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CSCI 599.

LECTURE NOTES → FEB 2nd 2011.

Previous lecture: Cloth Simulation.

Structured Deformable Objects: Hair.

⇒ Hair Simulation Videos.

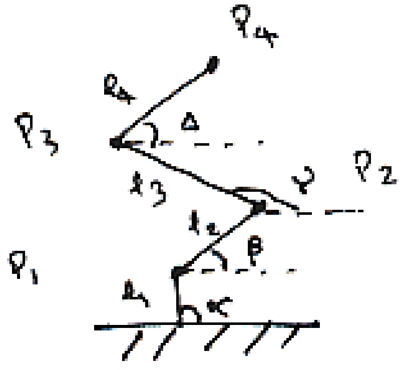
→ Hair if applied force in exactly opposite direction when in rest can buckle.

→ There is primary buckling as well as 2° buckling

→ In General, simulation has a choice between Reduced representation and Maximal representation.

Reduced Representation

Consider this hair strand, which is fixed at the bottom.



$P_1 = (l_1 \cos\alpha, l_1 \sin\alpha) = (x_1, y_1)$

$P_2 = (l_2 \cos\beta, l_2 \sin\beta) + P_1 = (x_2, y_2)$

$P_3 = (l_3 \cos\gamma, l_3 \sin\gamma) + P_2 = (x_3, y_3)$

$P_4 = (l_4 \cos\Delta, l_4 \sin\Delta) + P_3 = (x_4, y_4)$

$$q = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \Delta \end{bmatrix}$$

So q have 4 degrees of freedom.

velocity $\dot{q} = \begin{bmatrix} \dot\alpha \\ \dot\beta \\ \dot\gamma \\ \dot\Delta \end{bmatrix}$ There is 1-1 mapping between description and Degrees of freedom.

$a = \ddot{q} = f(q, \dot{q}, t)$

Most commonly used,

$$M\ddot{q} + D\dot{q} + Kq = F(t)$$

Maximal Representation

$$q = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \\ x_n \\ y_n \end{bmatrix}$$

- Maximal is easier to render.

Reduced $\xleftrightarrow{\text{interchangeable}}$ Maximal.

→ Maximal representation is constrained. if not then lengths can become unstable.

$$x_1^2 + y_1^2 = l_1^2$$
$$(x_1+x_2)^2 + (y_1+y_2)^2 = l_2^2$$
$$\vdots$$

$$a = \ddot{q} = G(q, \dot{q}, t)$$

$\Downarrow$

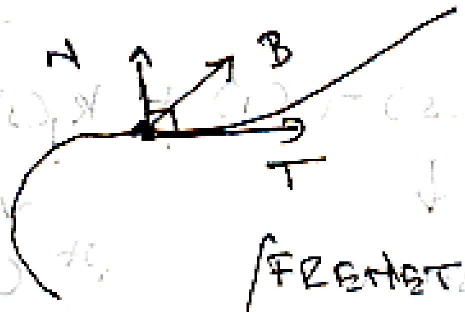
is a differential Algebraic Equation. (DAE).

FEATHER STORM's ALGORITHM

↳ helps in solving reduced representation.

Ragdoll physics uses this.

→ Hair is nothing but a curve.
Consider,



(FRENET FRAME)

Consider $P = P(s)$.

to understand s,

For circle, $x = R\cos s$

$y = R \sin s$

$z = 0$

For Helix,

$x = R\cos s$

$y = R\sin s$

$z = s$

$$\text{tangent} = \frac{ds/dt}{\|ds/dt\|} = T(s)$$

$$\text{Normal} = \frac{dT/ds}{\underbrace{\|dT/ds\|}_{\text{Curvature } K(s)}} = N(s)$$

Binormal: $B(s) = T \times N$

$n_0 = T$, $n_1 = N$, $n_2 = B$

$$n_i' = \underbrace{\Omega(s)}_{\text{DARBOO's Vector}} \times n_2$$

DARBOO's Vector.

$$\Omega(s) = \underbrace{\tau(s)}_{\text{Torsion}} T(s) + \underbrace{K_1(s)}_{\text{1st Curvature}} N(s) + \underbrace{K_2(s)}_{\text{2nd Curvature}} B(s)$$

Torsion 1st Curvature 2nd Curvature.

$$q = \begin{bmatrix} \xi_1 \\ K_{11} \\ K_{21} \\ \xi_2 \\ \vdots \end{bmatrix}$$

q_0 = Turned of all external forces.

$$E = \frac{1}{2}(q - q_0)^T A (q - q_0)$$

Mixing Matrix to stress some points.

Energy of the system.