The Jello Cube
Assignment 1, CS599, Spring 2011

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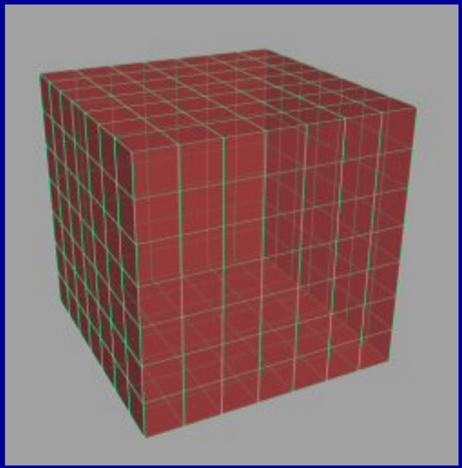
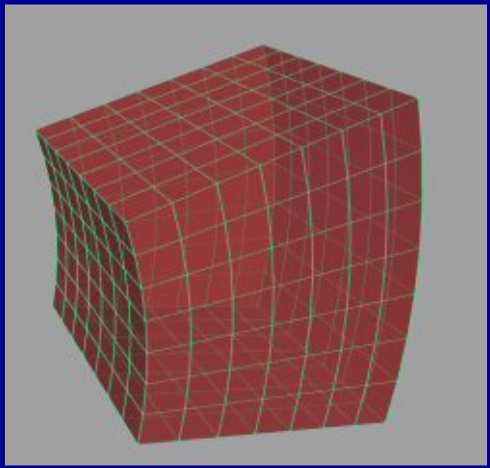
The jello cube



Undeformed cube



Deformed cube

- The jello cube is elastic,
- Can be bent, stretched, squeezed, …,
- Without external forces, it eventually restores to the original shape.

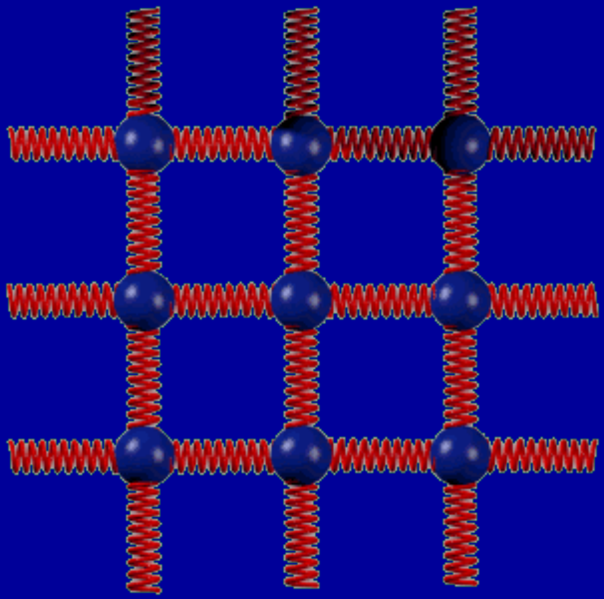
Mass-Spring System

- Several mass points
- Connected to each other by springs
- Springs expand and stretch, exerting force on the mass points
- Very often used to simulate cloth
- Examples:

  A 2-particle spring system

  Another 2-particle example

  Cloth animation example



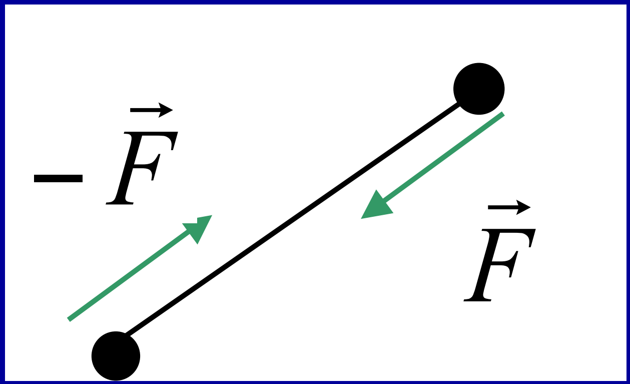
Newton's Laws

- Newton's 2nd law:

$$\vec{F} = m\vec{a}$$

  - Tells you how to compute acceleration, given the force and mass

- Newton's 3rd law: If object A exerts a force F on object B, then object B is at the same time exerting force -F on A.

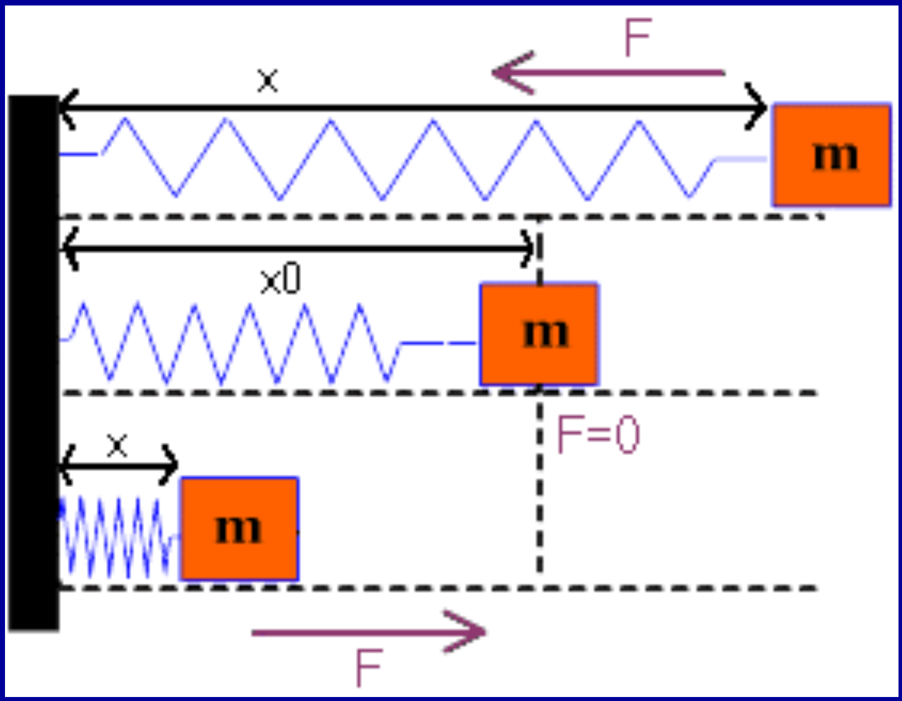


Single spring

- Obeys the *Hook's law*:

  $$F = k\,(x - x_0)$$

- x_0 = rest length
- k = spring elasticity (aka stiffness)
- For $x<x_0$, spring wants to extend
- For $x>x_0$, spring wants to contract



Hook's law in 3D

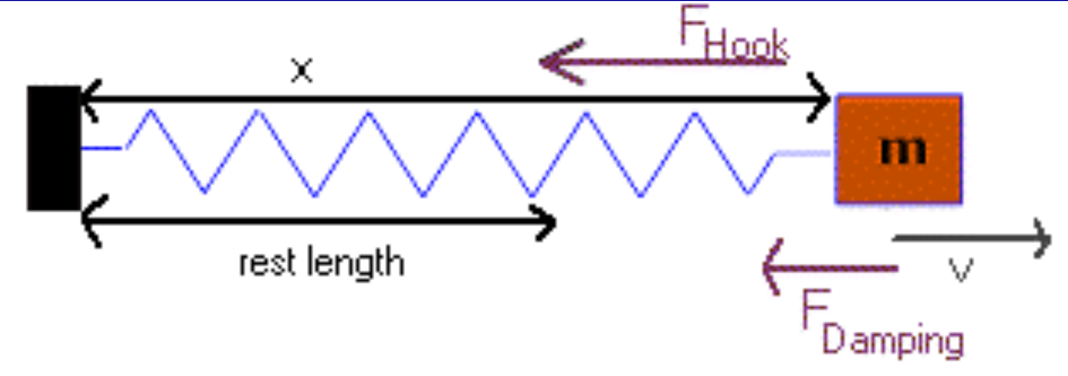
- Assume A and B two mass points connected with a spring.
- Let L be the vector pointing from B to A
- Let R be the spring rest length
- Then, the elastic force exerted on A is:

$$\vec{F} = -k_{Hook}(|\vec{L}| - R)\frac{\vec{L}}{|\vec{L}|}$$

Damping

- Springs are not completely elastic
- They absorb some of the energy and tend to decrease the velocity of the mass points attached to them
- Damping force depends on the velocity:

$$\vec{F} = -k_d \vec{v}$$



- k_d = damping coefficient
- k_d different than k_{Hook} !!

Damping in 3D

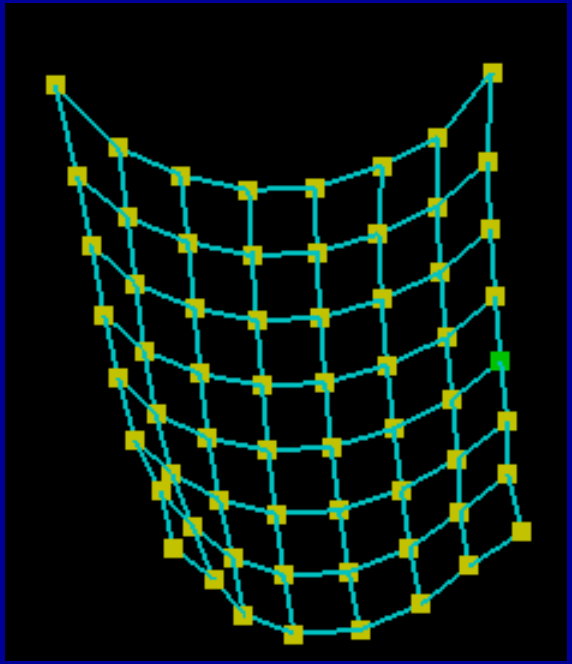
- Assume A and B two mass points connected with a spring.
- Let L be the vector pointing from B to A
- Then, the damping force exerted on A is:

$$\vec{F} = -k_d \frac{(\vec{v}_A - \vec{v}_B) \cdot \vec{L}}{|\vec{L}|} \frac{\vec{L}}{|\vec{L}|}$$

- Here v_A and v_B are velocities of points A and B
- Damping force always OPPOSES the motion

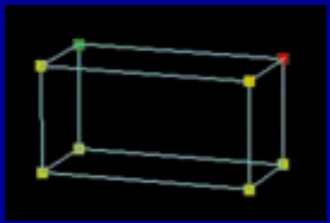
A network of springs

- Every mass point connected to some other points by springs
- Springs exert forces on mass points
  - Hook’s force
  - Damping force
- Other forces
  - External force field
    - » Gravity
    - » Electrical or magnetic force field
  - Collision force

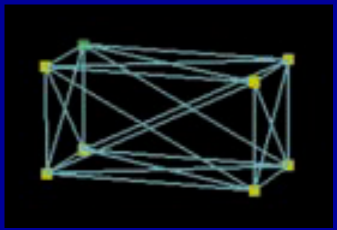


How to organize the network (for jello cube)

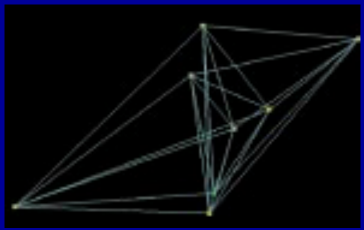
- To obtain stability, must organize the network of springs in some clever way
- Jello cube is a 8x8x8 mass point network
- 512 discrete points
- Must somehow connect them with springs



Basic network



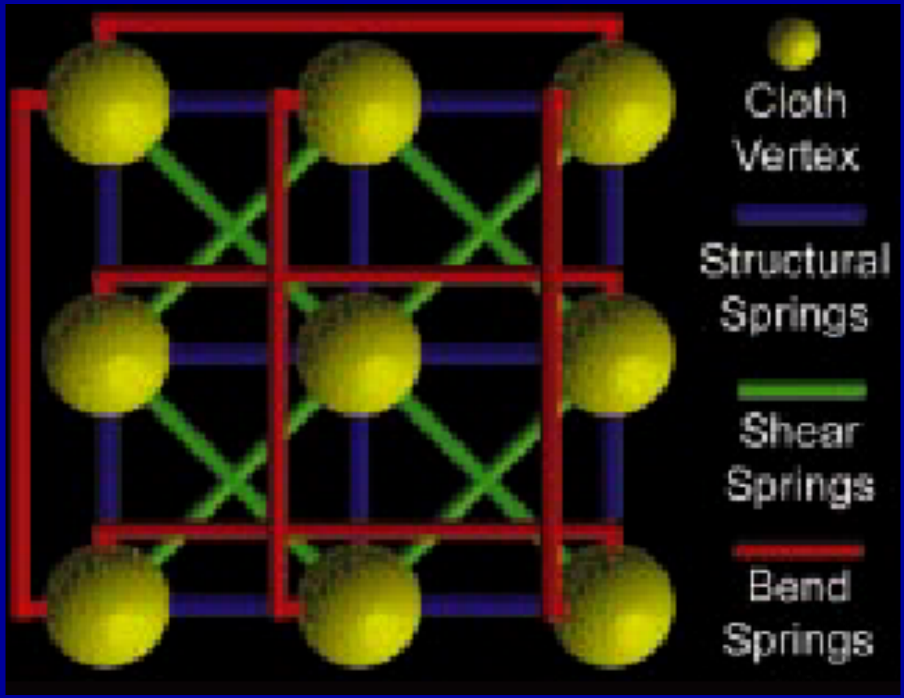
Stable network



Network out of control

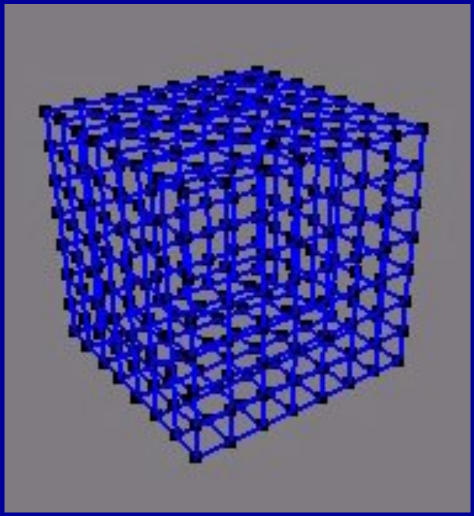
Solution:
Structural, Shear and Bend Springs

- There will be three types of springs:
  - Structural
  - Shear
  - Bend
- Each has its own function



Structural springs

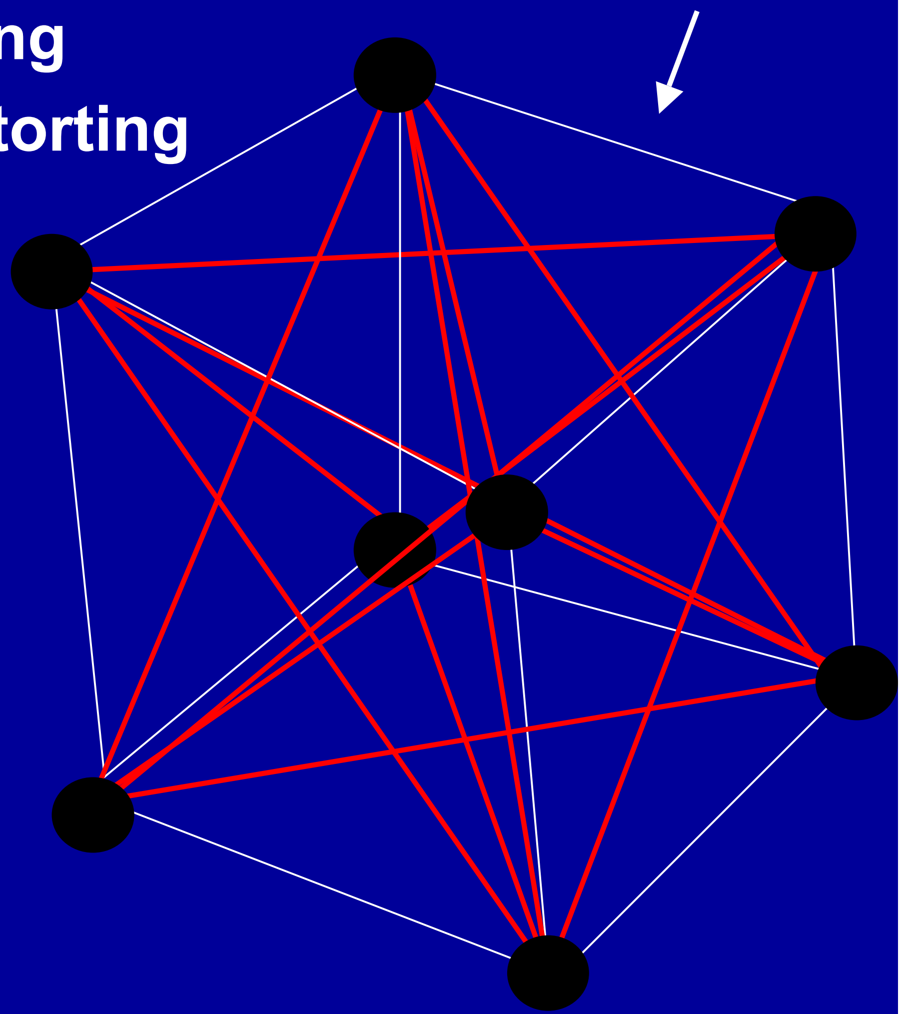
- Connect every node to its 6 direct neighbours
- Node (i,j,k) connected to
  - (i+1,j,k), (i-1,j,k), (i,j-1,k), (i,j+1,k), (i,j,k-1), (i,j,k+1)
    (for surface nodes, some of these neighbors might not exists)
- Structural springs establish the basic structure of the jello cube
- The picture shows structural springs for the jello cube. Only springs connecting two surface vertices are shown.

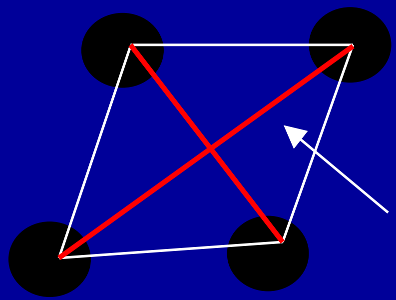


Shear springs

- Disallow excessive shearing
- Prevent the cube from distorting
- Every node (i,j,k) connected to its diagonal neighbors
- Structural springs = white
- Shear springs = red

A 3D cube
(if you can't see it immediately, keep trying)

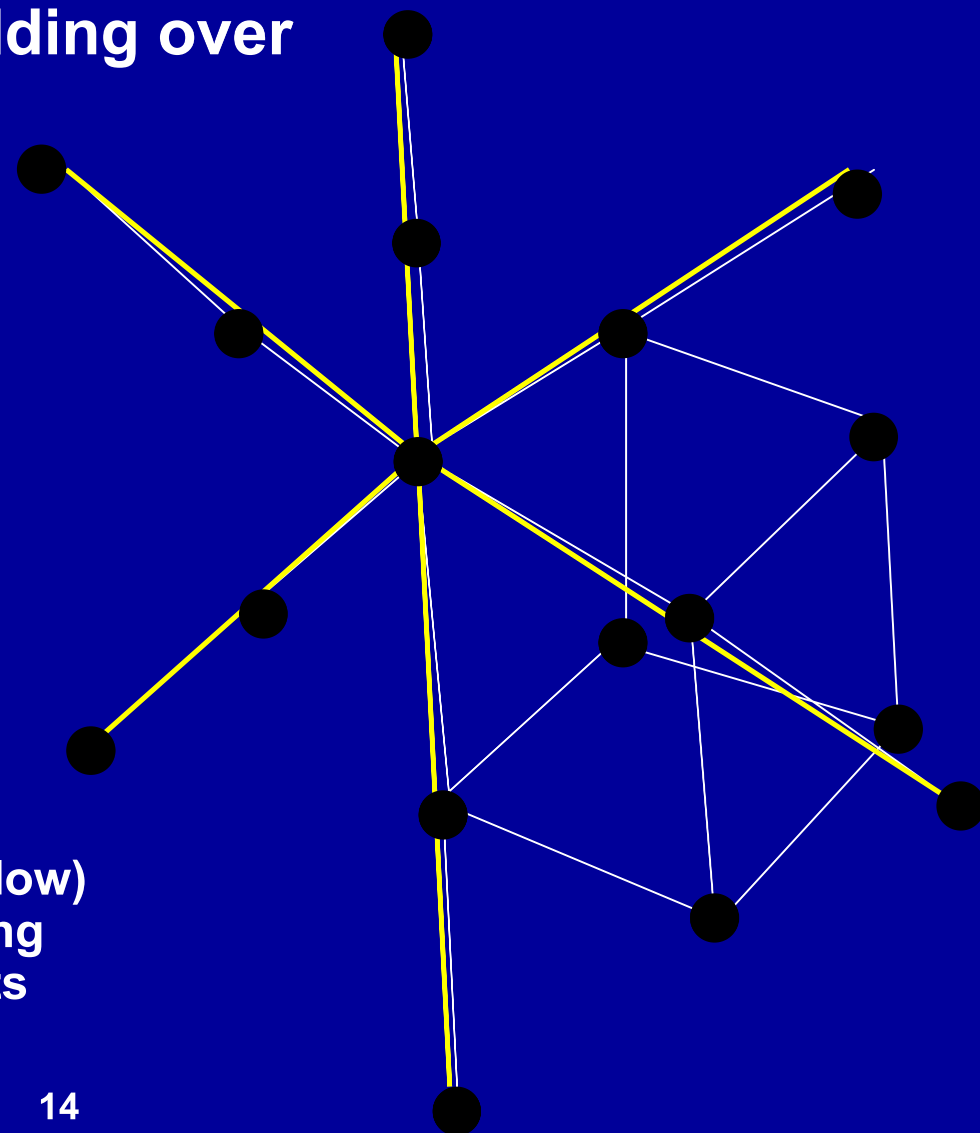


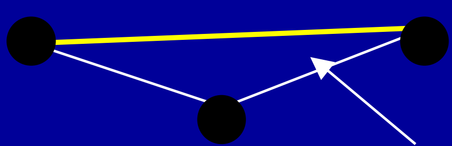


Shear spring (red) resists stretching and thus prevents shearing

Bend springs

- **Prevent the cube from folding over**
- **Every node connected to its second neighbor in every direction (6 connections per node, unless surface node)**
- **white=structural springs**
- **yellow=bend springs (shown for a single node only)**





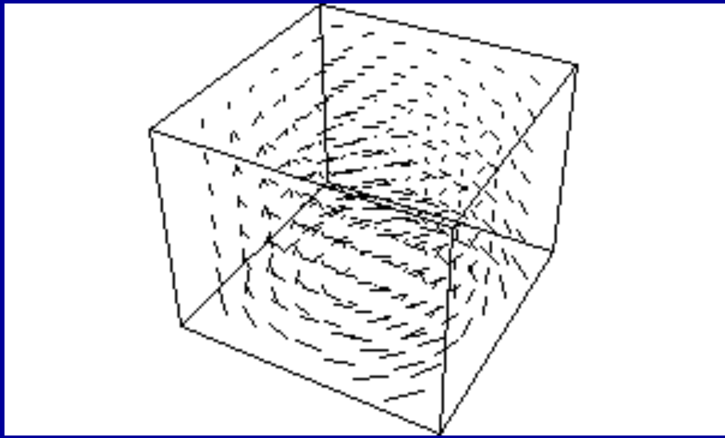
Bend spring (yellow) resists contracting and thus prevents bending

External force field

- If there is an external force field, add that force to the sum of all the forces on a mass point

$$\vec{F}_{total} = \vec{F}_{Hook} + \vec{F}_{damping} + \vec{F}_{force\ field}$$

- There is one such equation for every mass point and for every moment in time



Collision detection

- The movement of the jello cube is limited to a bounding box
- Collision detection easy:
  - Check all the vertices if any of them is outside the box
- Inclined plane:
  - Equation:

$$F(x,y,z) = ax + by + cz + d = 0$$

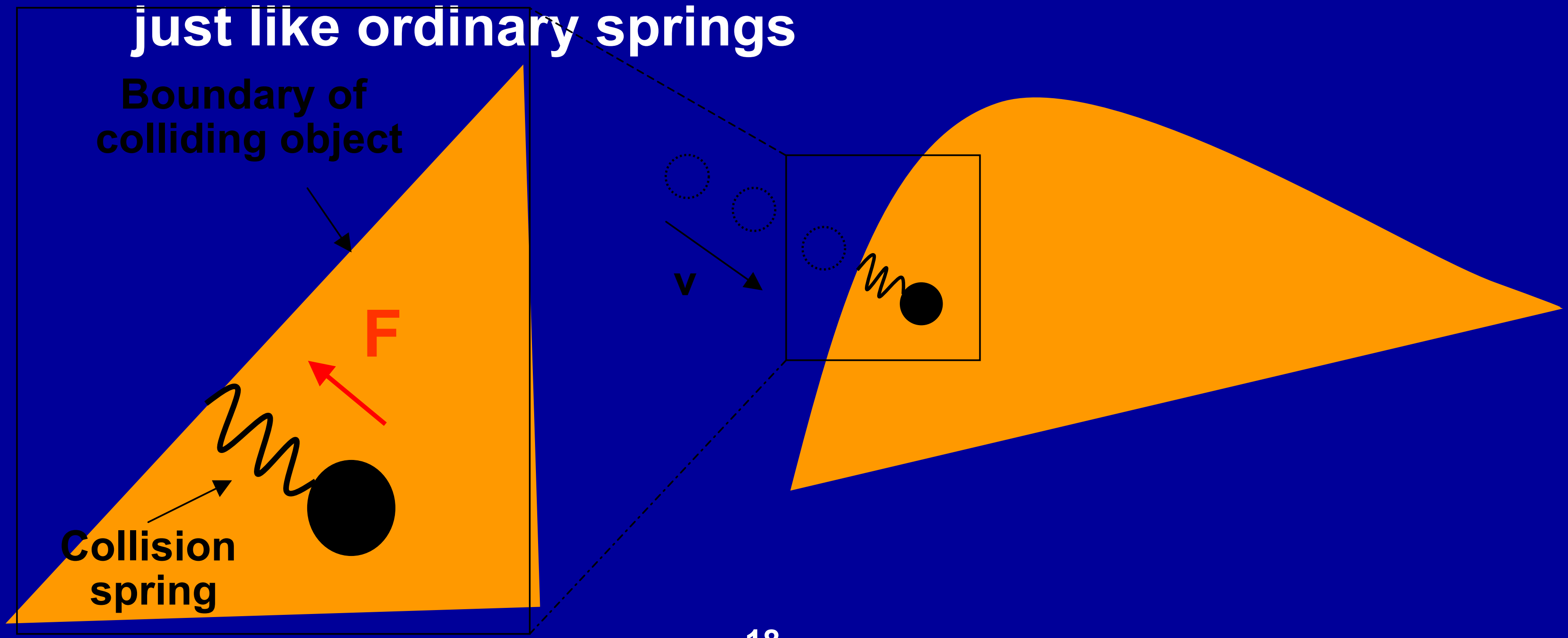
  - Initially, all points on the same side of the plane
  - $F(x,y,z)>0$ on one side of the plane and $F(x,y,z)<0$ on the other
  - Can check all the vertices for this condition

Collision response

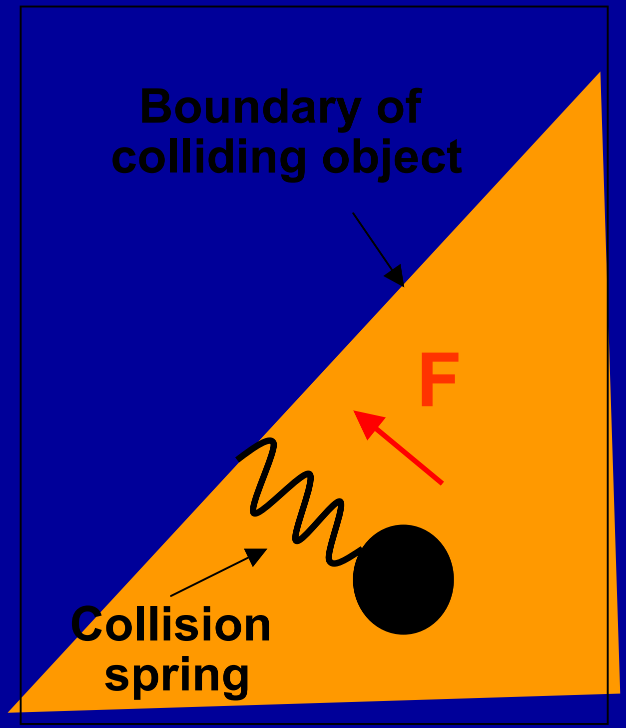
- When collision happens, must perform some action to prevent the object penetrating even deeper
- Object should bounce away from the colliding object
- Some energy is usually lost during the collision
- Several ways to handle collision response
- We will use the *penalty method*

The penalty method

- When collision happens, put an artificial *collision spring* at the point of collision, which will push the object backwards and away from the colliding object
- Collision springs have elasticity and damping, just like ordinary springs



Penalty force



- Direction is normal to the contact surface
- Magnitude is proportional to the amount of penetration
- Collision spring rest length is zero

Integrators

- Network of mass points and springs
- Hook's law, damping law and Newton's 2nd law give acceleration of every mass point at any given time
- F=ma
  - Hook's law and damping provide F
  - 'm' is point mass
  - The value for a follows from F=ma
- Now, we know acceleration at any given time for any point
- Want to compute the actual motion

Integrators (contd.)

- The equations of motion:

$$\frac{d\vec{x}}{dt} = \vec{v}$$

$$\frac{d^2\vec{x}}{dt^2} = \frac{d\vec{v}}{dt} = \vec{a}(t) = \frac{1}{m}(\vec{F}_{Hook} + \vec{F}_{damping} + \vec{F}_{force\ field})$$

- x = point position, v = point velocity, a = point acceleration
- They describe the movement of any single mass point
- F_{hook}=sum of all Hook forces on a mass point
- $F_{damping}$ = sum of all damping forces on a mass point

Integrators (contd.)

- When we put these equations together for all the mass points, we obtain a system of ordinary differential equations.
- In general, impossible to solve analytically
- Must solve numerically
- Methods to solve such systems numerically are called *integrators*
- Most widely used:
  - Euler
  - Runge-Kutta 2nd order (aka the midpoint method) (RK2)
  - Runge-Kutta 4th order (RK4)

Integrator design issues

- Numerical stability
  - If time step too big, method "explodes"
  - t = 0.001 is a good starting choice for the assignment
  - Euler much more unstable than RK2 or RK4
    - » Requires smaller time-step, but is simple and hence fast
  - Euler rarely used in practice
- Numerical accuracy
  - Smaller time steps means more stability and accuracy
  - But also means more computation
- Computational cost
  - Tradeoff: accuracy vs computation time

Integrators (contd.)

- RK4 is often the method of choice
- RK4 very popular for engineering applications
- The time step should be inversely proportional to the square root of the elasticity *k* [*Courant condition*]
- For the assignment, we provide the integrator routines (Euler, RK4)
  - void Euler(struct world * jello);
  - void RK4(struct world * jello);
  - Calls to there routines make the simulation progress one time-step further.
  - State of the simulation stored in ‘jello’ and automatically updated

Tips

- Use double precision for all calculations (double)
- Do not overstretch the z-buffer
  - It has finite precision
  - Ok: gluPerspective(90.0,1.0,0.01,1000.0);
  - Bad: gluPerspective(90.0,1.0,0.0001,100000.0);
- Choosing the right elasticity and damping parameters is an art
  - Trial and error
  - For a start, can set the ordinary and collision parameters the same
- Read the webpage for updates