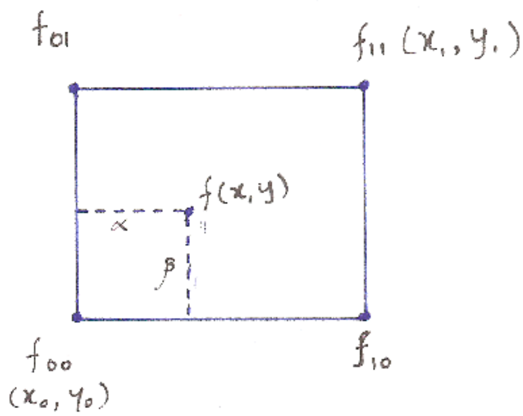
CSCI 599 Physically Based Modeling for Interactive Simulation and Games.

Topic: Cloth Simulation.

Interpolation Overview:

1) Trilinear Interpolation :
   If we know the color values at four points (viz. f00, f01, f10 and f11), how do we interpolate the color at f?

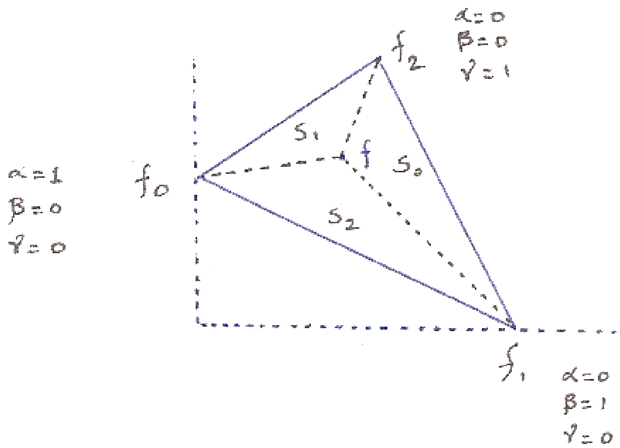


$$\alpha = \frac{x - x_0}{x_1 - x_0} \quad \ldots\ldots \text{(I)}$$

$$\beta = \frac{y - y_0}{y_1 - y_0} \quad \ldots\ldots \text{(II)}$$

$$f = (1-\alpha)\cdot(1-\beta)\, f_{00} + \alpha \cdot (1-\beta)\, f_{10} + \alpha \cdot \beta \cdot f_{11} + (1-\alpha)\cdot \beta\, f_{01}$$

2) Barycentric Interpolation:
This technique is used to interpolate color values in a triangle.



Property for α, β and γ

1) $\alpha+\beta+\gamma = 1$

2) $\alpha, \beta, \gamma \geqslant 0$

S_0, S_1, S_2 areas of triangles ; S is the total area.

$$\alpha = \frac{S_0}{S} \qquad \beta = \frac{S_1}{S} \qquad \gamma = \frac{S_2}{S}$$

So, the value at f is given as -

$$f = \alpha f_0 + \beta f_1 + \gamma f_2$$

Cloth Simulation (ref "*Large Steps in Cloth Simulation*" – Baraff and Witkin, SIGGRAPH 1998)

Problems in Cloth Simulation:

- Large Time step which affects numerical stability.
- Use of Explicit Integration techniques results in slower results.

This paper helps us in a better cloth simulation because:

- Use of implicit Integration method which enforces constraints on individual cloth particles.
- The technique used models cloth as triangular mesh, with internal cloth forces derived from a simple formulation.

Why Implicit Integration?

- Cloth strongly *resists* stretching motions while being comparatively permissive in *allowing* bending or shearing motions. This results in a "stiff" underlying differential equation of motion.
- Explicit methods are ill-suited to solving stiff equations because *they require many small steps to stably advance the simulation forward in time*.
- Thus, the computational cost of an explicit method greatly limits the realizable resolution of the cloth.

Various Forces on Cloth:

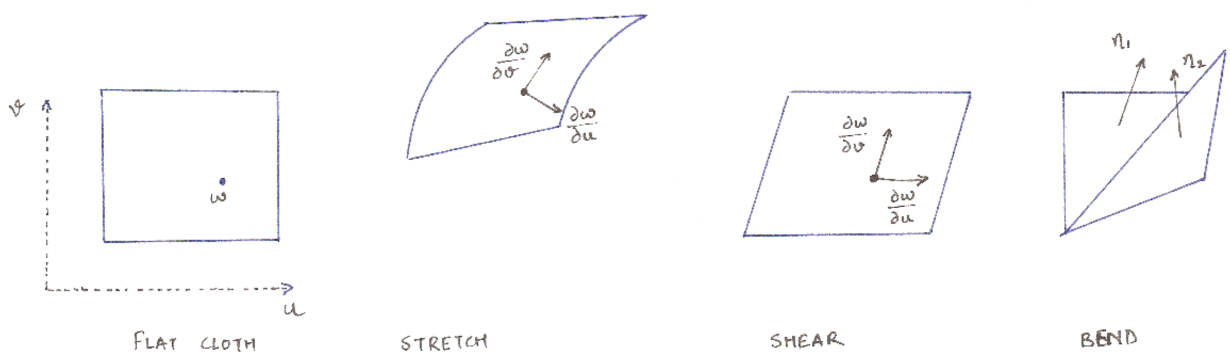
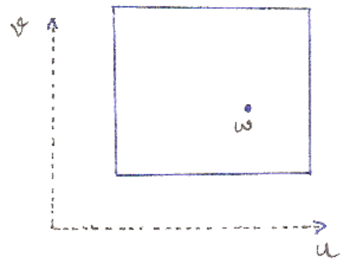


figure 1. Forces on cloth

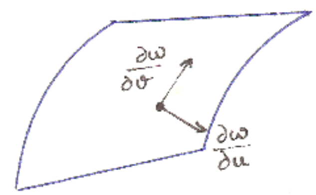


Consider every cloth particle has a changing position x in world space, and a fixed plane coordinate u - v.

Also consider we have a single continuous function **w** that maps from plane coordinates to world space.

1) Stretch Force:

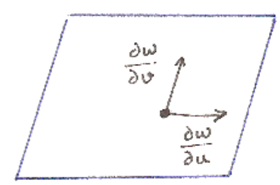
- Stretch can be measured at any point in the cloth surface by examining the derivatives $\frac{\partial w}{\partial v}$ and $\frac{\partial w}{\partial u}$ at that point.
- The magnitude of **w** describes the *stretch* or *compression* in the u direction (for e.g. the material is unstretched wherever ||w|| = 1)
- Thus the stretch energy is given by:



$$C(x) = a \cdot \left[\begin{array}{c} \left\| \frac{\partial w}{\partial v} \right\| - 1 \\ \hline \left\| \frac{\partial w}{\partial u} \right\| - 1 \end{array}\right]$$

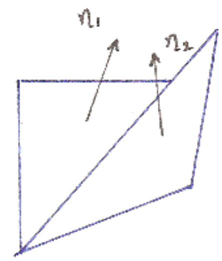
2) Shear Force:

- Cloth resists shearing in the plane.
- We can measure the extent to which cloth has sheared in a triangle by considering the inner product $\left(\frac{\partial w}{\partial u}\right)^T \cdot \frac{\partial w}{\partial v}$.
- In its rest state, this product is zero.
- Thus the shear energy us given by:



$$C(x) = b \cdot \left(\frac{\partial w}{\partial u}\right)^T \cdot \frac{\partial w}{\partial v}$$

3) Bend Force:
- Bend is measured in between pair of adjacent triangles.
- Let n1 and n2 denote the unit normal of two triangles and angle Θ between the two faces.
- Thus the Bend Energy can be given by:



$$C(x) = \theta$$

4) Damping Force:
This can be given as:

$$f = -k \cdot \frac{\partial C}{\partial x} \cdot C(x)$$

$$E = \frac{k}{2} \cdot C^T \cdot C(x)$$

And finally,

$$f = -\frac{\partial E}{\partial x}$$

Implicit Integration:

Considering the following notations

M = diagonal mass matrix.

F = Total Force.

We can write the acceleration function as :

$$M\dot{y} = F(y, \dot{y}, t) \quad \cdots \text{(I)}$$

- To transform equation (I) into a first-order differential equation,

$$x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} \qquad \text{Let } \begin{array}{l} y = x_1 \\ \dot{y} = x_2 \end{array}$$

Hence, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

now,

$$\dot{x} = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} x_2 \\ M^{-1} F(x_1, x_2, t) \end{bmatrix}$$

Let,

$$G(x_1, x_2, t) = \begin{bmatrix} x_2 \\ M^{-1} F(x_1, x_2, t) \end{bmatrix}$$

$$x_n = x_{n+1} - h \cdot G(x_{n+1})$$ by Newton Raphson method.

$$\left(I - h \cdot \frac{\partial G}{\partial x}\bigg|_{x = x_n} \right) \Delta x_n = h \cdot G(x_{n+t})$$

Where, $\Delta x_n = x_{n+1} - x_n$

References:

Baraff and Witkin “Large Steps in Cloth Simulation” http://www.cs.cmu.edu/~baraff/papers/sig98.pdf

Barycentric Interpolation:
http://www.soe.ucsc.edu/classes/cmps160/Fall10/resources/barycentricInterpolation.pdf