3/9/10 Topological QC equivalent to general QC

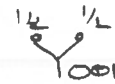
Last time Anyons

Examples: Fib $= SO(3)_3$ particle types $\{0, 1\}$ $1 \otimes 1 = 0 \oplus 1$

i.e. univalent vertices disallowed ↑trivial

$SU(2)_k$ particle types $\{0, \tfrac{1}{2}, \ldots, \tfrac{k}{2}\}$

Qubits $SU(2)$ with $\tfrac{1}{2}$s fixed

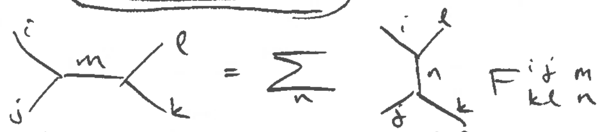


angular momentum basis

$n=1: |\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle$ $n=2: |0,0\rangle, |1,\pm 1\rangle, |1,0\rangle$

$n=3$:

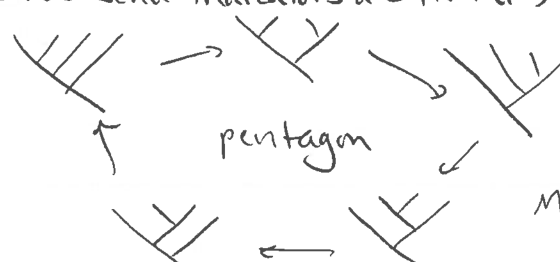
basis change: F moves



$= \sum_n F^{ij\,m}_{kl\,n}$

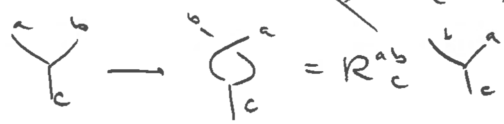
("if Frobenius-Schur indicators are trivial")

consistency



Mac Lane's thm.

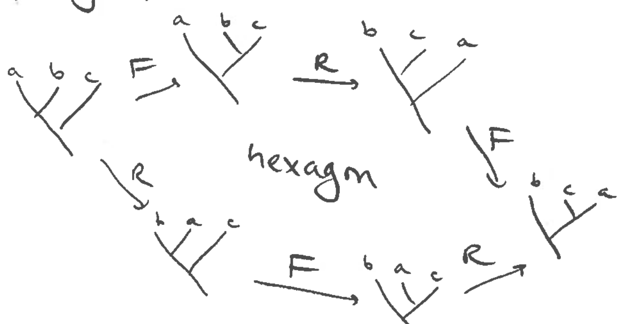
braiding



$= R^{ab}_{c}$

Bosons, Fermions, anyons, non-abelions

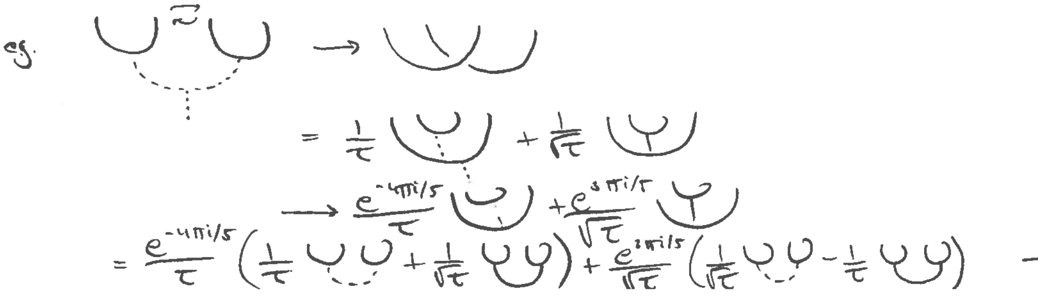
consistency



(in coordinates)

$$\text{Fib}: \begin{pmatrix} R^{11}_{0} & 0 \\ 0 & R^{11}_{1} \end{pmatrix} = \begin{pmatrix} e^{-4\pi i/5} & 0 \\ 0 & e^{3\pi i/5} \end{pmatrix}$$

e.g.

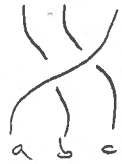


$= \frac{1}{\tau}(\ldots) + \frac{1}{\sqrt{\tau}}(\ldots)$

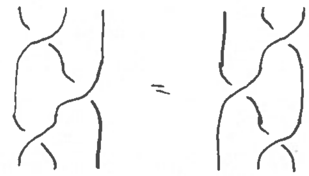
$\longrightarrow \frac{e^{-4\pi i/5}}{\tau}(\ldots) + \frac{e^{3\pi i/5}}{\sqrt{\tau}}(\ldots)$

$= \frac{e^{-4\pi i/5}}{\tau}\left(\frac{1}{\tau}(\ldots) + \frac{1}{\sqrt{\tau}}(\ldots)\right) + \frac{e^{3\pi i/5}}{\sqrt{\tau}}\left(\frac{1}{\sqrt{\tau}}(\ldots) - \frac{1}{\tau}(\ldots)\right)$ —

Anyon model defines a representation of the braid group



hexagon equation



Simulation by a quantum computer

- encoding
- initialization
- braiding in the standard basis
- measurement

Simulation <u>of</u> a q.c.

braiding to implement gates, avoiding leakage

<u>Theorem</u>: [Freedman, Larsen, Wang '02]: Fib w/ ≥ 3 particles, braids are dense. (up to phase)

[Freedman, Kitaev, Larsen, Wang '03]: $SU(2)_k$ dense

pure braids $n = 3$ $k \in \{3, 5, 6, 7\}$ or $k \geq 9$

$n \geq 4$ $k = 3$ or $k \geq 5$

[Simon, Bonesteel, Freedman, Petrovic, Hormozi '06]: ditto for pure weaves

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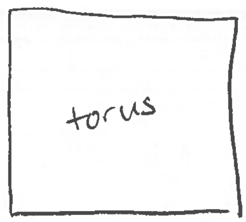
Implementations of anyon models

--- in physical systems. eg. FQHE

--- on qubit lattices

Toric code



basic code □ plaquettes

vertex & plaquette stabilizers

dimension of the codespace, logical operators

errors & error correction

anyons as excitations

Generalizations: other lattices, other surfaces

$v - e + f = 2 - 2 \cdot \text{genus} - \text{punctures}$