

Kernel Methods

Clustering

Problem 1

For this problem, you will prove why the following proposed function is or is not a valid kernel:

$$k(x, z) = (x^T z)^2 + e^{(-\|x\|^2 - \|z\|^2)}$$

Problem 2

What is the corresponding $\phi(x)$ for the Gaussian kernel of 2-dimensional vectors:

$$k(x, z) = e^{-\|x-z\|^2}$$

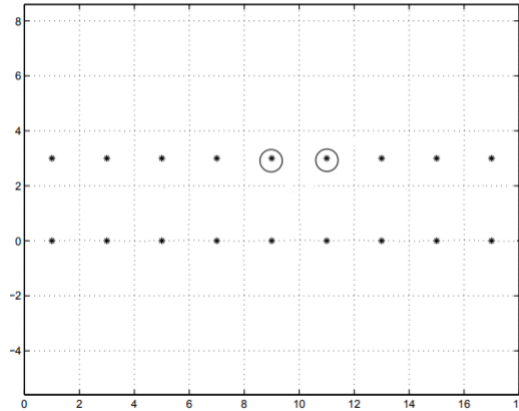
Problem 3

Kernels in Logistic Regression

- (1) Extend it to mapping ϕ .
- (2) Kernelize the algorithm by kernel k .

Problem 4

Consider the data plotted below, which consist of two rows of equally spaced points. k-means clustering ($k = 2$) is initialized with the two points that are circled. Indicate the final clusters obtained and their centroids.



Problem 5

Is it possible to get empty clusters during the k-means algorithm execution? If, so provide an example.

Problem 6

Can k-means algorithm be kernelized?

- (1) Given assignments, how will you update centers?
- (2) How do you compute the square distance between two data points.

Problem 7

k-means algorithm is an iterative algorithm that finds the centers that minimize the sum of squared distances to the points.

Let's consider another objective function: maximizing spacing. We define the spacing of a set of clusters as the distance between the closest pair of points in *different* clusters. For a good clustering, we would expect the spacing to be large.

Design an algorithm to solve this problem.