Due Th 10/17/24 (10am; on DEN Webpage)

1. Compute A^t and e^{At} for the matrices

$$A_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

2. For the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u$$

   a) Determine stability properties.
   b) Determine the transfer function.

3. For the nonlinear system

$$\begin{aligned} \dot{x}_1 &= -x_1^2 + x_1 x_2 \\ \dot{x}_2 &= -2x_2^2 + x_2 - x_1 x_2 + 2 \end{aligned}$$

   (a) Show that $\bar{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ is an equilibrium point.
   (b) Is $\bar{x}$ the only equilibrium point?
   (c) Linearize this system around $\bar{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ and find the resolvent and the state-transition matrix of the resulting linearized system.
   (d) What can you say about the stability properties of $\bar{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ for the original nonlinear system based on linearization?

4. Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= -\frac{x_2}{1+x_1^2} - 2x_1 \\ \dot{x}_2 &= \frac{x_1}{1+x_1^2}. \end{aligned}$$

   (a) Show that the origin is an equilibrium point.
   (b) Linearize the nonlinear system around the equilibrium point.
   (c) Obtain a suitable Lyapunov function for the linearized system by solving the Lyapunov equation

$$A^T P + PA = -Q,$$

   where

$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$