Lagrange Mechanics and FEM 02/09/2011

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Lagrange mechanics and FEM are used to model deformations of solid objects.

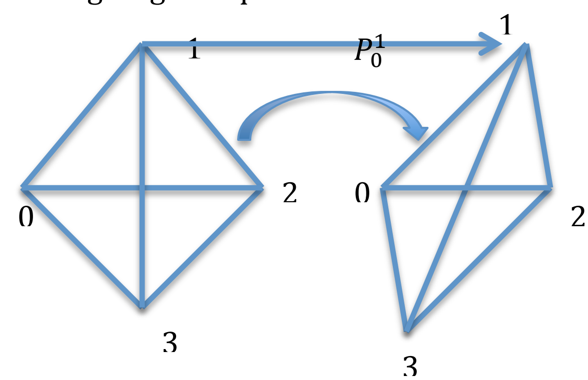
Legrangian Mechanics:

Generally for motion calculations we use the Newton's law of motion, $F = m\ddot{x}$.

Legrangian Mechanics is an extension of Newtonian mechanics. In Legrangian mechanics, the motion of a system of particles is described by solving the Lagrange Equations which use generalized coordinates. We obtain an equation of the form:

$q = [\alpha_1, \alpha_{2,} \dots, \alpha_n]^T$, $\ddot{q} = F(q)$

The equations of motion in Lagrangian Mechanics are derived from the principle of virtual work. These equations of motion are called Euler-Lagrangian equations and are of the form:



$M\ddot{u} + D(u, \dot{u}) + R(u) = f$

Here u is the displacement vector,

M is the mass matrix

$D(u, \dot{u})$ are damping forces

$R(u)$ are internal deformation forces

f is the external forces.

Such a system distributes its total mass over its vertices.

For deformation, we use the Lagrange equation

$M\ddot{u} + D\ddot{u} + fint(u) = fext(t)$

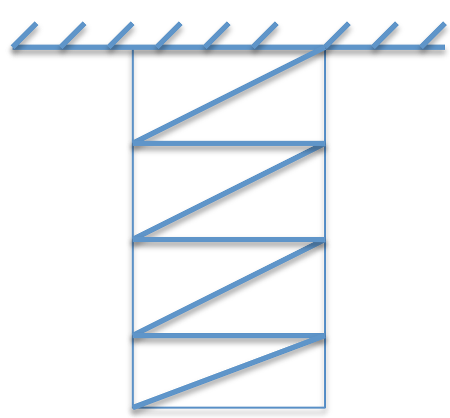
where $fint(u) = K.u$, K is 12 x 12

Also for rotation of each vertex,

$fint(u) = R_e.K\left(R_e{}^{-1}X - X_0\right)$

where $X_0 = \begin{bmatrix} P_0^0 \\ P_0^1 \\ P_0^2 \\ P_0^3 \end{bmatrix}$

and $X = X_0 + \mu$



Let

$C = [a_1, a_2, a_3, a_{4,}]^T$

$D = [Ra_1, Ra_2, Ra_3, Ra_{4,}]^T$

Solving for D = _ . C,

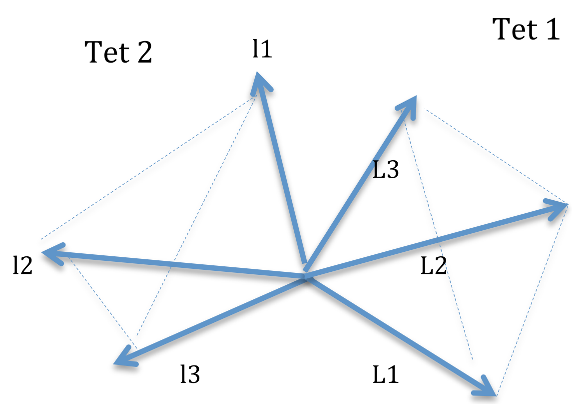
We get $D = \underbrace{\begin{bmatrix} R & & & \\ & R & & \\ & & R & \\ & & & R \end{bmatrix}}_{Re} . C$

Where Inverse of Rotation Matrix = Transpose of the Matrix

For Q – R Decomposition,

A . L_i = $l_{i=1,2,3}$

B. L_i = e_i



$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$L_i = B^{-1}. e_i$

$B^{-1} = C$

$L_i = C. e_i$

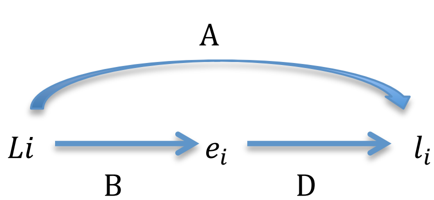
$L_1 = C. e_1$ = First Column of C

C = [L_1 L_2 L_3]

D. e_i= l_i , D = [l_1 l_2 l_3]

Also $B = C^{-1}$

First Apply B then D



A = D.B

= D. C^{-1}

For linear transformation that aligns Tet1 to Tet2, use polar decomposition,
Write any matrix as Rotation x Symmetric,
Here Symmetric matrix is the Stretch matrix for the object.

FEM:
To use lagrange mechanics for general FEM, use the equation:

$$x = X + \sum_{i=1}^{n} \psi_i \ (X) \ \mu_i$$

where ψ_i is the FEM shape function

$$\psi_i \ (X_j) = \ ð_{i\ j} \ = \begin{Bmatrix} 1, & if \ i = j \\ 0, & if \ i \ \neq j \end{Bmatrix}$$
