frac{1}{2}(F^T F - I)$$

The strain tensor can be considered as a measure displacement, just like Δx in Hooks principle. The relationship between 3D displacement E and force is according to material law. Function is decided according to specific material.

$$\textit{Material Law}: \quad S = \psi(\text{E,X,t})$$

S here represents the 2nd Piola stress Tensor. We also define 1st Piola stress Tensor P and Cauchy stress σ. The relationship between the three values is as following:

$$\text{P=F} \cdot \text{S}$$

$$\sigma = \frac{1}{\det(F)} \cdot F \cdot S \cdot F^T$$

Therefore, force exerted on the infinite small object:

$$f = \sigma \cdot \vec{n} \cdot s$$

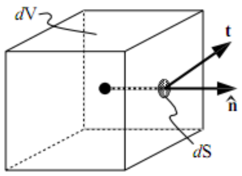
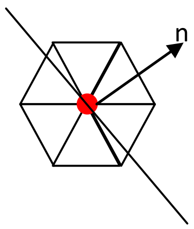


Figure 3: Given a point in the material, the traction, $\boldsymbol{t}$, that acts on the surface element, dS, of a differential volume, dV, centered around the point with outward unit normal, $\hat{\boldsymbol{n}}$, is given by $\boldsymbol{t} = \boldsymbol{\sigma}\,\hat{\boldsymbol{n}}$.

Here s represents the face area where the force is exerted. In the figure above, t is the same meaning as f.

So up to now we can get the force on the infinite small object after deformation. Then the key problem is how the judge whether this deformation would cause fracture. To do this, we compute the eigenvalue of σ. Find the largest eigenvalue λ. If $\lambda > \textit{threshold}$ then facture happens, if $\lambda < 0$, that means object is compression, it won't cause fracture.



Plane

If fracture really happens, find the eigenvector of the largest eigenvalue, represents it as direction n, this would yield a plane and intersect with the original object. Copy the vertices (the red dot), one to the upper plane, and one to the down plane. The object would re-mesh according to the plane. After that, repeating the above steps.

Summary

The steps are: Computing deformation gradient, then strain tensor, then 2nd Piola stress tensor then Cauchy stress and forces. According to Cauchy stress's eigenvalues, we can judge whether or not fracture happens. If happened, get the direction and the plane, re-mesh the object and repeat the whole process.

The merits of this method is a good visual effect, downsides is that if there is too much fractures, the efficient would be low.