CSCI 599 Physically Based Modeling for Interactive Simulation and Games

-------------------Structured deformable objects: hair

Part I. Implicit Integration

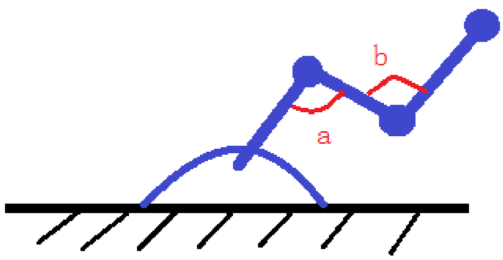
$M \cdot \ddot{y} = F(y, \dot{y}, t), \quad y \in R^n$

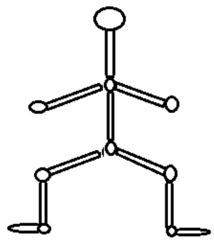
Non-linear, 2nd order

M can be identity, not have to be matrix

Eg: Mass – Spring system, Hair, FEM deformable object, cloth particular object
Notice: Water is simulate by first-order equation

Articular object, joints bend, links are rigid, say .. arm





Angle a and b will component with y

Angle $\alpha = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where x_1 is point state, x_2 is point velocity

$$\dot{x} = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} x_2 \\ M^{-1} \cdot F(x_1, x_2, t) \end{bmatrix} = \begin{matrix} G(x,t) \\ or G(x_1, x_2, t) \end{matrix}$$

Where F is variable by time t,

$\ddot{y}$: continue variable, state

x_0: initial condition;
x_1, x_2: continue state

Euler Integration

$$x_{n+1} = x_n + h \cdot \dot{x}_n = x_n + h \cdot G(x_n, t)$$

h: size of time step (seconds)

Explicit:
Unstable method, if time step h is too large, cube will blow up

Implicit: unconditionally stable
(Some times give out wrong answer)

Backward Euler

$$x_n = x_{n+1} - h\cdot\dot{x}_{n+1} = x_{n+1} - h\cdot G(x_{n+1},t)$$

Generally if f(x) = 0, x can have multiple answer if f(x) is non-linear
In order to get answer, use tangent to solve this problem

$$0 = f(x_{n+1}) = f(x_n + \triangle x_n) = f(x_n) + \frac{\delta f}{\delta x|_{x=x_n}}\cdot\triangle x_n + o(\triangle x_n{}^2)$$

$$\triangle x_n = |x_{n+1} - x_n|$$

$$J = \frac{\delta f}{\delta x|_{x=x_n}}$$

$$J\cdot\triangle x_n = -f(x_n)$$

$$G(x_{n+1},t) = G(x_n + \triangle x_n, t) = G(x_n) + \frac{\delta G}{\delta x|_{x=x_n}}\cdot\triangle x_n$$

$$x_n = x_{n+1} - h\cdot(G(x_n,t) + \frac{\delta G}{\delta x|_{x=x_n}}\cdot\triangle x_n)$$

$$\because x_{n+1} - x_n = \triangle x_n$$

$$\therefore 0 = \triangle x_n - h\cdot(G(x_n,t) + \frac{\delta G}{\delta x|_{x=x_n}}\cdot\triangle x_n)$$

$$(I - h\cdot\frac{\delta G}{\delta x|_{x=x_n}})\cdot\triangle x_n = h\cdot G(x_n,t)$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Where *I* is a unit matrix

Use Jacob to compute $\frac{\delta G}{\delta x}$, and need $\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y}$

Usually, $\dot{y}$ is the Damping force

In reality, x and y are not independent, but during calculation, we can treat them as independent.

Part II. Hair

Geometry different

Over 150,000 numbers of hairs on human head.

See SIGGRAPH paper demo.

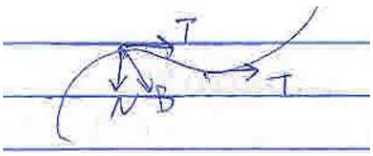
Buckling: The phenomenon of abrupt and radical changes is referred to as buckling.

Figure 2: Left, close view of a hair fiber (root upwards) showing the cuticle covered by overlapping scales. Right, bending and twisting instabilities observed when compressing a small wisp. [1]

1. Dynamics[1]

   Super-helicis: new dynamic model

   A curve in 3D

   FRENET FRAME

   T---tangent; N---normal; S---arc-length

   $$T = \frac{\delta x}{\delta s}$$

   $$N = \frac{\frac{\delta T}{\delta s}}{\left\| \frac{\delta T}{\delta s} \right\|}$$

   Bi-Normal: $B = T \times N$

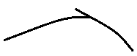
   Radiant: $R = \frac{1}{\left\| \frac{\delta T}{\delta s} \right\|}$

2. Collision detection and contract with friction

   Select around 100 strands of hair to calculate their physical states

3. Simulate 10^5 hairs based on the previous 100 strands of hair

4. Render:

    ,

   When two hairs contact

   it has penalty affect: compute over-lap -> push each other opposite out

Torque, is the tendency of a force to rotate an object about an axis, fulcrum, or pivot. It is can be thought as a twist[2].

Reference:

[1]F. Bertails et al.: Super-Helices for Predicting the Dynamics of Natural Hair, SIGGRAPH 2006

[2]Torque explanation: http://en.wikipedia.org/wiki/Torque