Homework #2
CS672 Spring 2017

Due Monday, March 20, at 3:10pm

General Instructions The following assignment is meant to be challenging. Feel free to discuss with fellow students, though please write up your solutions independently and acknowledge everyone you discussed the homework with on your writeup. I also expect that **you will not attempt to consult outside sources, on the Internet or otherwise,** for solutions to any of these homework problems. Finally, unless otherwise stated please provide a formal mathematical proof for all your claims.

Note In order to disincentivize skipping class to finish the homework, I have made it due within 10 minutes of the beginning of class. If you need more time, consider using one of your late days.

Problem 1. (10 points)
W&S Problem 3.6

Problem 2. (5 points)
W&S Problem 3.9

Problem 3. (10 points)
(Vazirani Problem 8.3) Derive an FPTAS for the following optimization problem: Given n positive integers $a_1 < \ldots, < a_n$, find two disjoint non-empty sets $S_1, S_2 \subseteq \{1, \ldots, n\}$ with $\sum_{i \in S_1} a_i \geq \sum_{i \in S_2} a_i$ such that the ratio

$$\frac{\sum_{i \in S_1} a_i}{\sum_{i \in S_2} a_i}$$

is minimized.

Problem 4. (10 points)
(Vazirani Problem 3.2) Consider the following optimization problem. You are given an undirected graph $G = (V, E)$ with nonnegative costs c_e for edges $e \in E$. Moreover S, the *senders*, and R, the *receivers*, are given disjoint subsets of V. The problem is to find a minimum cost subgraph of G that has a path connecting each receiver to a sender (any sender suffices).

(a) [3 points]. Show that this problem is solvable in polynomial time when $S \cup R = V$.

(b) [3 points]. Show that this problem is NP-hard in general.

(c) [4 points]. Derive a 2-approximation algorithm for this problem.

Hint: Consider adding a new vertex which is connected to each sender by a zero cost edge. Consider the new vertex and all receivers as required and the remaining vertices as Steiner, and find a minimum cost Steiner tree.

Problem 5. (10 points)
(Vazirani Problem 4.2) A natural greedy algorithm for computing a multiway cut is the following. Starting with G, compute the minimum $s_i - s_j$ cuts for all pairs s_i, s_j that are still connected and remove the lightest of these cuts; repeat this until all pairs s_j, s_j are disconnected. Prove that this algorithm also achieves a guarantee of $2 - 2/k$.