Homework #3
CS675 Fall 2023

Due Monday Sept 18, by midnight

General Instructions The following assignment is meant to be challenging. Feel free to discuss with fellow students, though please write up your solutions independently and acknowledge everyone you discussed the homework with on your writeup. I also expect that you will not attempt to consult outside sources, on the Internet or otherwise, for solutions to any of these homework problems — doing so would be considered cheating.

Several of these problems are drawn from the following texts, each of which is linked on the course website: Luenberger and Ye (4th edition), Korte and Vygen (5th edition), and Boyd and Vendenberghe. Please make sure you are using the correct edition of each of the books by using the links on the course website.

We request that you submit your homework as a pdf file, by email to the TA.

Finally, whenever a question asks you to "show" or "prove" a claim, please provide a formal mathematical proof.

Problem 1. (5 points)
B&V Exercise 2.3.

Problem 2. (5 points)
B&V Exercise 2.12.

Problem 3. (5 points)
B&V Exercise 2.15.

Problem 4. (5 points)
B&V Exercise 2.21.

Problem 5. (5 points)
B&V Exercise 2.26.

Problem 6. (5 points)
The *strict separating hyperplane theorem* states that whenever $A, B \subseteq \mathbb{R}^n$ are disjoint closed convex sets, and at least one of them is compact, there is a hyperlane *strictly* separating them; i.e. there is a vector $a \in \mathbb{R}^n$ and $b \in \mathbb{R}$ such that $a^T x < b$ for every $x \in A$, and $a^T x > b$ for every $x \in B$. The Farkas Lemma states that for every matrix $A \in \mathbb{R}^{n \times m}$ and vector b, either the system $Ax = b$ has a nonnegative vector solution $x \succeq 0$, or else there is a vector y such that $y^T A \succeq 0$ and $y^T b < 0$. Show that the Farkas lemma follows from the strict separating hyperplane theorem.