

# Homework #5

## CS675 Fall 2023

Due Monday Oct 9, by midnight

**General Instructions** The following assignment is meant to be challenging. Feel free to discuss with fellow students, though please write up your solutions independently and acknowledge everyone you discussed the homework with on your writeup. I also expect that you will not attempt to consult outside sources, on the Internet or otherwise, for solutions to any of these homework problems — doing so would be considered cheating.

Several of these problems are drawn from the following texts, each of which is linked on the course website: Luenberger and Ye (4th edition), Korte and Vygen (5th edition), and Boyd and Vandenberghe. Please make sure you are using the correct edition of each of the books by using the links on the course website.

We request that you submit your homework as a pdf file, by email to the TA.

Finally, whenever a question asks you to “show” or “prove” a claim, please provide a formal mathematical proof.

**Problem 1. (4 points)**

B&V Exercise 4.25.

**Problem 2. (4 points)**

B&V Exercise 4.30.

**Problem 3. (6 points)**

B&V Exercise 4.43.

**Problem 4. (4 points)**

B&V Exercise 4.56. (Hint: perspective of a convex function is convex)

**Problem 5. (4 points)**

B&V Exercise 5.3.

**Problem 6. (4 points)**

B&V Exercise 5.12.

**Problem 7. (4 points)**

B&V Exercise 5.18.

**Problem 8. (6 points)**

Let  $f_1, \dots, f_m : \mathbb{R}^n \rightarrow \mathbb{R}$  be convex functions. Let  $g : \mathbb{R}^n \rightarrow \mathbb{R}$  be the greatest convex function lower-bounding their point-wise minimum, i.e.  $g(x) = \text{convexhull}(\min_{i=1}^m f_i(x))$ . Formulate the problem of evaluating  $g(x)$  for a given  $x$  as a convex program. The number of variables in your convex program should be polynomially bounded in  $n$  and  $m$ .