1/5/2010

Lecture 1:

- Fault-tolerance problem
- Noise threshold intuition
- Noise & tradeoff w/ resources
- Recent progress
- Quantum EC codes
- Discrete vs. continuous errors

Fault-tolerance problem:

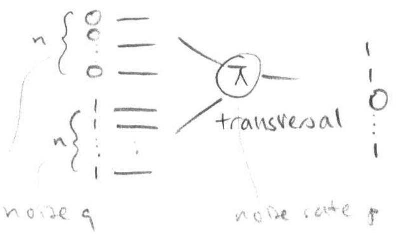
implement scalable qu. computers in presence of noise

1% noise ⇒ only 100 operations

factoring n bit # uses $72n^3$ op's

Classically: How does your brain work?

Obvious solution: Encode the data



$> \wedge \quad > \wedge$

$\Rightarrow$ M+Js

p 0s $\quad 1-p$ 1's

$\Rightarrow \underbrace{p^3 + 3p^2(1-p)}_{O(p^2)}, \quad 1-q$

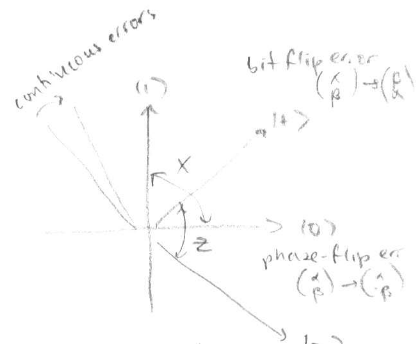
p small enough steady-state error prob. of deviations $e^{-\Omega(n)}$

p too large ⇒ errors grow out of control

T-gate circuit choose $n = \log T$

Quantumly: - more noise, different types of noise

- more constrained resources



measurements that can collapse state

Operational def. of quantum ECC:

data $|\psi\rangle \xrightarrow{\text{encode}}$ encoded data $\xrightarrow{\text{noise}} N^{\otimes n}(\mathcal{E}(|\psi\rangle)) \xrightarrow{}$ append ancilla & copy "syndrome" into it

undo errors on encoded data & invert the encoding

$\downarrow$ decode

$|\psi\rangle$

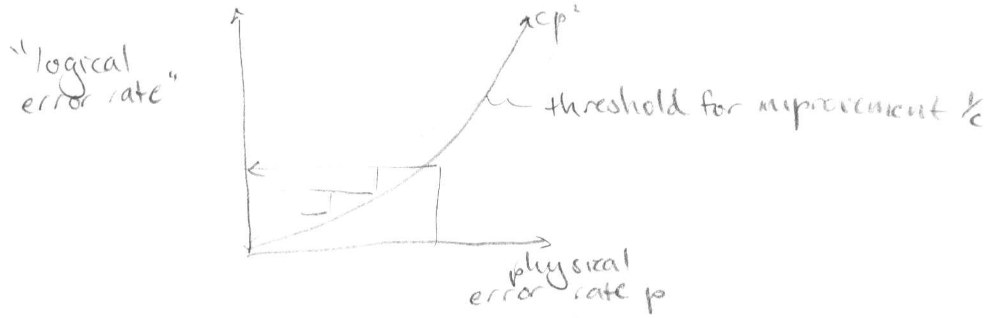
Using QECCs: * need operations as well as memory

* error recovery must be resilient to faults

* how to encode into them in the first place ??

Concatenated coding: start small

* compile ideal ckt into fault-tolerant version using small QECC



* concatenate (repeat) for arbitrarily improved reliability so long as start below the threshold

ℓ levels $\Rightarrow$ $\frac{1}{c}(cp)^{2^\ell}$

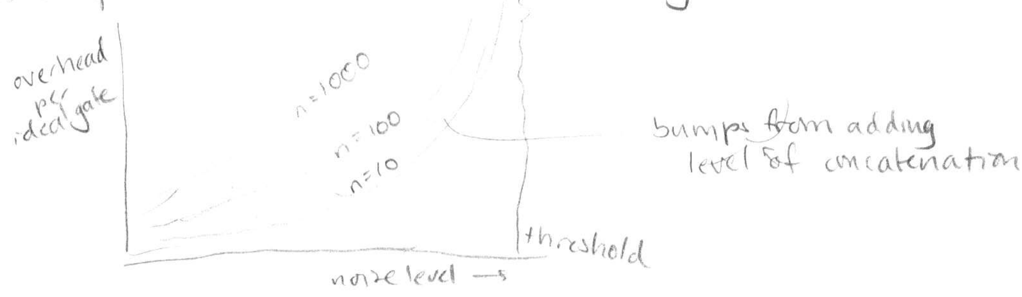
$p, \; cp^2, \; c(cp^2)^2, \ldots$

* overhead: to get effective error rate $\frac{1}{n}$, need $\log\log n$ levels,

$\Rightarrow$ $O(\log n)$ overhead

Basic problem:

- given noise model, maximize threshold
- given noise & physical & resource constraints, how large a computation can be simulated reliably?



Other approaches: * Topological QC

* Self-correcting systems

summary of known threshold results

$\sim 10^{-3}$ proven 1% w/ high overhead

$\sim 10^{-4}$

What kind of noise?

Not global: 1% chance/step of an asteroid destroying the earth

But global control can overcome local noise

- e.g., independent or w/ bdd dependencies

  then highly coordinated errors are exponentially unlikely

  Chernoff: X_i independent Bernoulli, $\mathbb{E}X_i = v_i$, $v = \sum_i v_i$

  $$\Pr\left[|X = \sum_i (X_i - v_i)| \geq t\right] \leq e^{-ct^2/2(v+t)}$$

* local Hamiltonians

- ok but gives quadratically lower thresholds than stochastic models

  threshold given in terms of the norm of the interaction H

  (but "probability of error is quadratically smaller")

* local stochastic noise

any r specific physical operations fail w/ prob. $\leq \varepsilon^r$

otherwise adversarial

* depolarizing noise (generic stochastic)

$$S_p(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$$

$$= (1 - \tfrac{4}{3}p)\rho + \tfrac{4}{3}\tfrac{p}{2} I$$

* dephasing: w/prob p, a phase error Z — often dominates other noise on resting qubits (symmetrical to X)

"T_2" time $\quad p = 1 - e^{-t/T_2}$

channel capacity is $>18\%$ and $\leq 25\%$ — not the bottleneck is coding

* amplitude damping

relaxation from $|1\rangle$ to $|0\rangle$ "T_1 time" $T_2 \leq T_1$

$$S(\rho) = A_0\rho A_0^\dagger + A_1\rho A_1^\dagger \qquad A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma^2} \end{pmatrix} \quad A_1 = \begin{pmatrix} 0 & \gamma \\ 0 & 0 \end{pmatrix}$$

* erasure errors: an extra classical register created saying whether or not error occurred — e.g., qubit loss

leakage errors

Qu error-correcting codes (expt & can be based on classical linear codes)

Classical ECC:

$C \subseteq \{0,1\}^n$

$\forall x,y \in C,\ x \neq y,\ d(x,y) = |x \oplus y| \geq d$

$[n,k,d] \Rightarrow |C| = 2^k$

eg. $[3,1,3]$: $\{000, 111\}$

Quantumly: more kinds of errors (continuous), more careful EC

also works quantumly

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \mapsto \alpha|000\rangle + \beta|111\rangle = |\bar{\psi}\rangle$

protects against $\{I, X_1, X_2, X_3\}$

$X_1|\bar{\psi}\rangle = \alpha|100\rangle + \beta|011\rangle \quad (\neq \alpha|000\rangle + \beta|011\rangle)$

① Find the error (w/o collapsing data)

f: $000, 111 \mapsto 0$

$100, 011 \mapsto 1$

$010, 101 \mapsto 2$

$001, 110 \mapsto 3$

$\rightarrow \alpha|100\rangle|1\rangle + \beta|011\rangle|1\rangle = X_1|\bar{\psi}\rangle \otimes |1\rangle$

② Correct it $\quad |1\rangle\langle 1| \otimes X_1 + |2\rangle\langle 2| \otimes X_2 + |3\rangle\langle 3| \otimes X_3$

but no protection against dephasing

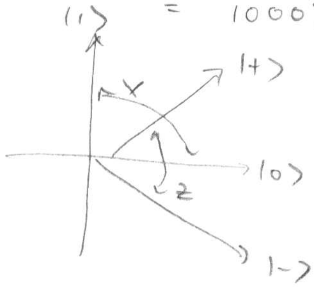
also works against cont. errors

eg. $U = c_0 \mathbb{1} + c_1 X_1 + c_2 X_2 + c_3 X_3$

$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$\sqrt{2} Z_1 |\bar{+}\rangle = Z_1(|000\rangle + |111\rangle)$

$(1) = |000\rangle - |111\rangle = \sqrt{2}|\bar{-}\rangle$



duality: use $|+\rangle \mapsto |+++\rangle = \sum_{x \in \{0,1\}^3} |x\rangle / \sqrt{8}$

$|-\rangle \mapsto |---\rangle$

Code concatenation:

$|+\rangle \mapsto |+\rangle^{\otimes 3} \mapsto (|000\rangle + |111\rangle)^{\otimes 3}$

$|-\rangle \mapsto |-\rangle^{\otimes 3} \mapsto (|000\rangle - |111\rangle)^{\otimes 3}$

works on linear combs of errors: Paulis form a basis