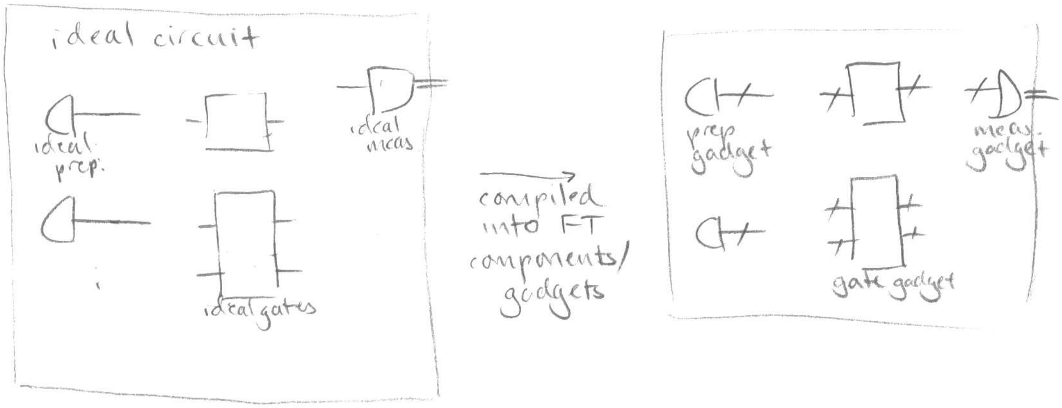
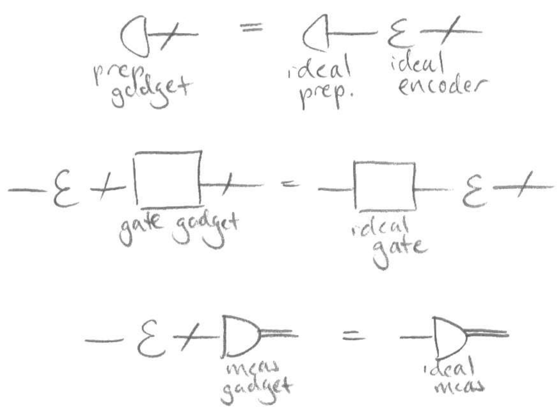
Lecture 7 2/2/10 Fault-tolerant error correction

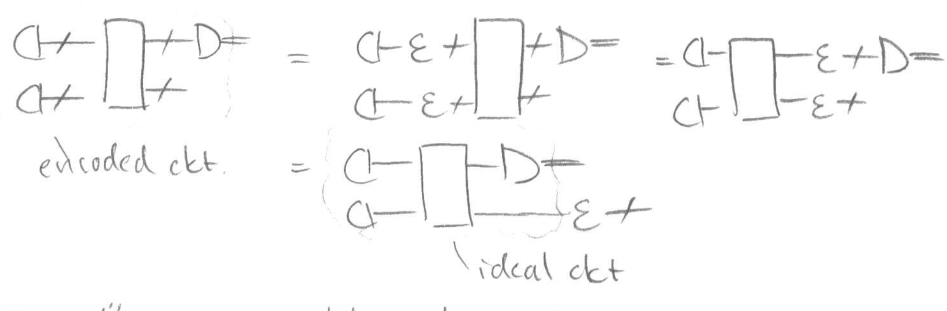
Goal of fault tolerance



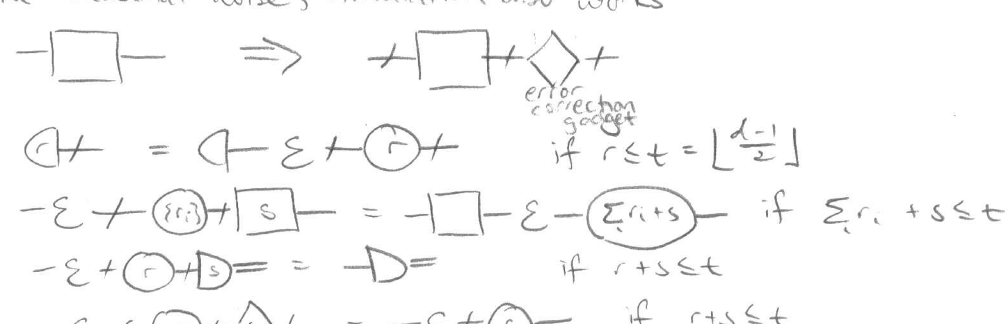
① With 0 noise, compiled ckt should simulate an ideal ckt.



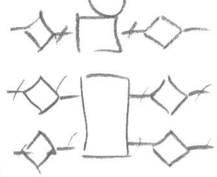
Thus:

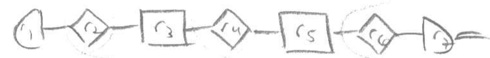


② With "occasional" noise, simulation also works



Call an "extended rectangle" (exRec) a gadget plus the preceding and following ECs





$\Rightarrow$ If every exRec has $\le t$ noise locations, the simulation works.

- we'll talk more about this definition later

③ In general, need to be able to recover noisy code blocks...

$$-\mathcal{E}+(r)+\langle s\rangle+ \;=\; -(1)-\mathcal{E}+(s)+ \quad \text{if } s \le t$$

Today: Fault-tolerant error correction

$$-\mathcal{E}+(r)+\langle s\rangle+ \;=\; -\mathcal{E}+(s)+ \quad \text{if } r+s \le t$$

$$-\mathcal{E}+(r)+\langle s\rangle+ \;=\; -(1)+\mathcal{E}+(s)+ \quad \text{if } s \le t$$

(Note: = can be replaced by $\le$.)

Example: Biased noise only bit-flip (X) errors

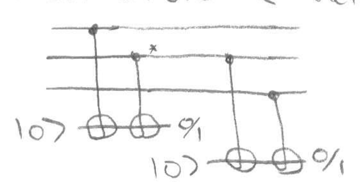
$0 \longrightarrow 000$ $\qquad ZZ1$

$1 \longrightarrow 111$ $\qquad 1ZZ$

$\qquad\qquad \bar{X} = XXX$

$d=3,\ t=1$ $\qquad \bar{Z} = Z11$

correct errors $\Leftarrow$ determine the syndrome $\Leftarrow$ measure the stabilizers



Corrections

	0	1
0	111	11X
1	X11	1X1

Fault tolerant? Rule 2 ✓ obvious since $d=3$

Rule 1 $r=1, s=0$ ✓ obvious

$r=0, s=1$ ✗ an error at * $\to$ 1XX error

Solutions? $r=0, s=0$ ✓ stabilizer algebra

- for an FT EC proc. based on syndrome meast.

s_1 ZZ1 $\quad$ s_1 ZZ1

s_2 1ZZ $\;\longrightarrow\;$ s_2 1ZZ

Z1 $\quad$ ZZ1Z1

1Z $\quad$ 1ZZ1Z

XXX $\quad$ XXX

ZZZ $\quad$ ZZZ

- repeat both measurements 2x

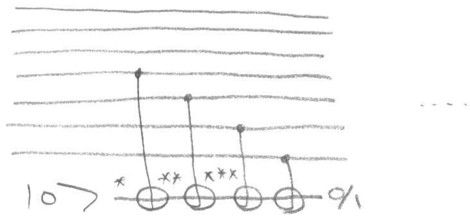
apply correction only if all measts agree

$r=0,\ s=1 \Rightarrow$ no correction applied, one error survives

(note: suffices to repeat first stabilizer meas. only)

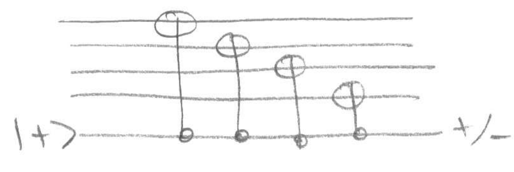
② Steane code

0001111
0110011
1010101



repeated syndrome extraction

for correcting Z errors, meas X stabilizer syndromes:



use EC

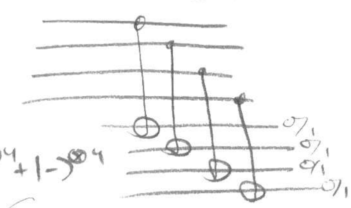
Fault tolerant?

X EC FT vs. X errors ✓

Z errors? -on ancilla * ✓ ** ✓ *** X

- interacting diffuse encoded data w/ single-qubit ancilla ×

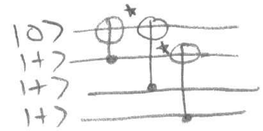
Solutions? "Shor EC"



$|+\rangle^{\otimes 4} + |-\rangle^{\otimes 4}$

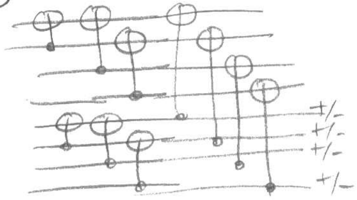
ZZZZ

ZZZZ
XX11
1XX1
11XX

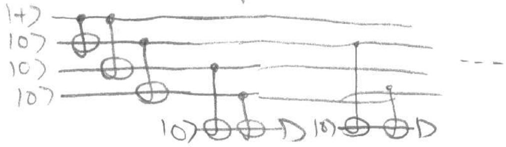


Not FT!

- Verify the ancilla before using it.

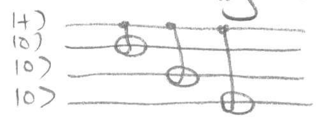


OR: compare bits



Optimized Shor EC:

→ Note: Only some correlated errors possible.



one X error can create

X 1 1 1
1 X 1 1
1 1 X 1
1 1 1 X
X 1 1 X

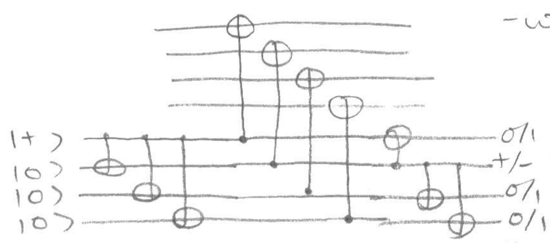
not X X 1 1 ...

①

⇒ 

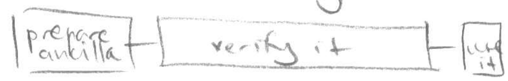
suffices?

② FT QC w/ slow measurements [Aliferis & DiVincenzo] 0607047



= syndrome

– want one bit, measuring 4



if ancilla qubits 1 & 4 are both flipped, flip data bits 1 & 4

still need repeated syndrome extraction

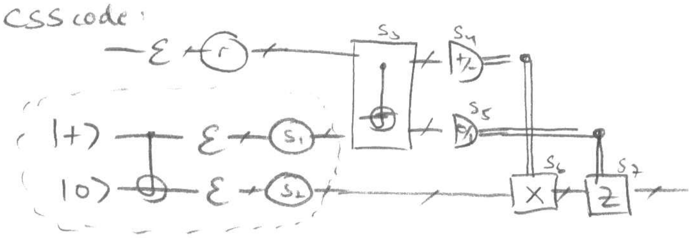
Main disadvantage: works poorly for larger codes

Next: Steane EC, Knill EC

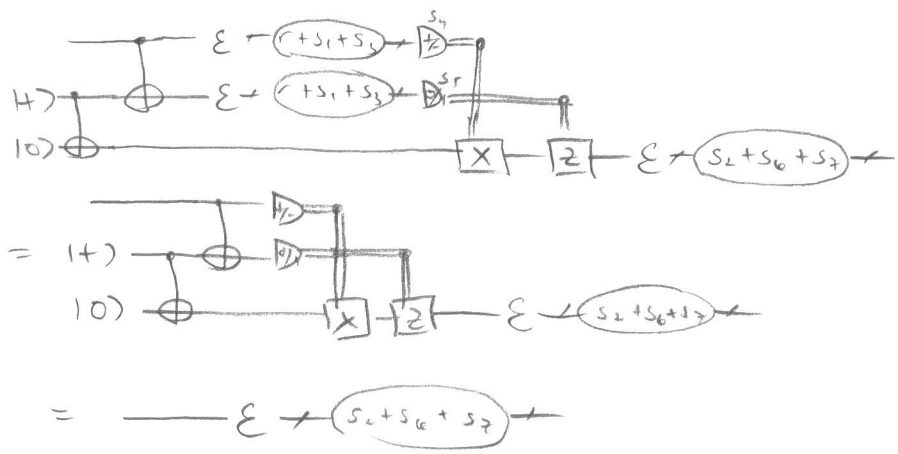
– also reduce FT EC to FT state prep?

Knill EC: reduces FT EC to FT prep of encoded Bell pairs

use a CSS code:



$r+s_1+s_2+s_3+s_4+s_5 \leq t$
$+s_6+s_7$



Steane EC: Assume we have FT prep of $|\bar{0}\rangle$ and $|\bar{+}\rangle$.