Homework #1
CS699 Fall 2017

Due Wednesday October 4, by 5:00pm

General Instructions The following assignment is meant to be challenging. Feel free to discuss with fellow students, though please write up your solutions independently and acknowledge everyone you discussed the homework with on your writeup. We also expect that **you will not attempt to consult outside sources, on the Internet or otherwise,** for solutions to any of these homework problems. Finally, unless otherwise stated please provide a formal mathematical proof for all your claims.

Note To submit the homework, you may either email it to Li, hand it to him in person, or drop it off in the box which we will make available in SAL 246 on Wednesday. If you need more time, consider using one of your late days.

Problem 1. (10 points)
Consider the following 2-player 2-action game.

	A	B
A	1 4	0 0
B	3 3	4 1

Describe the set of dominant strategy equilibria, pure Nash equilibria, and mixed Nash equilibria. Find a correlated equilibrium whose expected social welfare exceeds that of any Nash equilibrium. The social welfare is defined is as the sum of utilities of all players.

Problem 2. (10 points)
Prove that every finite Bayesian game of incomplete information, under the common prior assumption, admits a (mixed) Bayes-Nash equilibrium. (Hint: invoke Nash's theorem, which states that every finite game of *complete* information admits a mixed Nash equilibrium)

Problem 3. (10 points)
VC-dimension of simple concept classes

(a) *Parity Functions:* For a set $I \subseteq \{1, 2, \ldots, n\}$, we define a parity function $h_I : \{0,1\}^n \to \{0,1\}$ as follows: On a binary vector $x = (x_1, \ldots, x_n) \in \{0,1\}^n$, we have:

$$h_I(x) = \left(\sum_{i \in I} x_i \right) \bmod 2 \, .$$

(That is, h_I computes the parity of bits in I.) What is the VC-dimension of the class of all such parity functions, $\mathcal{H}^n_{\text{parity}} = \{h_I : I \subseteq \{1, 2, \ldots, n\}\}$?

(b) *Axis-aligned rectangles*: For two vectors $a, b \in \mathbb{R}^n$ with $a \leq b$ (coordinate-wise), we define an axis-aligned rectangle $h^{a,b}_{\text{rec}} : \mathbb{R}^n \to \{0,1\}$ as $h^{a,b}_{\text{rec}}(x) = 1$ if $a \leq x \leq b$ and $h^{a,b}_{\text{rec}} = 0$ otherwise. Let $\mathcal{H}^n_{\text{rec}}$ be the class of all axis-aligned rectangles in $\mathbb{R}^n$. What is the VC-dimension of the class $\mathcal{H}^n_{\text{rec}}$?

(c) *Boolean conjunctions*: Let $\mathcal{H}^n_{\text{con}}$ be the class of Boolean conjunctions over the variables $x_1, \ldots, x_n$. What is the VC-dimension of the class $\mathcal{H}^n_{\text{con}}$?

Problem 4. (10 points)
Properties of VC dimension

(a) Monotonicity of VC dimension: Do Exercise 1 in Chapter 6 of the Shalev-Shwartz–Ben-David textbook.

(b) VC dimension versus log of class size: Do Exercise 7 in Chapter 6 of the Shalev-Shwartz–Ben-David textbook.

(c) VC dimension of union: Do Exercise 11 in Chapter 6 of the Shalev-Shwartz–Ben-David textbook.