

Huanyu Chen, Danyong Zhao, Jernej Barbic: Capturing Animation-Ready Isotropic Materials Using Systematic Poking, ACM Transactions on Graphics 42(6) (SIGGRAPH Asia 2023), Dec 2023.

The below table gives several elastic materials, as well as their f, g and h functions, wherever applicable.

| Material Model Name | Material Energy Density Function | Parameters | Val.-Landel expressible | f(x) | f''(x) | g(x) | g''(x) | h(x) | h''(x) |
|--|---|--|-------------------------|--|--|--|--|------------------------------|----------------------------------|
| Linear Corotational (Seth-Hill $\alpha = 1$) | $\mu((\lambda_1 - 1)^2 + (\lambda_2 - 1)^2 + (\lambda_3 - 1)^2) + \frac{\lambda}{2}(\lambda_1 + \lambda_2 + \lambda_3 - 3)^2$ | μ, λ | Yes | $\frac{\mu}{2}(x - 1)^2 + \frac{\lambda}{2}(x - 1)(x - 5)$ | $2\mu + \lambda$ | $\lambda(x - 1)$ | 0 | 0 | 0 |
| St. Venant-Kirchhoff (Seth-Hill $\alpha = 2$) | $\frac{\mu}{4}((\lambda_1^2 - 1)^2 + (\lambda_2^2 - 1)^2 + (\lambda_3^2 - 1)^2) + \frac{\lambda}{8}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)^2$ | μ, λ | Yes | $\frac{\mu}{4}(x^2 - 1)^2 + \frac{\lambda}{8}(x^2 - 1)(x^2 - 5)$ | $\frac{\mu}{2}(3x^2 - 1) + \frac{3}{2}\lambda(x^2 - 1)$ | $\frac{\lambda}{4}(x^2 - 1)$ | $\frac{\lambda}{2}$ | 0 | 0 |
| Hencky (Seth-Hill $\alpha = 0$) | $\mu(\log^2 \lambda_1 + \log^2 \lambda_2 + \log^2 \lambda_3) + \frac{\lambda}{2} \log^2(\lambda_1 \lambda_2 \lambda_3)$ | μ, λ | Yes | $\mu \log^2 x$ | $2\mu \frac{1 - \log x}{x^2}$ | 0 | 0 | $\frac{\lambda}{2} \log^2 x$ | $\lambda \frac{1 - \log x}{x^2}$ |
| Seth-Hill family | $\frac{\mu}{\alpha^2}((\lambda_1^\alpha - 1)^2 + (\lambda_2^\alpha - 1)^2 + (\lambda_3^\alpha - 1)^2) + \frac{\lambda}{2\alpha^2}(\lambda_1^\alpha + \lambda_2^\alpha + \lambda_3^\alpha - 3)^2$ | μ, λ, α | Yes | $\frac{\mu}{\alpha^2}(x^\alpha - 1)^2 + \frac{\lambda}{2\alpha^2}(x^\alpha - 1)(x^\alpha - 5)$ | $(2\mu + \lambda) \frac{2\alpha - 1}{\alpha} x^{2\alpha - 2} - (2\mu + 3\lambda) \frac{\alpha - 1}{\alpha} x^{\alpha - 2}$ | $\frac{\lambda}{\alpha^2}(x^\alpha - 1)$ | $\lambda \frac{\alpha - 1}{\alpha} x^\alpha$ | 0 | 0 |
| Hill family | $\mu(f^2(\lambda_1) + f^2(\lambda_2) + f^2(\lambda_3)) + \frac{\lambda}{2}(f(\lambda_1) + f(\lambda_2) + f(\lambda_3))^2$ | μ, λ, f | No | | | | | | |
| Neo-Hookean (standard version) | $\frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) - \mu \log(\lambda_1 \lambda_2 \lambda_3) + \frac{\lambda}{2} \log^2(\lambda_1 \lambda_2 \lambda_3)$ | μ, λ | Yes | $\frac{\mu}{2}(x^2 - 1) - \mu \log x$ | $\mu \left(1 + \frac{1}{x^2}\right)$ | 0 | 0 | $\frac{\lambda}{2} \log^2 x$ | $\lambda \frac{1 - \log x}{x^2}$ |
| Neo-Hookean (Ogden) | $\frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) - \mu \log(\lambda_1 \lambda_2 \lambda_3) + \frac{\lambda}{2}(\lambda_1 \lambda_2 \lambda_3 - 1)^2$ | μ, λ | Yes | $\frac{\mu}{2}(x^2 - 1) - \mu \log x$ | $\mu \left(1 + \frac{1}{x^2}\right)$ | 0 | 0 | $\frac{\lambda}{2}(x - 1)^2$ | λ |
| Stable Neo-Hookean ([Smith 2018]) | $\frac{\mu_{\text{pixar}}}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) - \frac{\mu_{\text{pixar}}}{2} \log(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + 1) + \frac{\lambda_{\text{pixar}}}{2}(\lambda_1 \lambda_2 \lambda_3 - \alpha)^2$ | $\mu_{\text{pixar}}, \lambda_{\text{pixar}}$ $\alpha = 1 + \frac{3\mu}{4\lambda}$ | No | | | | | | |
| STS material ([Pai 2018]) | $\frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) - \mu \log(\lambda_1 \lambda_2 \lambda_3) + \frac{\lambda}{2} \log^2(\lambda_1 \lambda_2 \lambda_3) + \frac{\mu_4}{8}((\lambda_1^2 - 1)^4 + (\lambda_2^2 - 1)^4 + (\lambda_3^2 - 1)^4)$ | μ, μ_4, λ | Yes | $\frac{\mu}{2}(x^2 - 1) - \mu \log x + \frac{\mu_4}{8}(x^2 - 1)^4$ | $\mu \left(1 + \frac{1}{x^2}\right) + \mu_4(x^2 - 1)^2(7x^2 - 1)$ | 0 | 0 | $\frac{\lambda}{2} \log^2 x$ | $\lambda \frac{1 - \log x}{x^2}$ |
| Ogden | $\sum_{p=1}^N \frac{\mu_p}{\alpha_p} (\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3)$ | μ_p, α_p | Yes | $\sum_{p=1}^N \frac{\mu_p}{\alpha_p} (x^{\alpha_p} - 1)$ | $\sum_{p=1}^N \mu_p (\alpha_p - 1) x^{\alpha_p - 2}$ | 0 | 0 | 0 | 0 |
| Mooney-Rivlin | $C_1(\lambda_1 \lambda_2 \lambda_3)^{-2/3}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) + C_2(\lambda_1 \lambda_2 \lambda_3)^{-4/3}(\lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 - 3)$ | C_1, C_2 | No Yes if $J = 1$ | $C_1(x^2 - 1)$ | | $C_2(x^2 - 1)$ | | | |