CSCI699: Topics in Learning and Game Theory

Lecture 9 (Part-1)

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1 Recap on Myerson's Auction

Continuation from the last class regarding Myerson's Auction. We were considering a single item auction. The seller knows the exact distribution of value of the biddders (but not the actual outcomes), and wants to maximize the expectation of revenue by solving an optimization problem. This is like a second price auction on virtual values. The item is given to the bidder with the highest (non-negative) virtual value.

Now, we are interested in the empirical form of Myerson's auction. In this case, the exact distribution is not known, rather some samples drawn from that distribution are known to the seller.

Recommended reading: *Cole and Roughgarden 2014* (Sample Complexity of Revenue Maximization)

2 Empirical Myerson Auction

When the seller does not know the exact distribution of bidder's valuation, and only has some samples drawn from that distribution, the most natural thing to do would be:

- Construct empirical distribution of valuation $\hat{F}_i$'s from the drawn samples. $\hat{F} = \hat{F}_1 \times \hat{F}_2 \times ... \times \hat{F}_k$.
- Run Myerson's auction on $\hat{F}_i$'s.

If this approach works, it can be computationally efficient. But how many samples would we need for this? The answer is $poly(k, \frac{1}{\epsilon})$, where k is the number of bidders. Note: if the distributions are IID, k does not matter; otherwise it does if they are not identical.

However, this above mentioned approach actually does not work, but some variant of this method works.

Why the empirical Myerson auction does not work? Why is this problem hard? The distribution can be very complex, and there can be more complications (some described below).

There exists a true product distribution on valuation $F = F_1 \times F_2 \times ... \times F_k$.

Assumption for sample case: F_i's are 'regular' distribution. Recall that distribution F_i is called regular if virtual value $\phi(v_i)$ is non-decreasing. $\phi(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$

For a regular distribution, item is given to the bidder with highest virtual value. If the distribution is not regular, 'ironed' virtual value is used instead.

The problem with the empirical method is that even if F_i's are regular, $\hat{F}_i$'s can be non-regular. However, even if $\hat{F}_i$'s are regular or we have many samples, the empirical method still will not work. The reason is that F_i's and $\hat{F}_i$'s are not pointwise close, they are close in a more complicated way. And- the error needs to be bounded as well.

There may be two bidders with very close values and virtual values. Thus somehow the sampled version of distribution may result in a slightly different virtual value and give the item to a different person.

3 Variant of Empirical Myerson Auction

Below is a variant of the empirical method which works. Here,
F_i = true distribution of bidder i; k = number of bidders

3.1 Algorithm

1. Draw m IID samples from each F_i. Let v_{ij} sample values, where i = bidder's id, j = sample id, and $1 \leq j \leq m\dot{\text{O}}$rder the samples such that $v_{i1} \geq v_{i2} \geq v_{im}$.
2. Throw away $\left\lfloor \hat{\xi} \cdot m - 1 \right\rfloor$ samples, for some $\hat{\xi} = \frac{a}{m} > 0$. Let S denote the set of remaining samples.
3. For each $v_{ij} \in S$, plot a point $(\frac{2j-1}{2m}, \frac{2j-1}{2m} v_{ij})$. (The 'empirical quantile' of v_{ij} is $\frac{2j-1}{2m}$).

4. Add points (0,0), (1,0)
5. Define 'empirical revenue curve' $R_i(\bar{q})$: the curve with straight line segments joining the points m step 3 and 4.
6. Take the convex hull of this point set. This gives the 'ironed empirical revenue curve' $\bar{R}(q)$. This has constant slope between any two consecutive empirical quantile of points in S.
7. Define Empirical Ironed virtual values:

   For $v > v_i$, $\hat{\xi}_m \Rightarrow \bar{\phi(v)} = v$. For $v \leq v_i, \hat{\xi_m}$, find the two samples $v_{ij}, v_{i,j+1} \in S$ that 'sandwich' v. The empirical ironed virtual valuation (EIVV) of point v is the slope of the curve $\bar{R}_i$ in the interval defined by the empirical quantiles of $v_{ij}, v_{i,j+1}$.
8. Run Myerson on the EIVV's.

3.2 Analysis

We need to relate empirical expected valuation with actual expected valuation.

1. All but bottom $\hat{\xi}$ fraction of the empirical quantile are good multiplicative approximates to their expected values (with high probability).
2. Empirical ironed virtual value is approximated to the true virtual value.