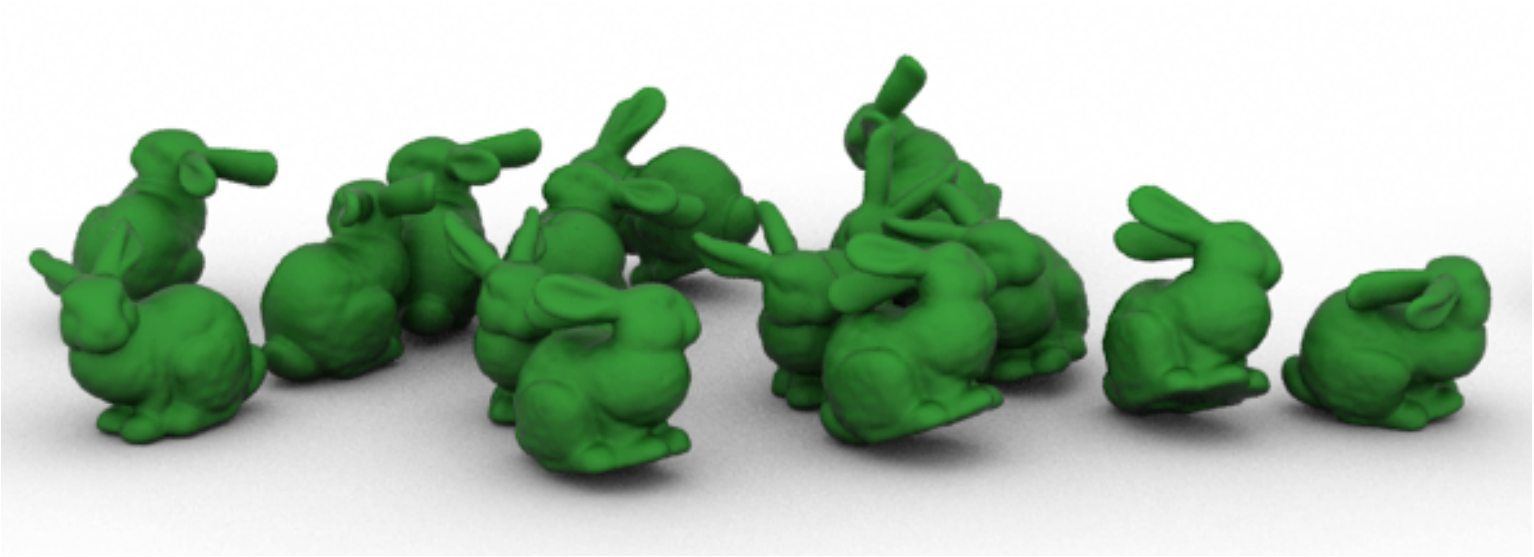
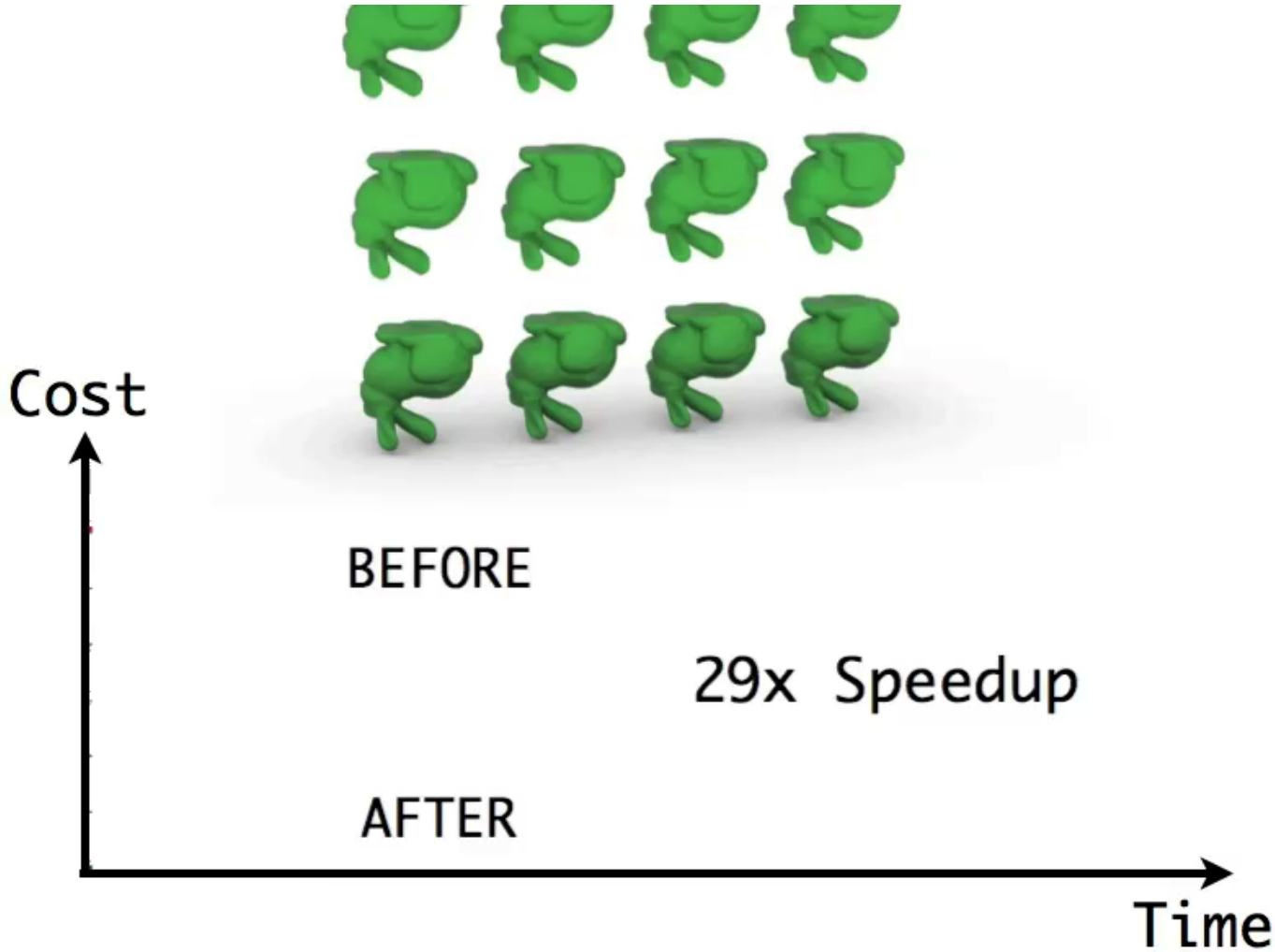


Subspace Self-Collision Culling

Jernej Barbic, University of Southern California

Doug James, Cornell University

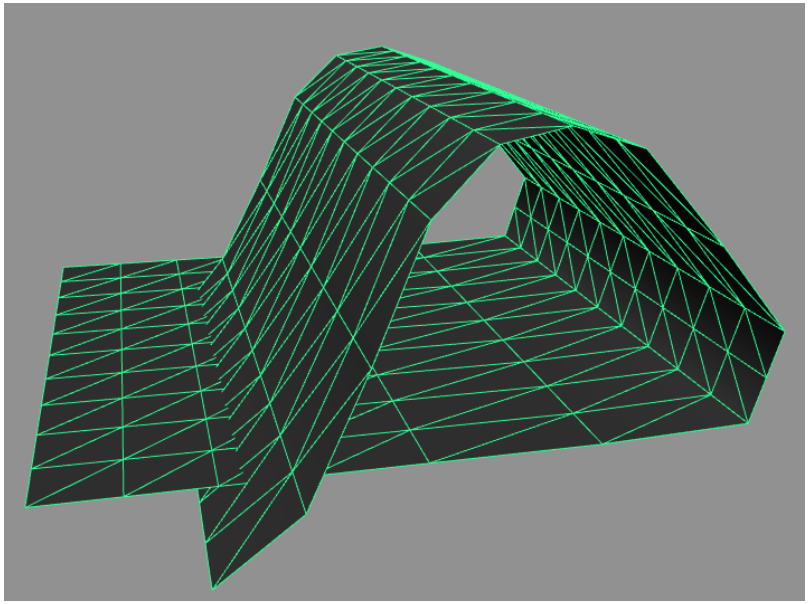
Fast self-collision detection for reduced deformable models



Self-collisions: definition

Deformable model is self-colliding iff

there exist non-neighboring intersecting triangles.



Self-collision detection (SCD)

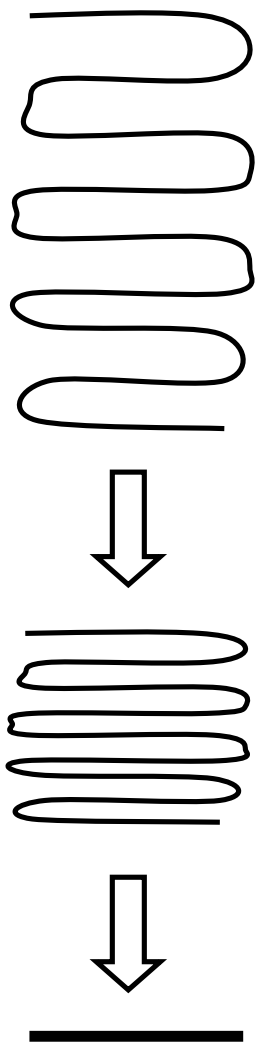
INPUT:

Triangle mesh + a deformation

OUTPUT:

List of colliding non-neighboring triangles

Self-collision detection is O(N^2) in the worst case



N **triangles total**

compress to plane

O(N^2) triangles in contact

Under a structured deformation field, model is less prone to self-contact

Linear modal simulation

Intuition

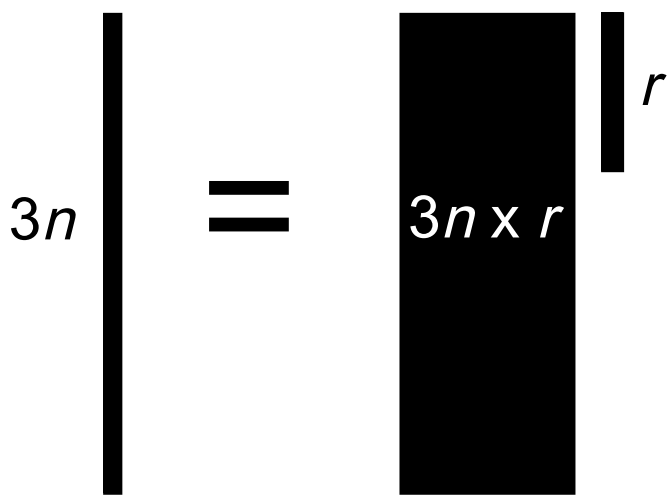
If the mesh does not deform enough, it cannot self-collide.

Do not need to look at all triangles

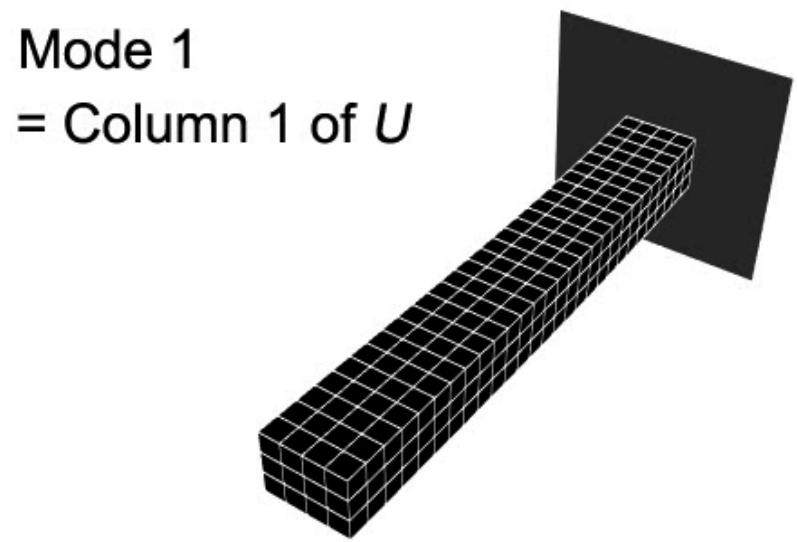
$\Longrightarrow$ sublinear cost.

Reduced-order deformations

$$\mathbf{u} = \mathbf{U}\mathbf{q}$$

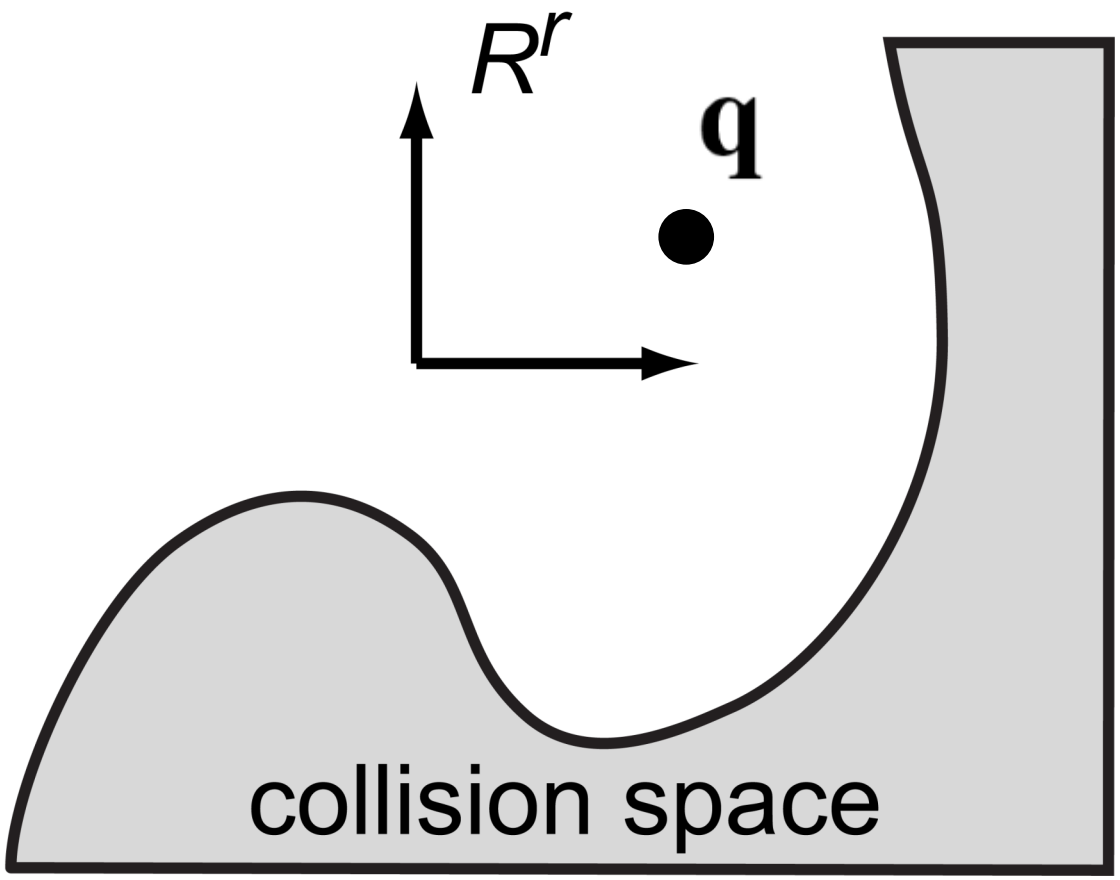


$r << 3n$



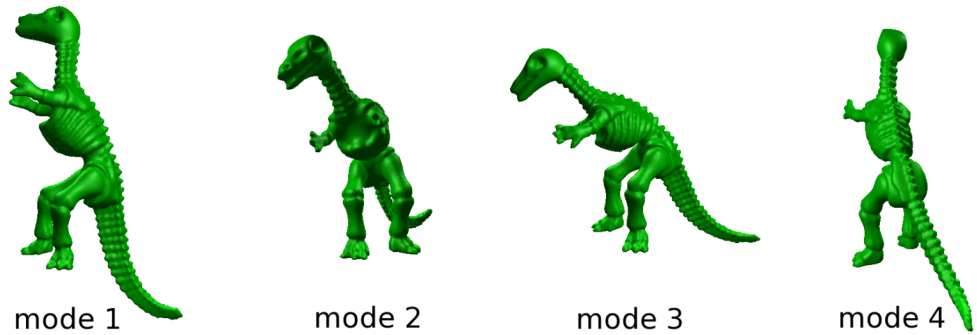
Examples:
Linear & nonlinear modal analysis,
PCA, polynomial deformers,
FFD/embedded, splines, subdivision, etc.

Reduced-dimensional configuration space



Subspace Self-Collision Culling

Arbitrary triangle models
and low-rank deformations



Fast culling for
moderate deformations

Supports continuous collision detection

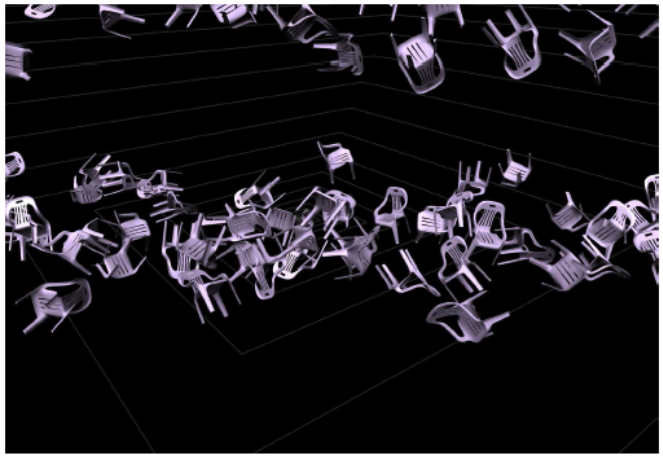
SCD without looking at all the triangles!

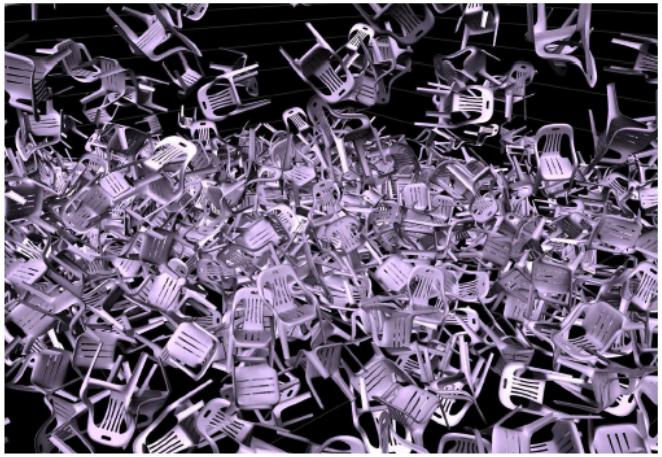
Related Work: Bounded-Deformation Trees

[James and Pai 2004]

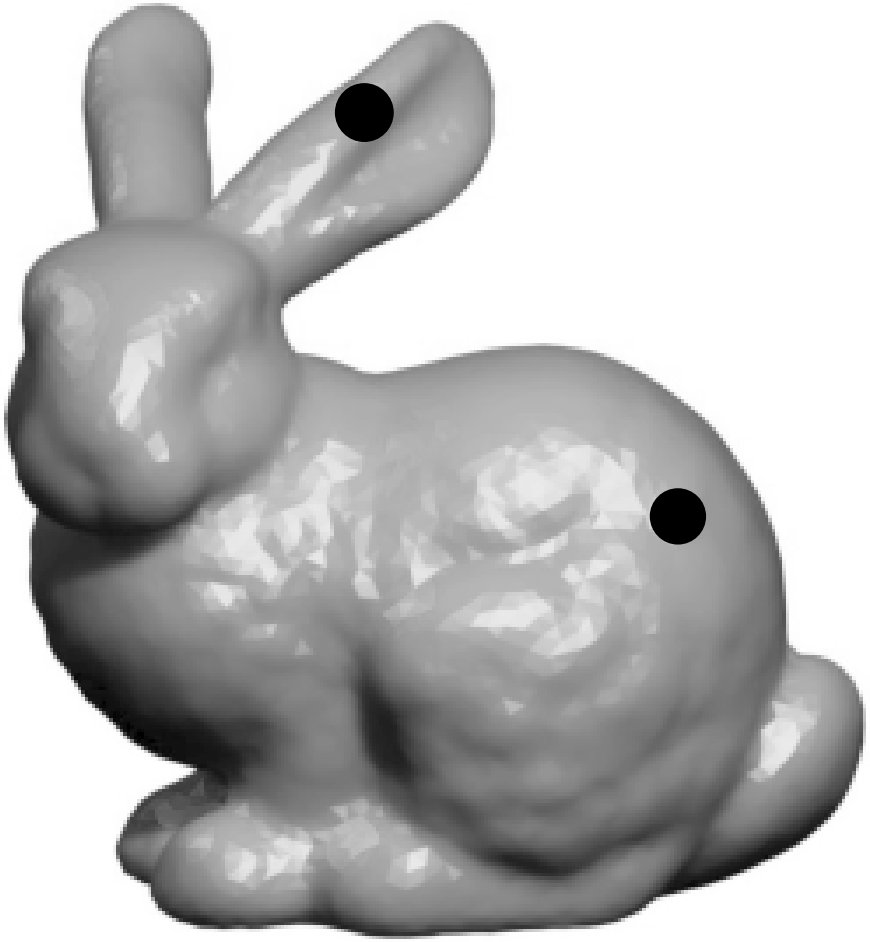
Exploit deformation structure for **inter-object** collision detection

Cannot be applied to SCD directly



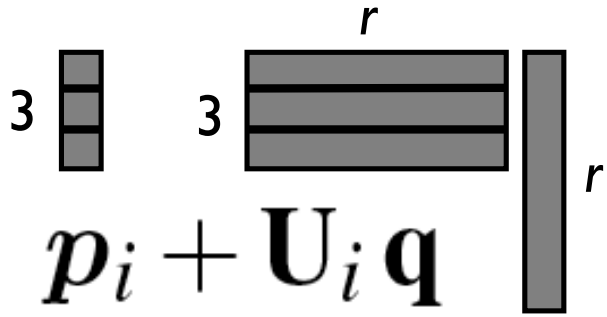


Key idea

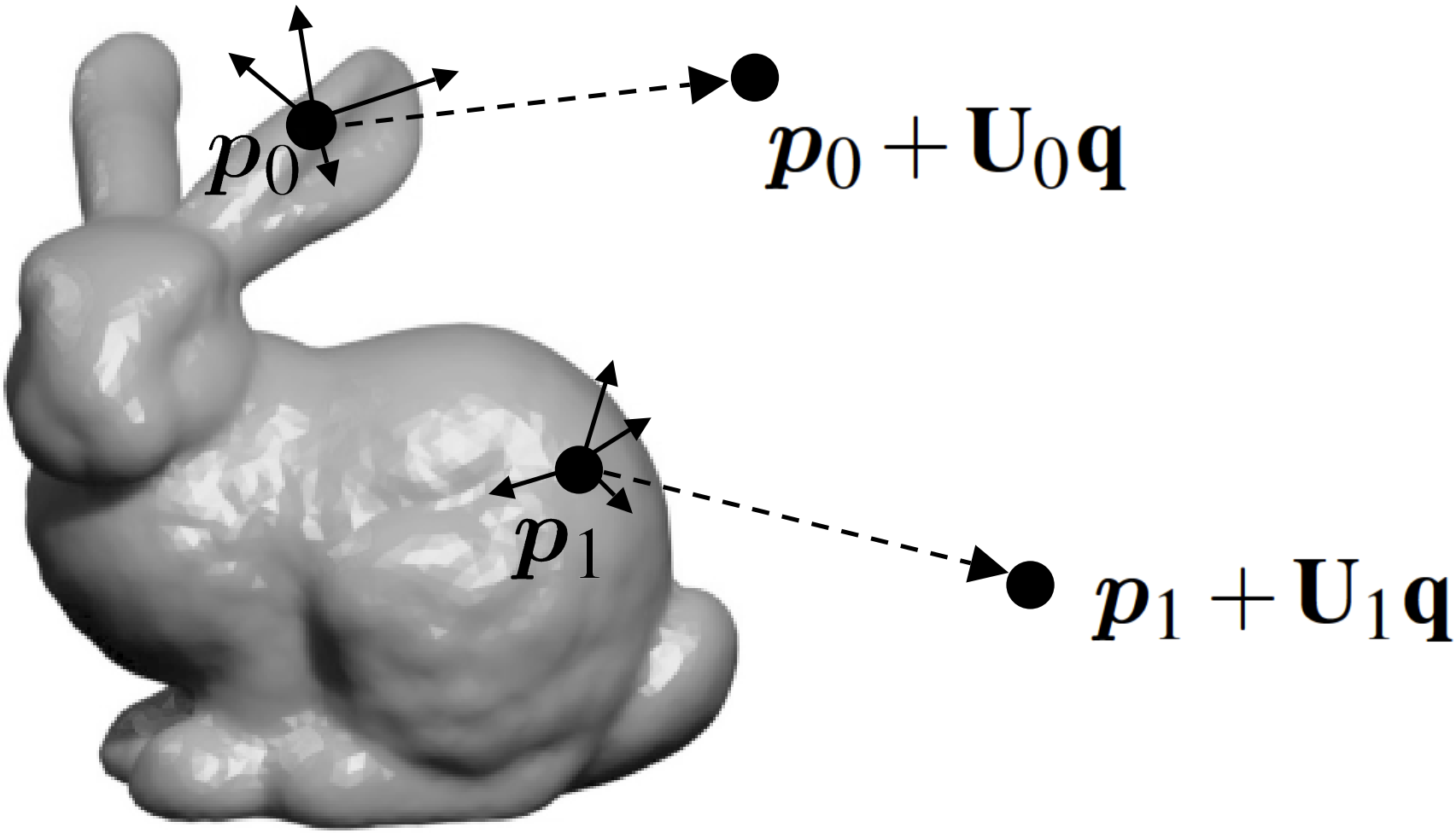


How can two points collide?

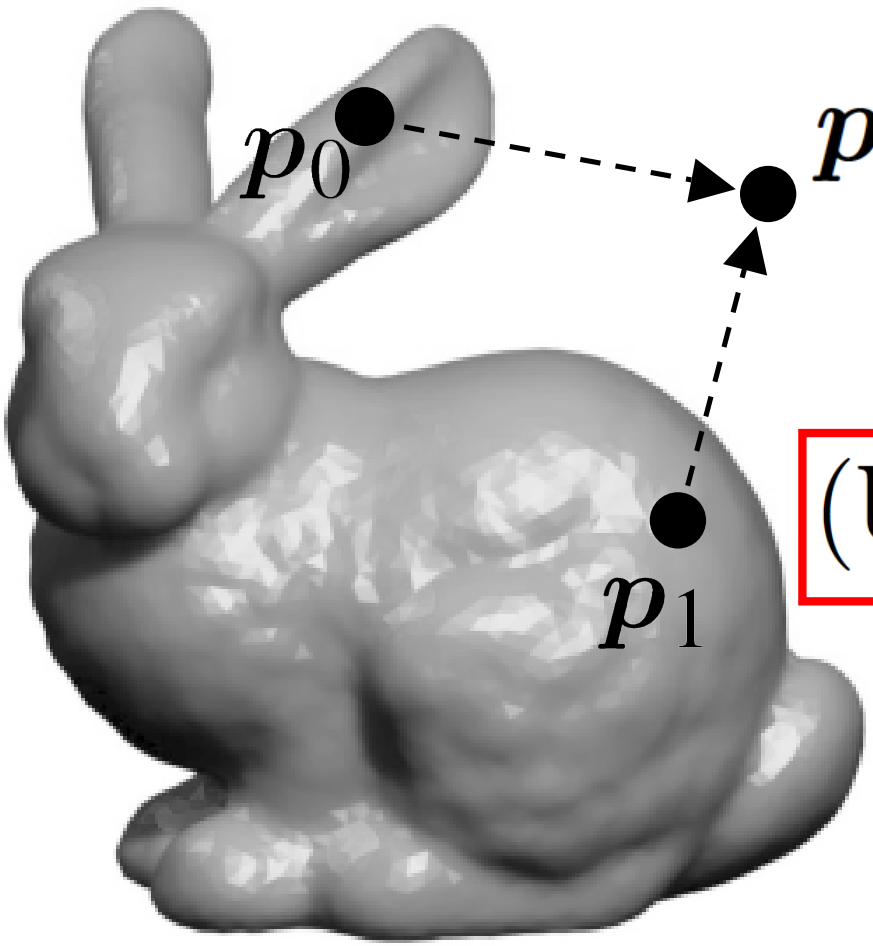
Low-rank deformations



$$\boldsymbol{p}_i + \mathbf{U}_i\,\mathbf{q}$$



Contact condition

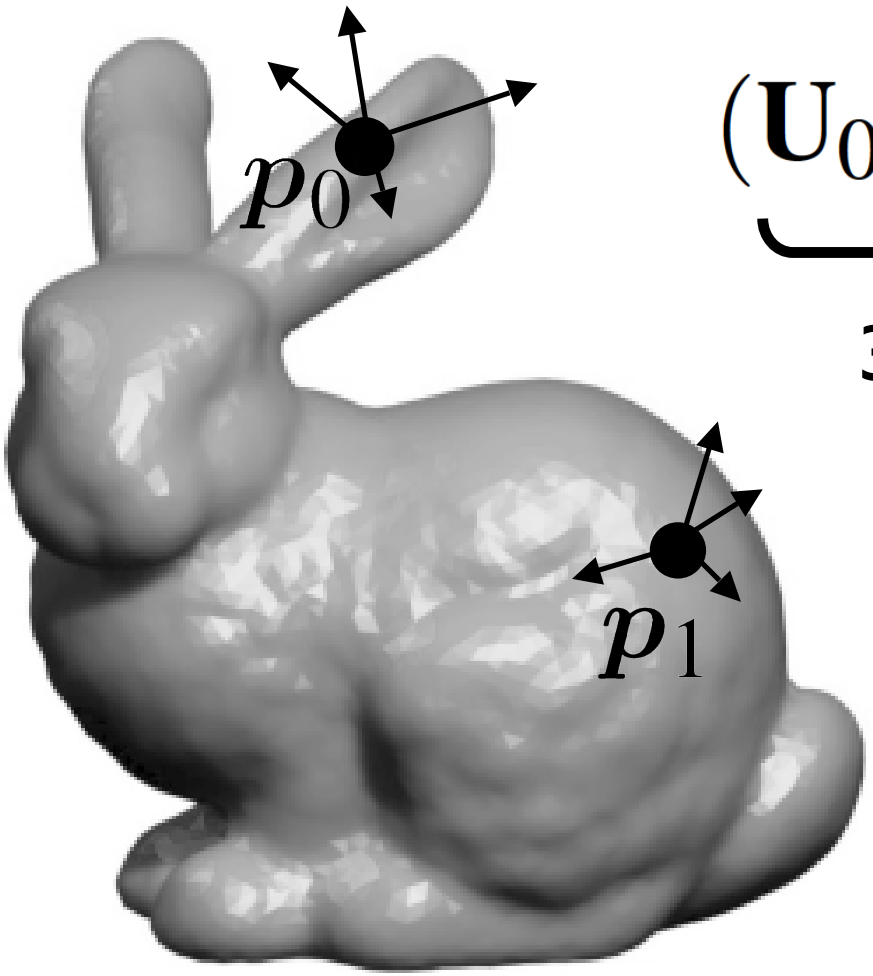


$$\boldsymbol{p}_0 + \mathbf{U}_0\mathbf{q} = \boldsymbol{p}_1 + \mathbf{U}_1\mathbf{q}$$

$$\Updownarrow$$

$$\boxed{(\mathbf{U}_0 - \mathbf{U}_1)\mathbf{q} = (\boldsymbol{p}_1 - \boldsymbol{p}_0)}$$

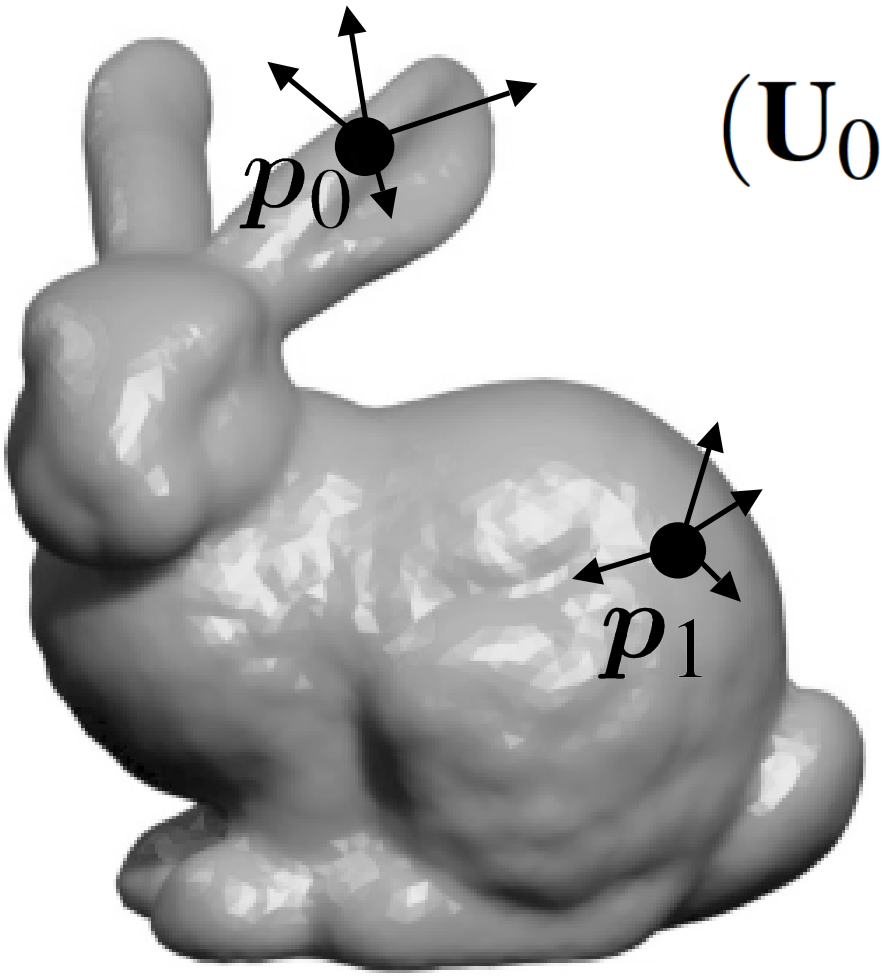
Three equations for *r* unknowns



$$\underbrace{(\mathbf{U}_0 - \mathbf{U}_1)}_{3 \times r}\underbrace{\mathbf{q}}_{r} = \underbrace{(\boldsymbol{p}_1 - \boldsymbol{p}_0)}_{3}$$

Compute min-norm solution $\mathbf{q}^*$.

Three equations for *r* unknowns

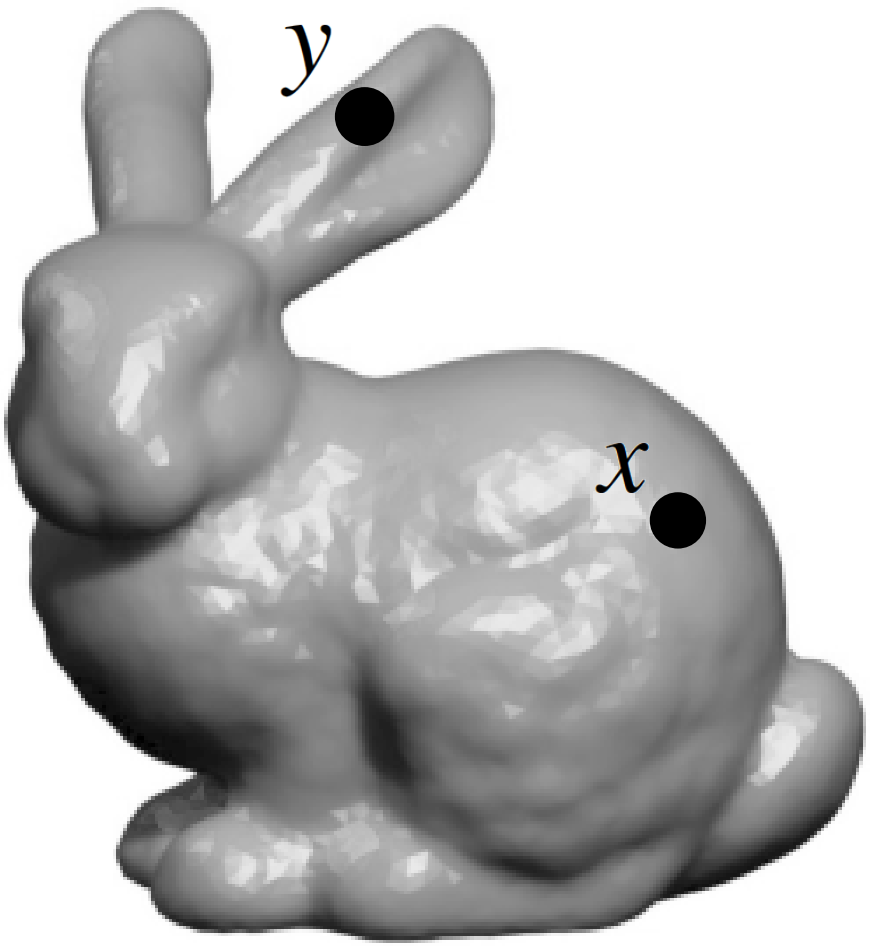


$$(\mathbf{U}_0 - \mathbf{U}_1)\mathbf{q} = (\boldsymbol{p}_1 - \boldsymbol{p}_0)$$

Compute min-norm solution $\mathbf{q}^*$.

If $\|\mathbf{q}\| < \|\mathbf{q}^*\|$, the two points cannot collide.

Global collision bound



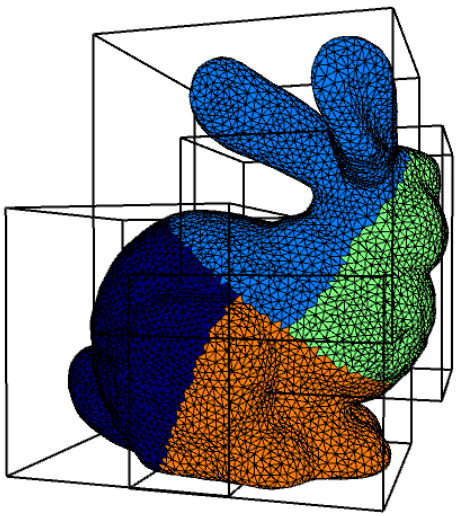
$$\mathcal{R} = \min_{x \neq y} ||\mathbf{q}^*(x, y)||$$

Self-collision-free if

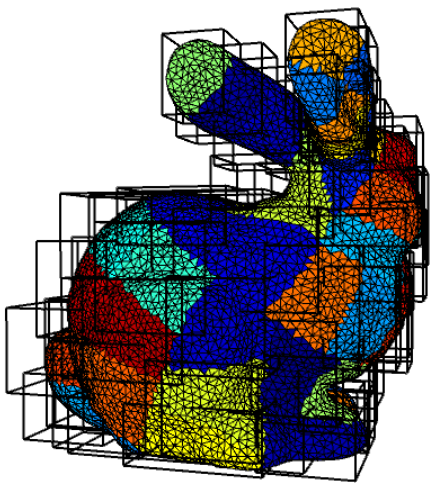
$$\|\mathbf{q}\| < \mathcal{R}$$

Hierarchical Subspace Self-Collision Culling

Related work: Bounding volume hierarchies



AABBs
Level 1



AABBs
Level 3

[Hubbard 1995]

[Gottschalk et al. 1996]

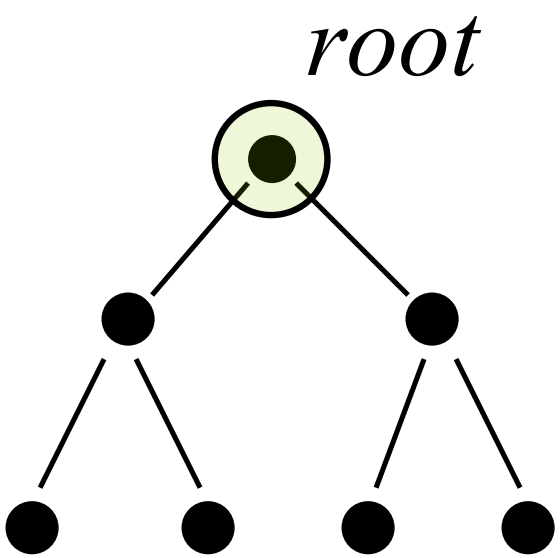
[van den Bergen 1997]

[Bridson et al. 2002]

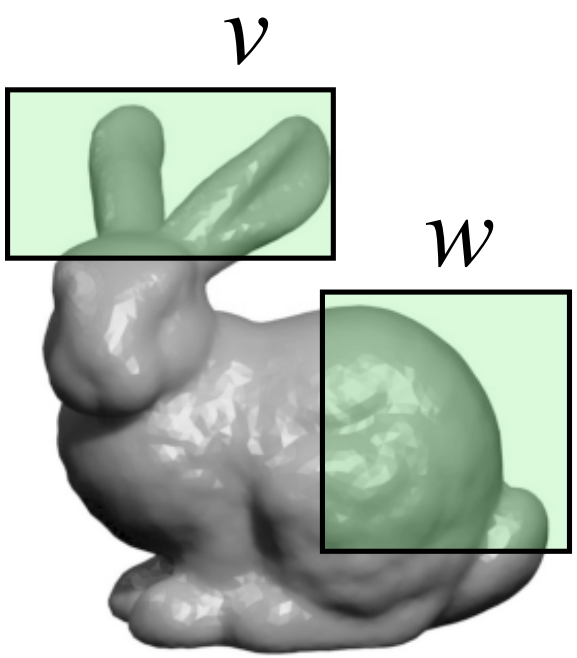
[Teschner et al. 2002]

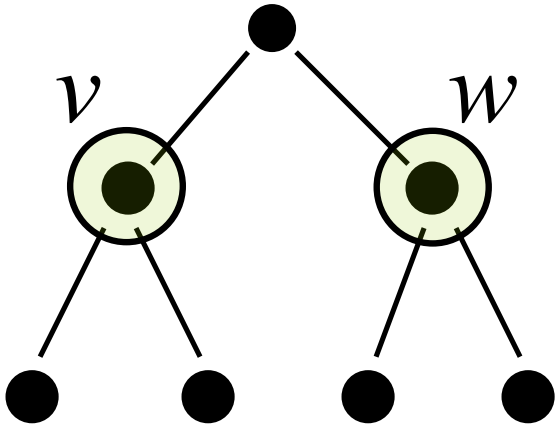
[Govindaraju et al. 2005]

Bounding volume hierarchy



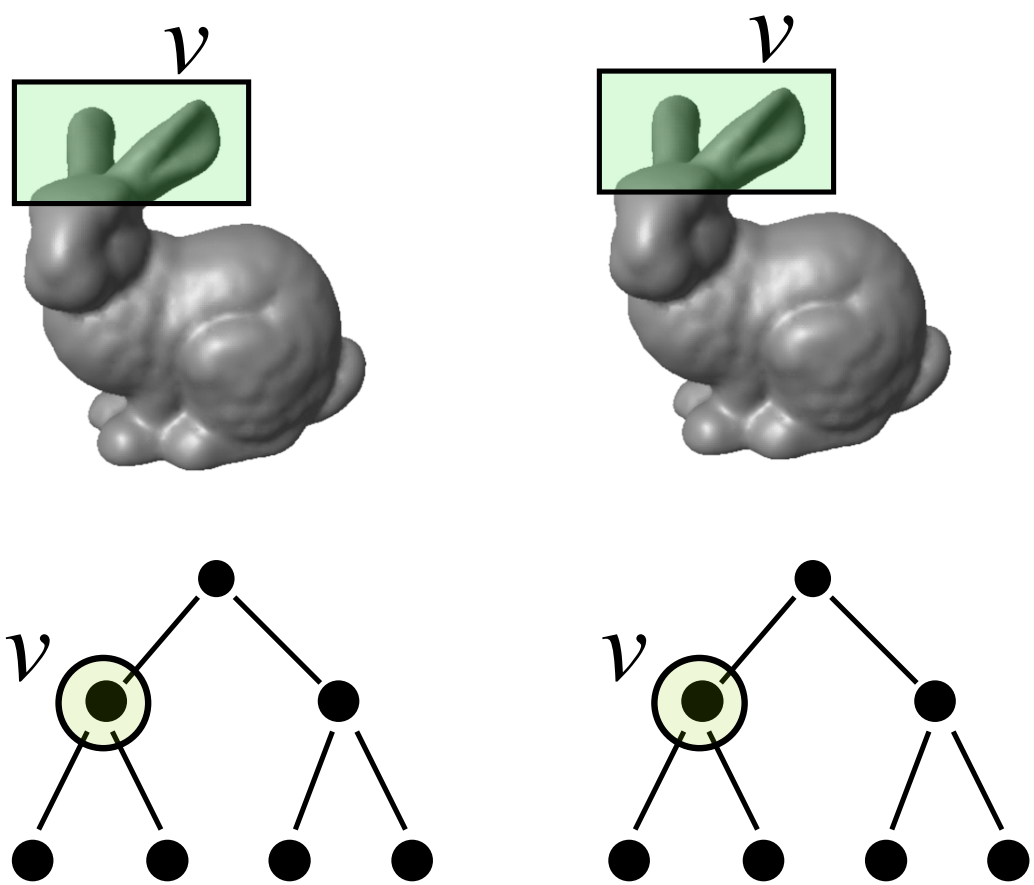
Bounding volume hierarchy



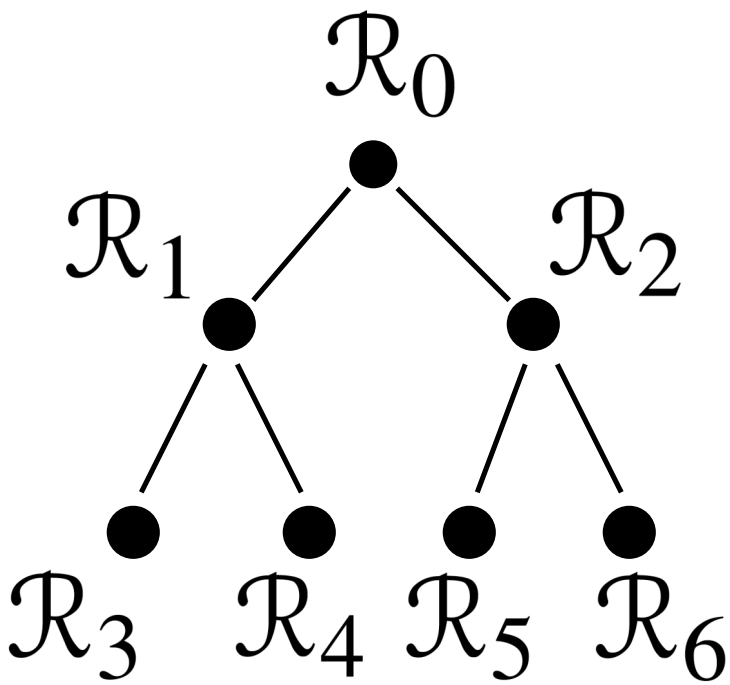


Self-collision detection

Test the tree against *itself* :



Certificates

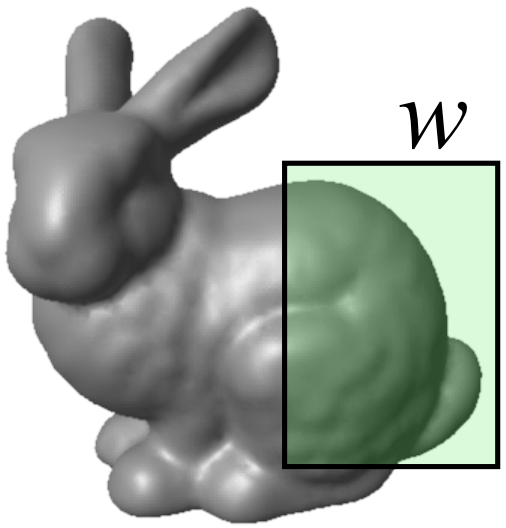


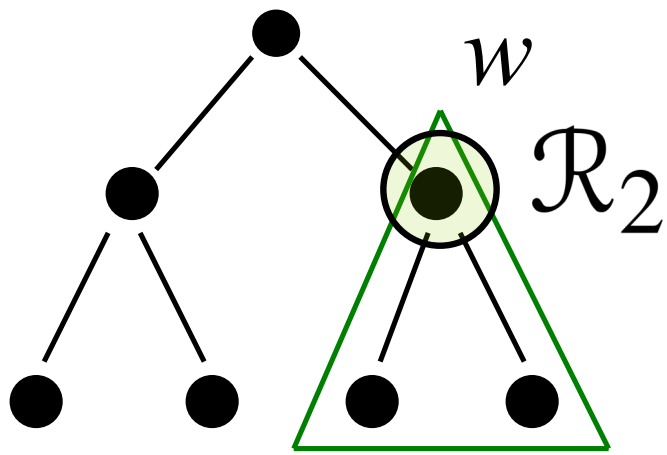
No collisions in subtree if

$$\|\mathbf{q}\| < \mathcal{R}_i$$

Kinetic data structures
[Guibas 2004]

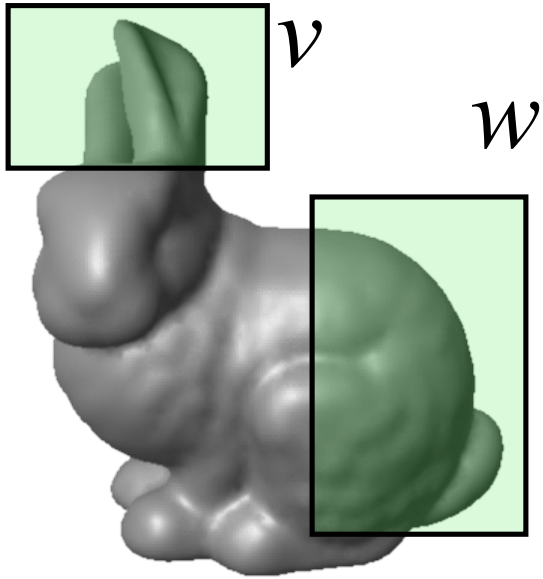
Use the certificates to accelerate tree traversal

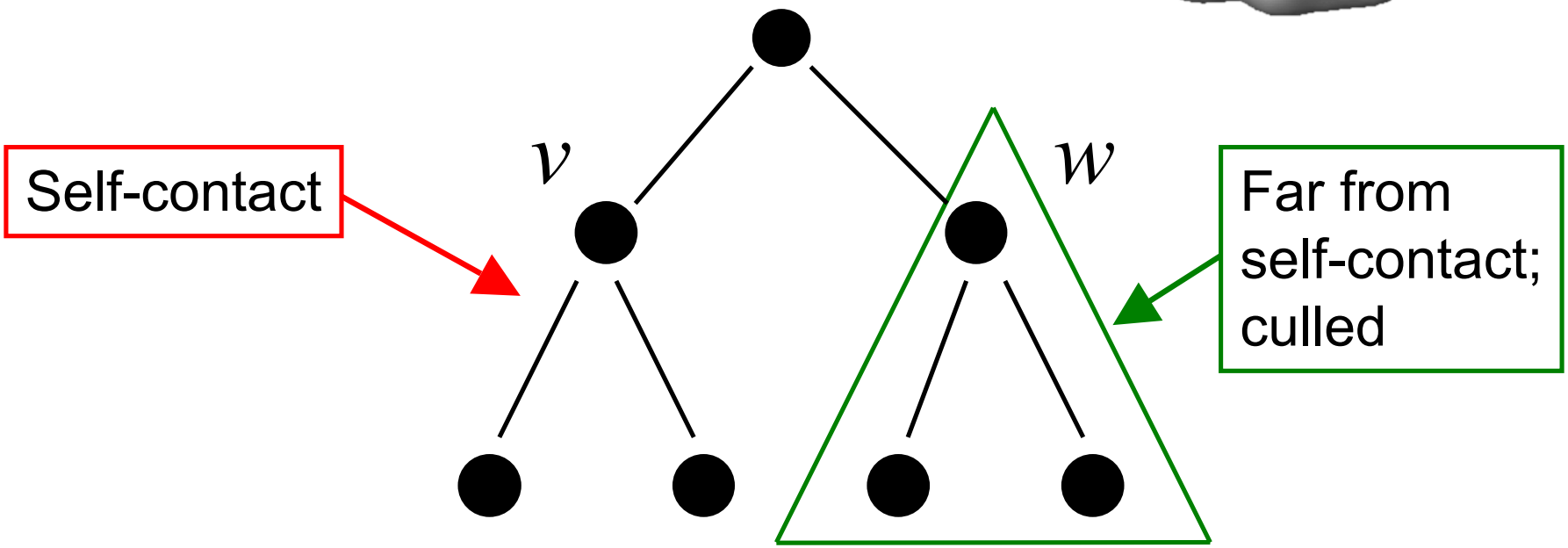




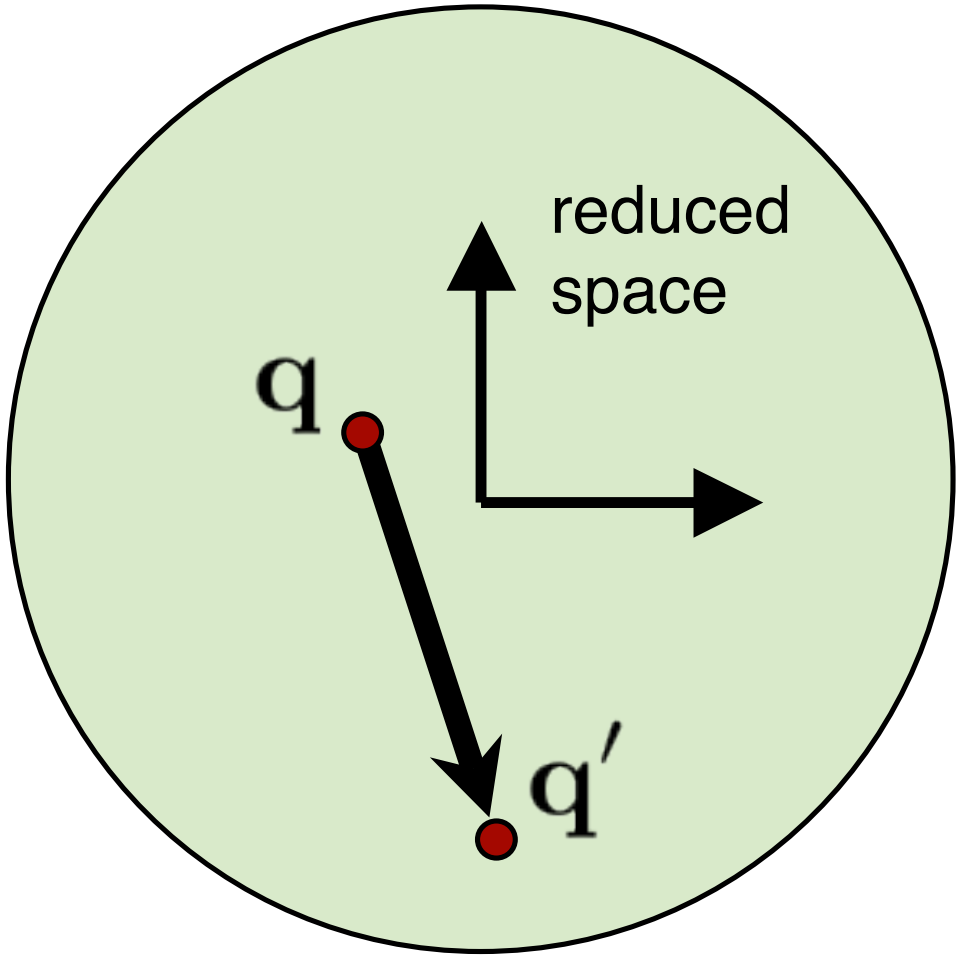
Cull if $\|\mathbf{q}\| < \mathcal{R}_2$

Certificates useful even with self-contact





Supports continuous SCD



Speedup (bunnies):

Discrete SCD: **29x**

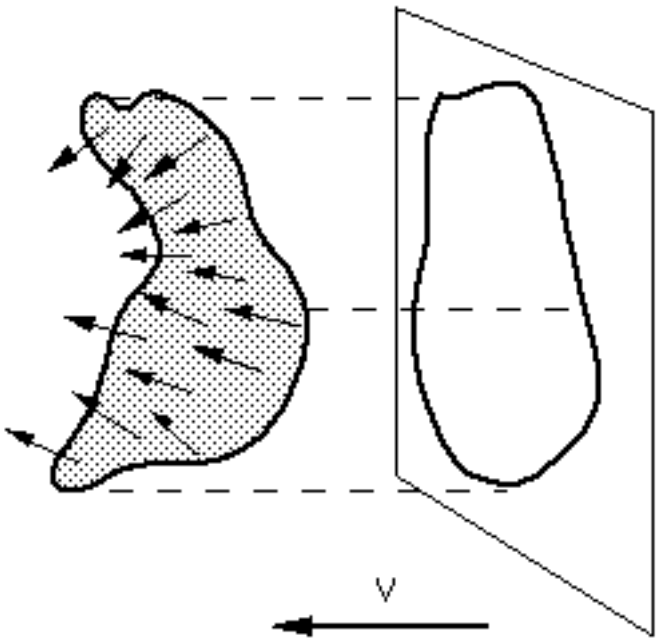
Continuous SCD: **32x**

Handles nonsmooth geometry

Rest Smooth Noisy

Largest deformation culled by root node

Related work: Curvature bounds and normal cones



[Volino and Magnenat-Thalmann 1994]

[Provot 1997]

[Grinspun and Schroeder 2001]

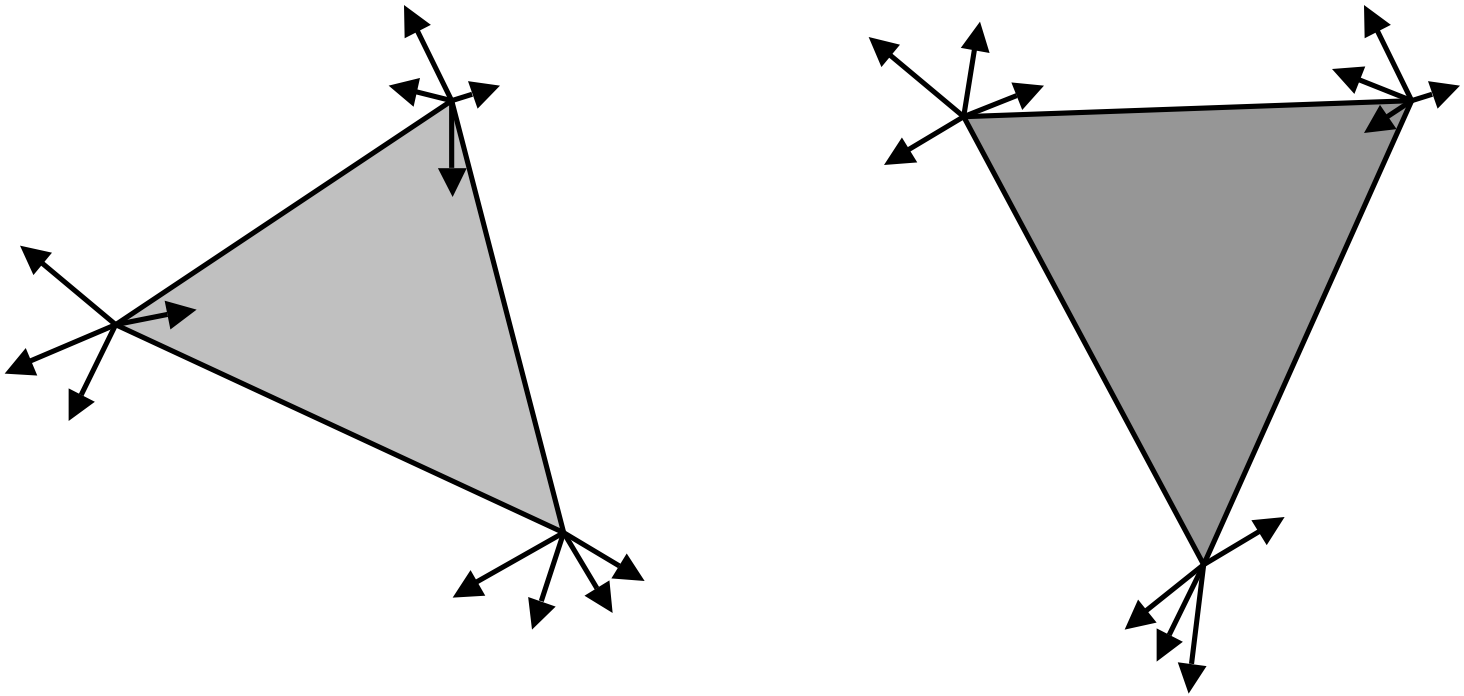
[Schvartzman et al. 2009]

[Tang et al. 2009]

[Schvartzman et al. 2010]

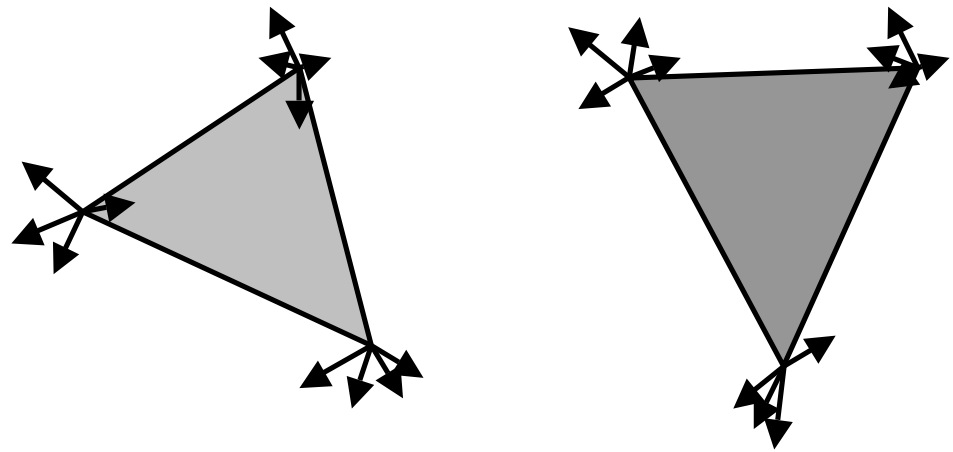
Certificate precomputation

Two triangles



$$\mathcal{R} = ?$$

Two triangles



Min ||q|| s.t. contact:

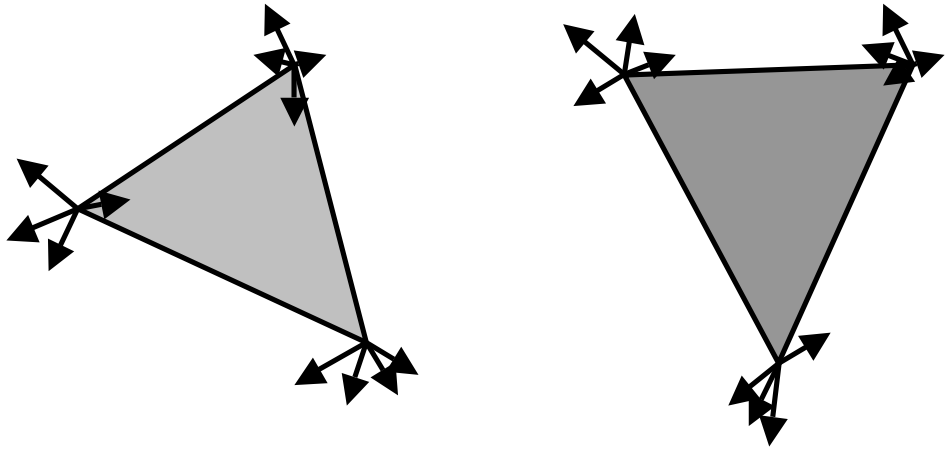
$$\text{minimize} \quad \|\mathbf{q}\|^2, \quad \text{over all } \mathbf{q}, \alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2,$$

$$\text{subject to } \alpha_i + \beta_i + \gamma_i = 1,\ \alpha_i \geq 0,\ \beta_i \geq 0,\ \gamma_i \geq 0,\ i = 1, 2,$$

$$\alpha_1 \boldsymbol{p}_1^1 + \beta_1 \boldsymbol{p}_1^2 + \gamma_1 \boldsymbol{p}_1^3 + \left(\alpha_1 \mathbf{U}_1^1 + \beta_1 \mathbf{U}_1^2 + \gamma_1 \mathbf{U}_1^3\right)\mathbf{q} =$$

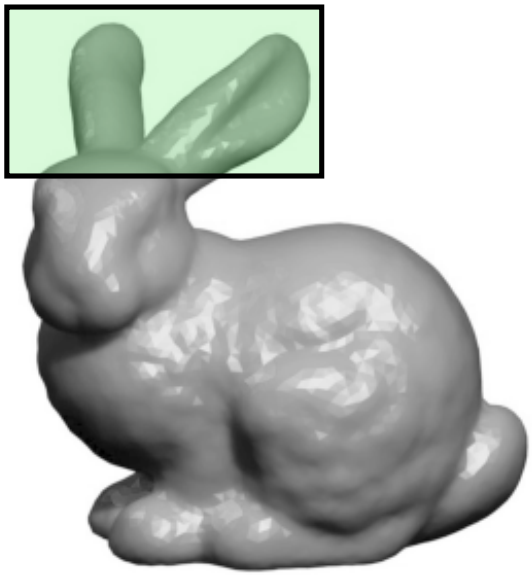
$$\alpha_2 \boldsymbol{p}_2^1 + \beta_2 \boldsymbol{p}_2^2 + \gamma_2 \boldsymbol{p}_2^3 + \left(\alpha_2 \mathbf{U}_2^1 + \beta_2 \mathbf{U}_2^2 + \gamma_2 \mathbf{U}_2^3\right)\mathbf{q}.$$

Two triangles

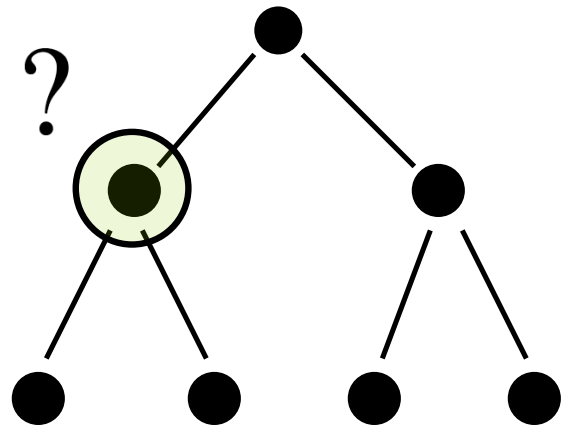


- Quadratic constraints, quadratic objective
- NP-hard optimization problem
- We designed a fast approximation algorithm (tight, with a proof)

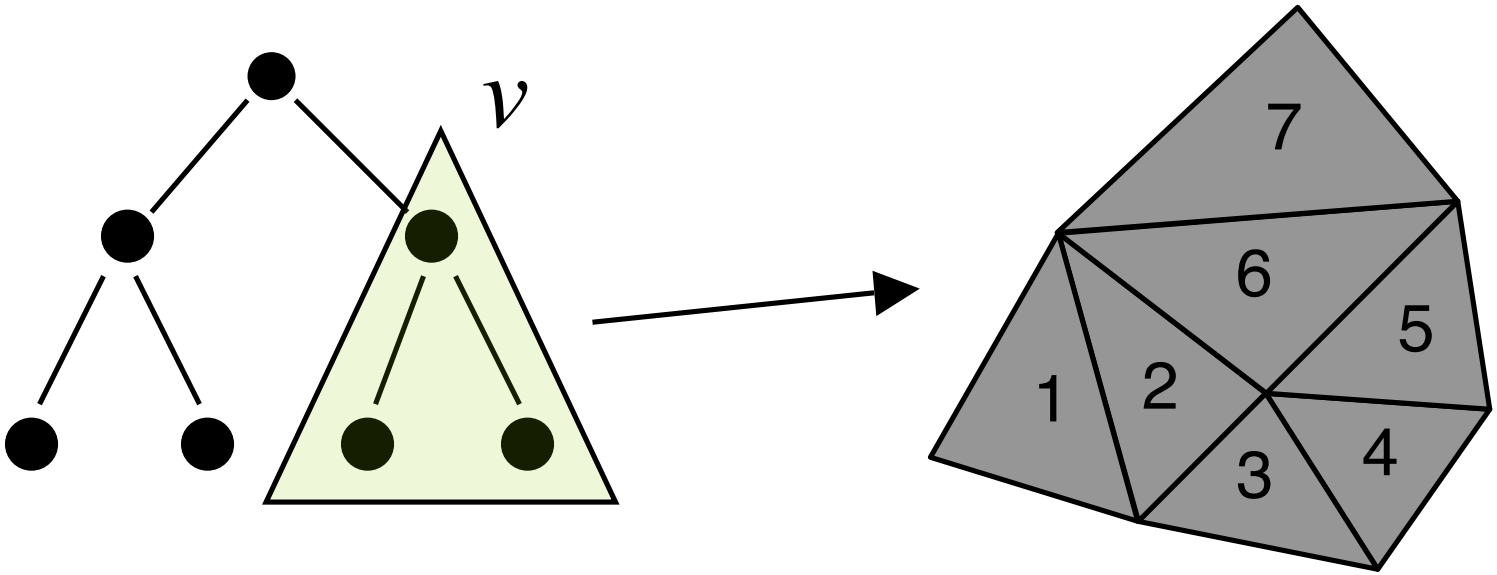
Certificates for the hierarchy



$$\mathcal{R} = ?$$

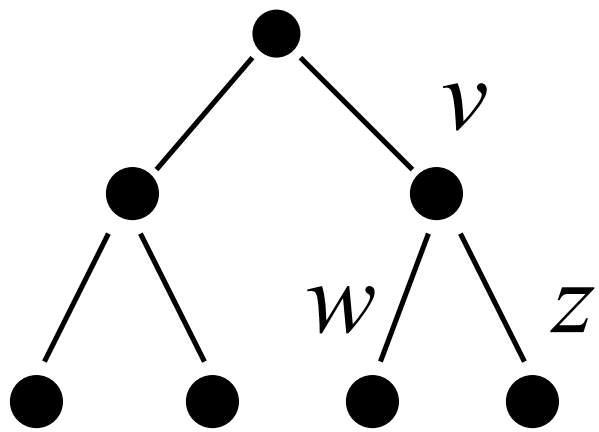


Certificates for the hierarchy



$$\mathcal{R}(v) = \min \underbrace{\left\{ \mathcal{R}_{14}, \mathcal{R}_{15}, \mathcal{R}_{37}, \mathcal{R}_{47} \right\}}_{\text{non-neighboring pairs}}$$

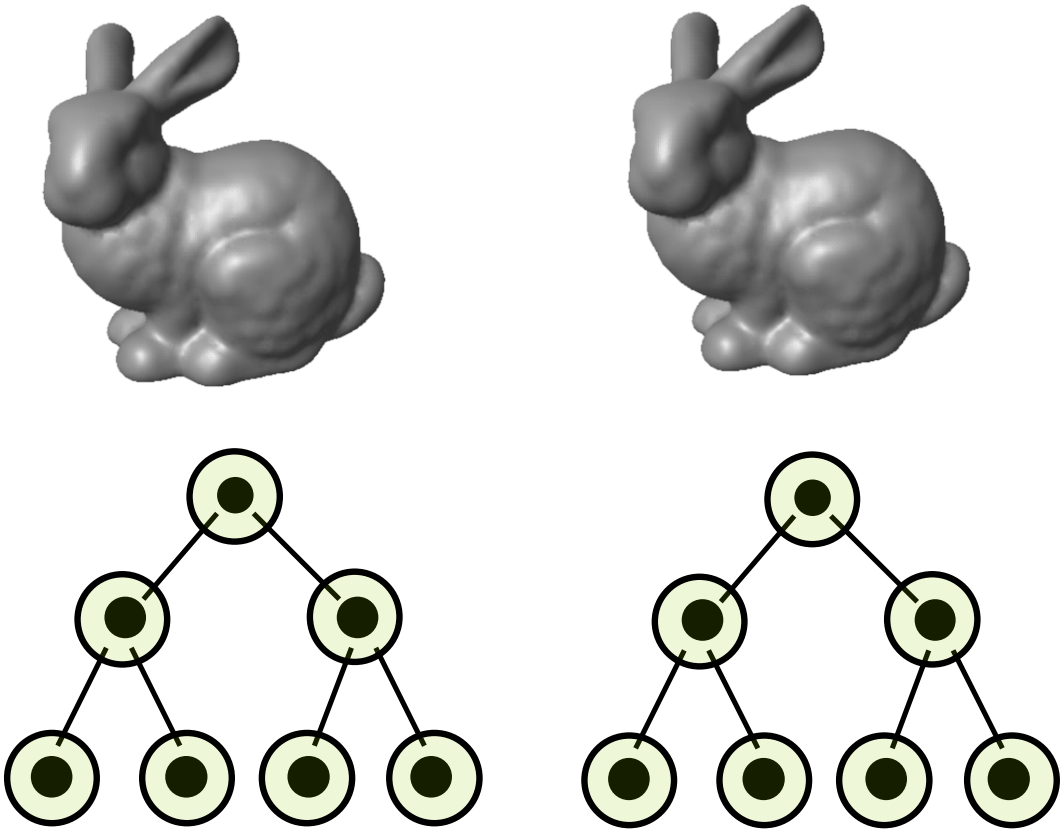
Hierarchical computation of certificates



$$\mathcal{R}(v) = \min\{\mathcal{R}(w),\ \mathcal{R}(z),\ \mathcal{R}(w,z)\}$$

Sublinear cost of SCD

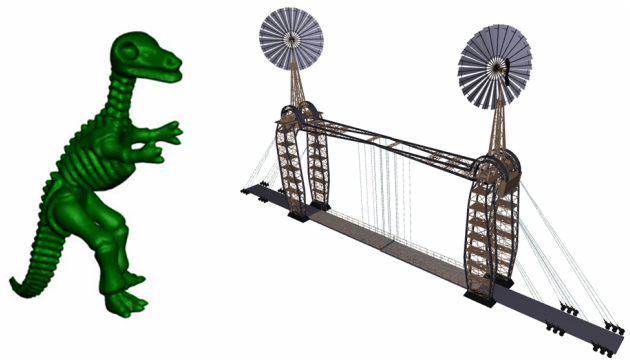
Standard SCD traverses *all* tree nodes



Collisions between two objects

ARE TYPICALLY EASIER THAN

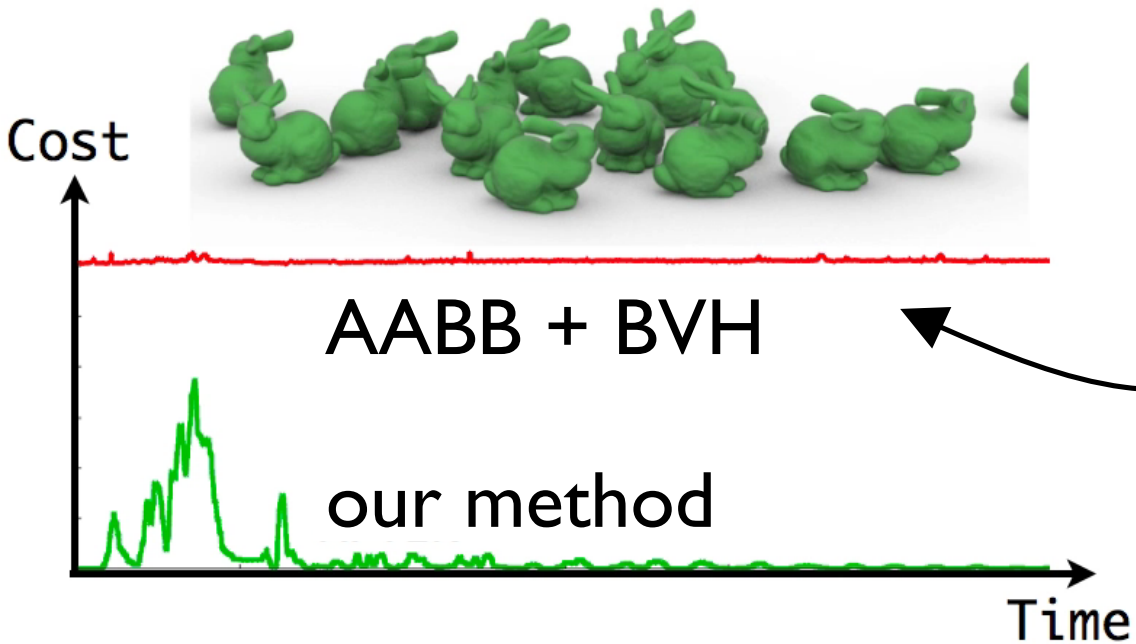
Self-collisions



A few isolated hot-spots

Every triangle is a hot-spot, always !

Standard SCD traverses *all* tree nodes



$\Omega(N)$ cost

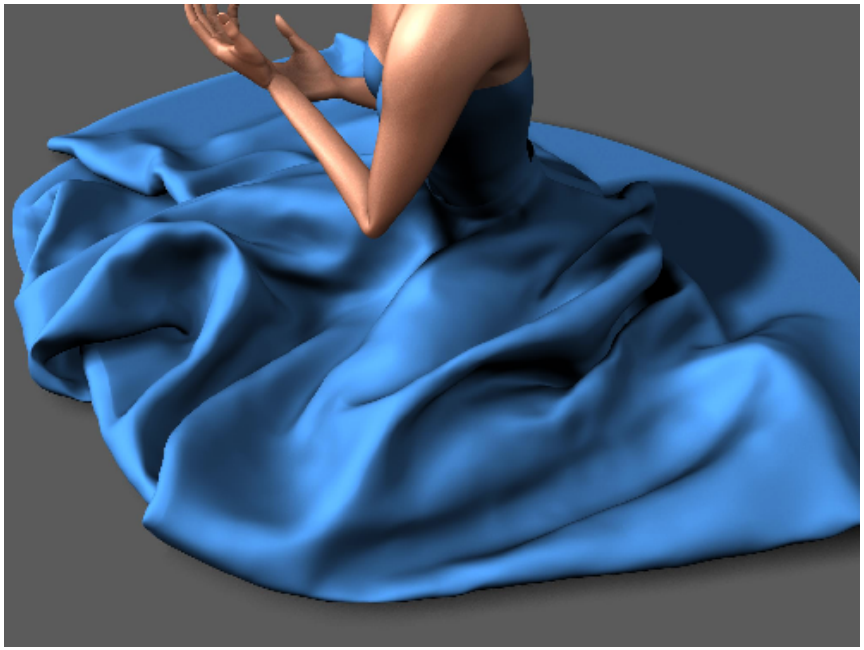
Even when no self-collisions!

Improved Runtime Complexity

$$\Omega(N) \Longrightarrow O(r)$$

SCD without looking at all the triangles!

Related work: GPU methods



+ general
+ highly deformable
- inherent $\Omega(N)$ cost

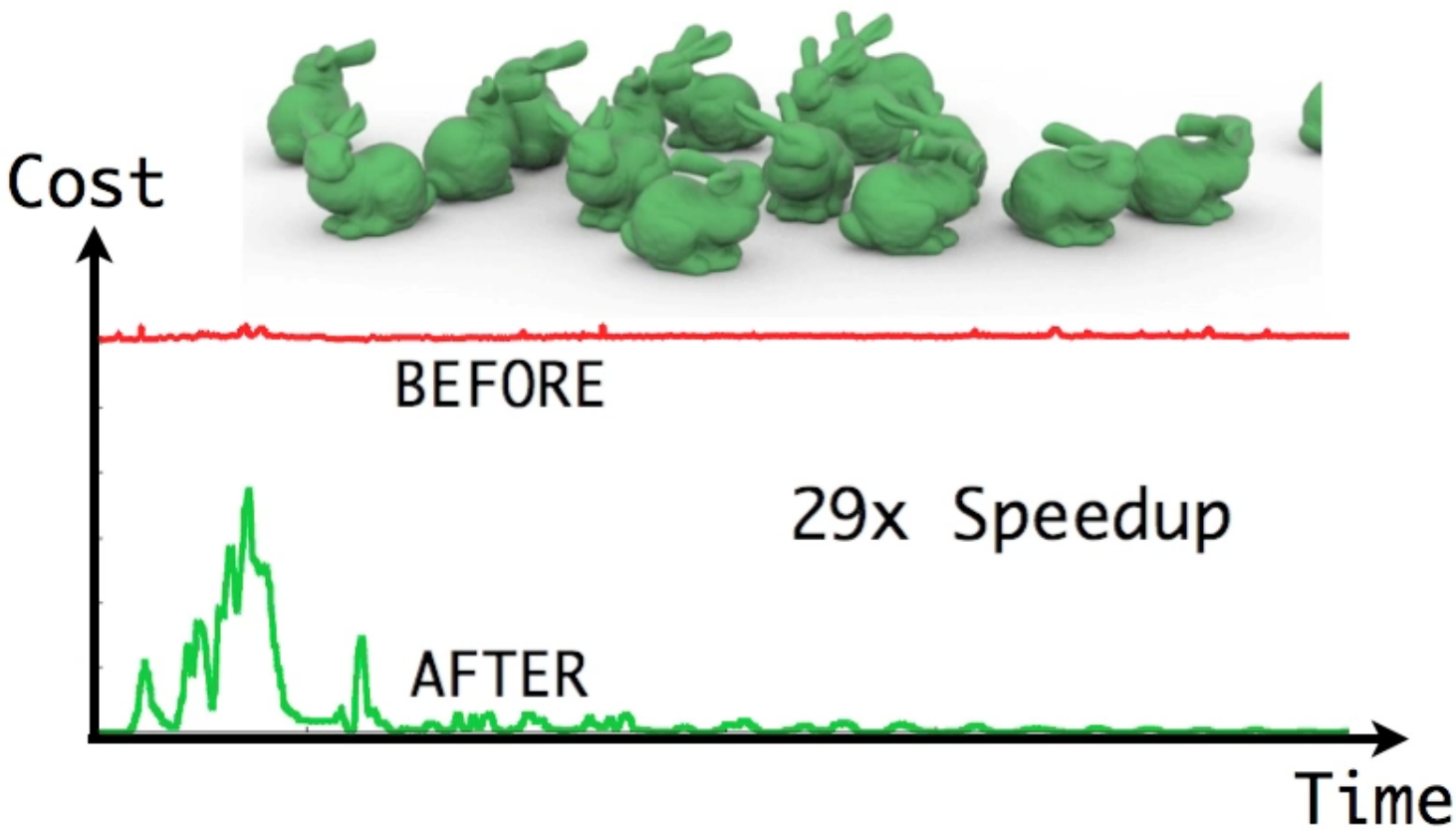
[Heidelberger et al. 2004]

[Govindaraju et al. 2005]

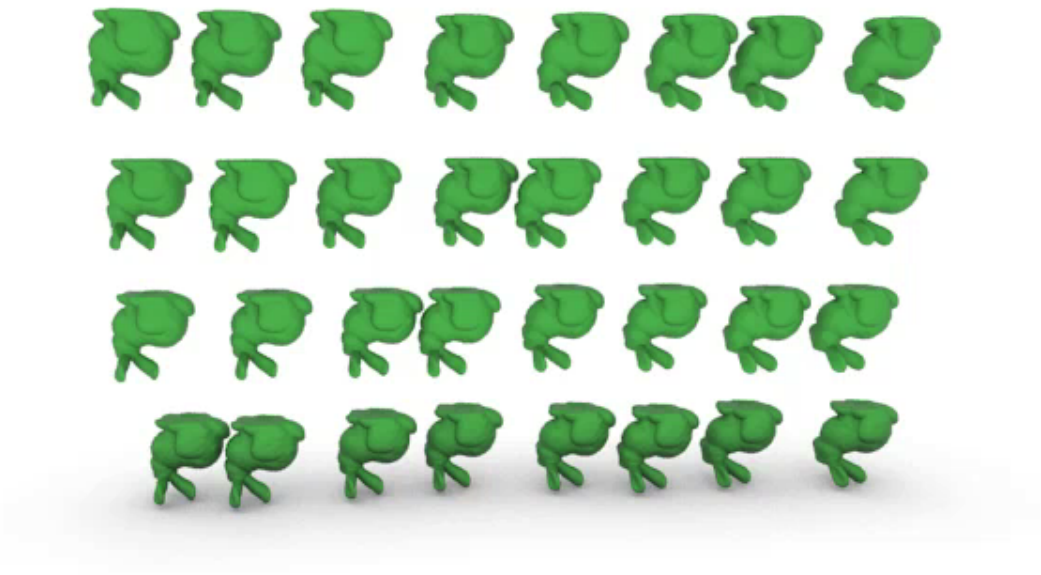
[Sud et al. 2006]

Results

Example: 16 bunnies



Example: 128 bunnies



15x Speedup

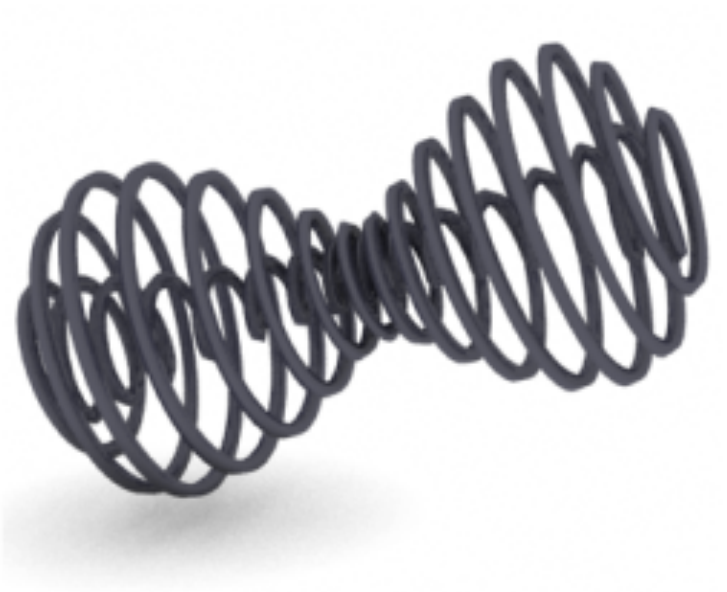
Bunny model:
#Modes(r): 20
#Triangles: 15k
Nonlinear reduced FEM

128 bunnies, 1,915,000 triangles

Self-collision sounds

SCD at audio rates
(44,100 timesteps / sec)

SCD is 98% of computation



double helix
16,000 triangles

Self-collision sounds
(127x speedup)

Self-contacts

Many self-contacts

16x Speedup

#Modes(r): 30
#Triangles: 77k
Nonlinear reduced FEM

Conclusion

Exploit reduced deformations to accelerate SCD

$\Omega(N) \Rightarrow O(r)$

Minimal change to standard BVH traversal

Supports non-smooth geometry

Future work

Extension to skinning (characters)

[Lewis et al. 2000]

General deformations

Shorten precomputation time

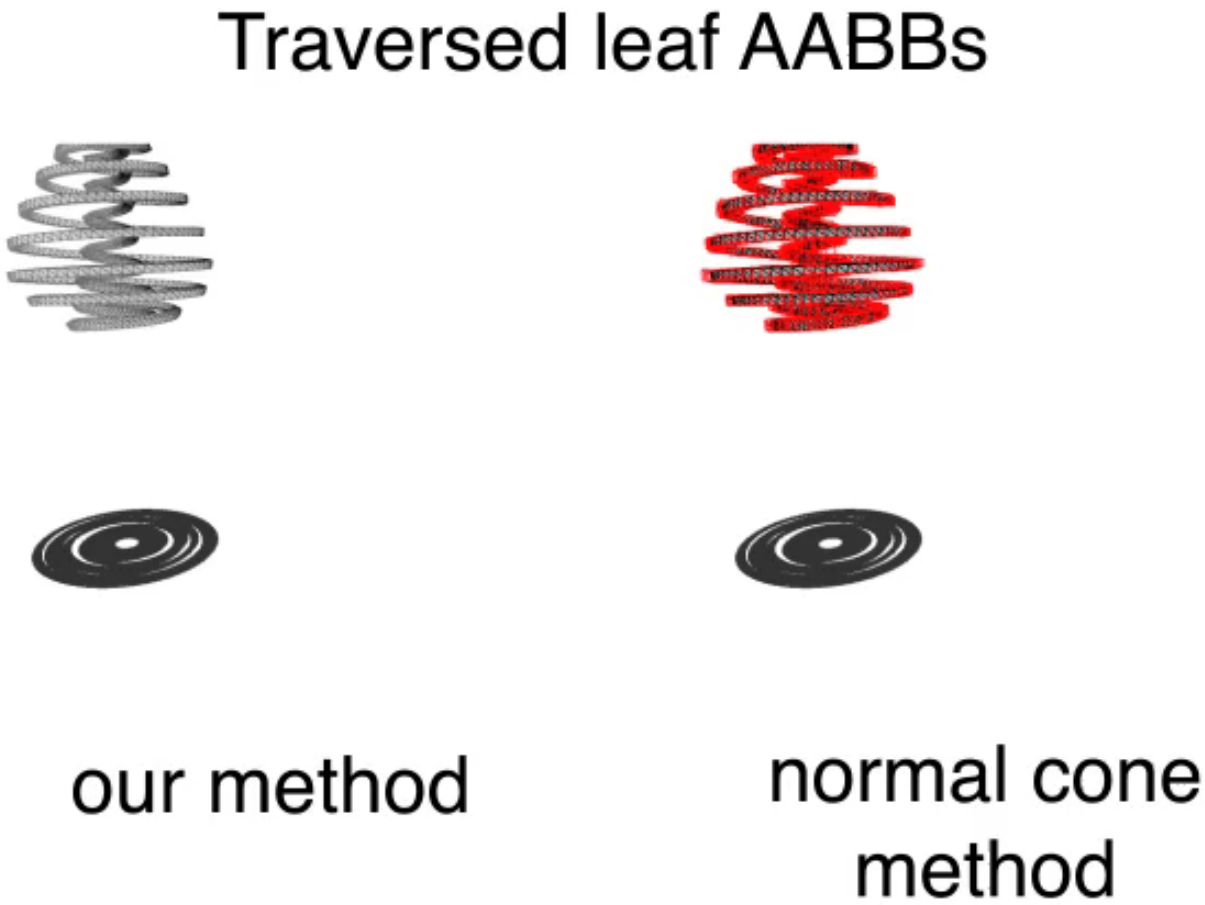
Extra slides

Geometric basis
(quadratic polynomials in space)

102x speedup

Comparison to normal cones

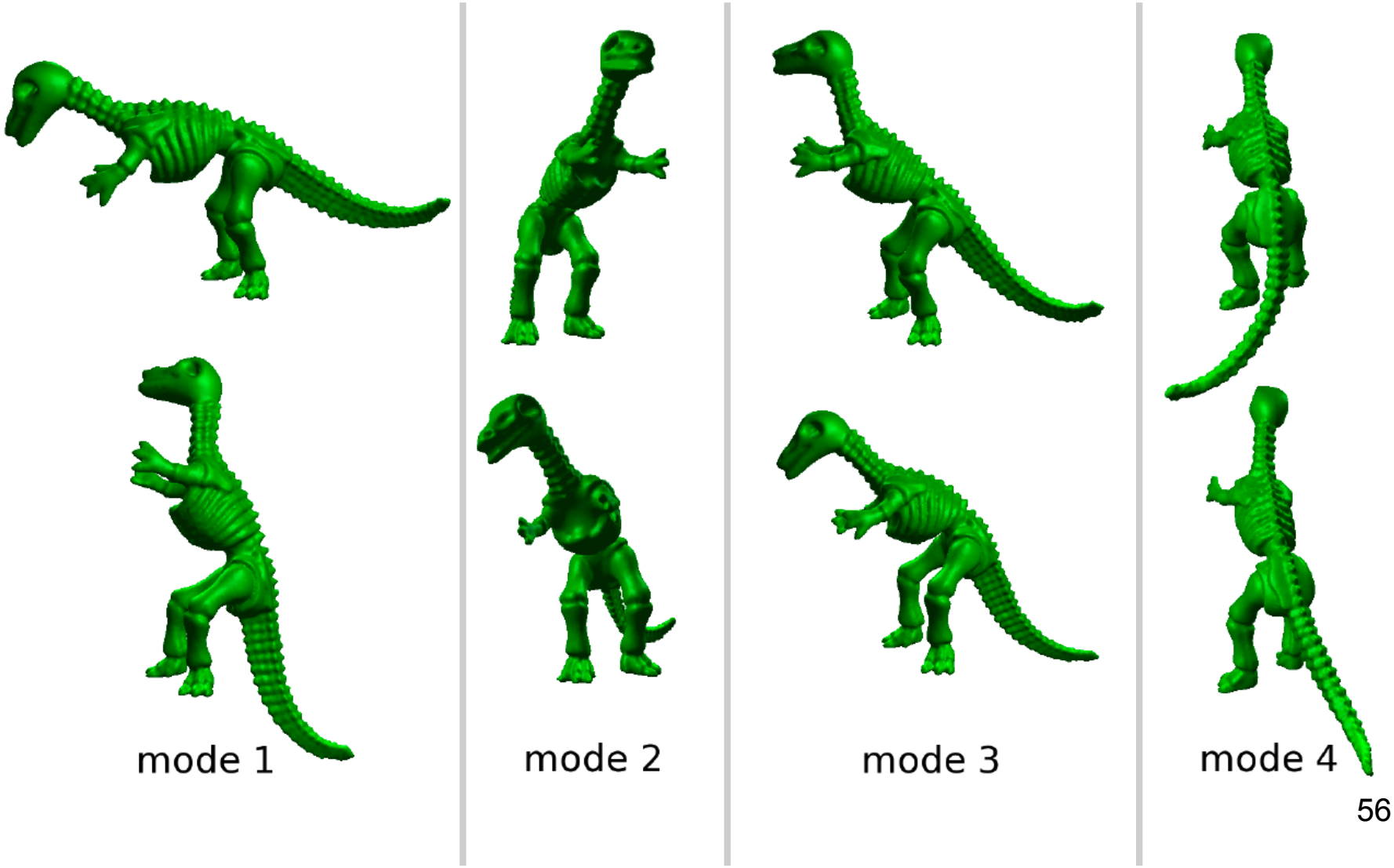
[Volino and Magnenat-Thalmann 1994]



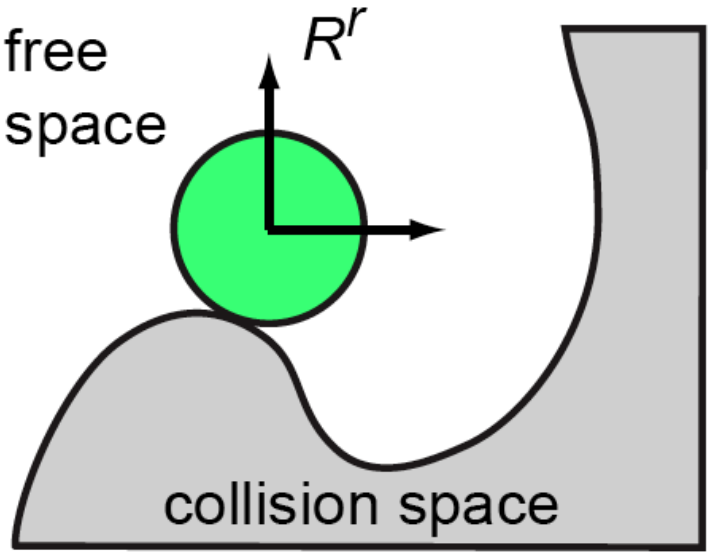
Self-collision sounds (OFF)

Self-collision detection turned off

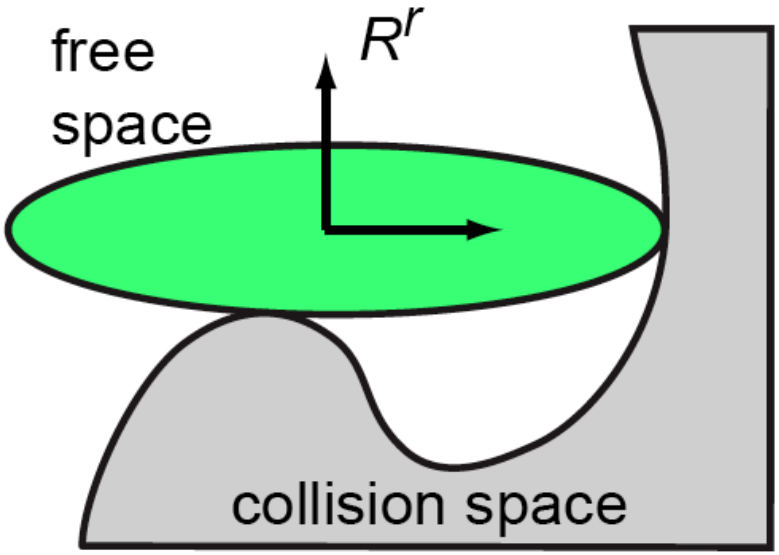
Extremal Deformations of Root Collision-free Certificate



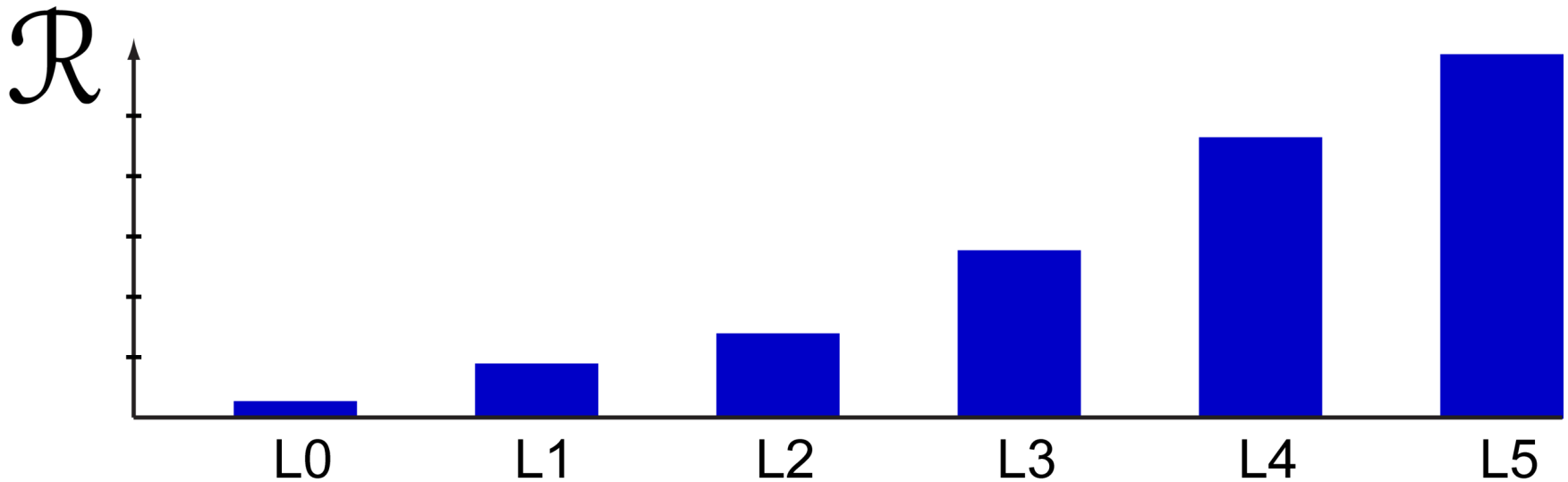
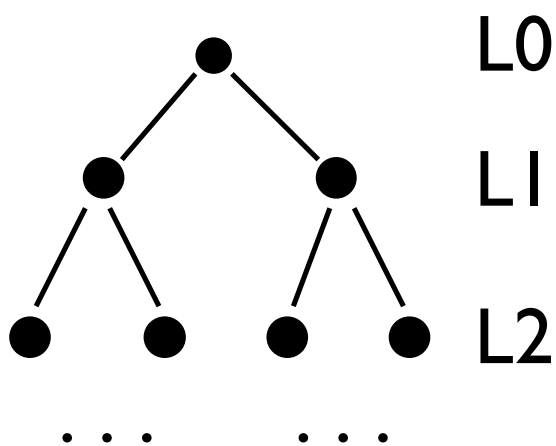
Ellipsoidal bounds



$$\|\mathbf{q}\|_2 < \mathcal{R}$$



$$\|\mathbf{q}\|_{\mathbf{M}} < \mathcal{R}$$

$$\|\mathbf{q}\|_{\mathbf{M}} \equiv \sqrt{\mathbf{q}^T \mathbf{M} \mathbf{q}}$$

Bounds increase with tree level