

QIC 710 Problem Set 1

1. Prove that the tensor product of two unitary operators is unitary.
2. Alice and Bob share an arbitrarily long common random string. Alice is given as input a random bit x_A and Bob a random bit x_B . Without communicating with each other, Alice and Bob wish to output bits a and b , respectively, such that $x_A \wedge x_B = a \oplus b$. Prove that any protocol that Alice and Bob follow has success probability at most $3/4$.
3. Alice, Bob and Charlie share a GHZ state, of three qubits:

$$\frac{1}{2}(|000\rangle - |011\rangle - |101\rangle - |110\rangle).$$

They are given as input bits X_A, X_B, X_C , respectively, which satisfy the condition $X_A \oplus X_B \oplus X_C = 0$. They wish to output a, b, c such that $X_A \vee X_B \vee X_C = a \oplus b \oplus c$, but cannot communicate. Give a quantum protocol to achieve this with certainty. What is the maximum probability with which you can achieve this classically?

4. You are given one of two quantum states of a single qubit, either $|\varphi\rangle = |0\rangle$ or $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$. What measurement best distinguishes between these two states? If the state you are presented is either $|\varphi\rangle$ or $|\psi\rangle$ with 50% probability each, what is the probability that your measurement correctly identifies the state?