Lyapunov-based state-feedback distributed control of systems on lattices

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Abstract

We investigate the properties of systems on lattices with spatially distributed sensors and actuators. These systems arise in a variety of applications such as the control of vehicular platoons, formation of unmanned aerial vehicles, arrays of microcantilevers, and satellites in synchronous orbit. We use a Lyapunov-based framework as a tool for stabilization/regulation/asymptotic tracking. We first present results for nominal design and then describe the design of adaptive controllers in the presence of parametric uncertainties. These uncertainties are assumed to be temporally constant, but are allowed to be spatially varying. We show that the design yields decentralized distributed controllers with the passage of information determined by the interactions between different plant units. We also provide several examples and validate derived results using computer simulations of systems containing a large number of units.

1 Introduction

Systems on lattices are encountered in a wide range of modern technical applications. Typical examples of such systems include: platoons of vehicles ([1, 2]), arrays of microcantilevers [3], unmanned aerial vehicles in formation [4], and satellites in synchronous orbit [5]. These systems are characterized by the interactions between different subsystems which often results in surprisingly complex behavior. A distinctive feature of this class of systems is that every single unit is equipped with sensors and actuators. The controller design problem is thus dominated by architectural questions such as localized versus centralized control, and the information passing structure in both the plant and the controller. This is in contrast with 'spatially lumped' control design problems, where the dominant issues are optimal and reduced order controller design.

A framework for considering spatially distributed systems is that of a spatio-temporal system [6]. In the specific case of systems on discrete spatial domains, signals of interest are functions of time and a spatial variable $n \in \mathbb{F}$, where $\mathbb{F}$ is a discrete spatial lattice (e.g. $\mathbb{Z}$ or $\mathbb{N}$).

In this paper, we study distributed control of systems on lattices. We use a Lyapunov-based approach to provide stability/regulation/asymptotic tracking of nominal systems and systems with parametric uncertainties. In the latter case, we assume that the unknown parameters are temporally constant, but are allowed to be spatially varying. We design adaptive Lyapunov-based estimators and controllers to guarantee boundedness of all signals in the closed-loop in the presence of parametric uncertainties. In addition to that, the adaptive controllers provide convergence of the states of the original system to their desired values. We also show that the distributed design results in controllers whose information passing structure is similar to that of the original plant. This means, for example, that if the plant has only nearest neighbor interactions, then the distributed controller also has only nearest neighbor interactions.

Stabilizing controllers are designed using the technique of *backstepping*. Backstepping is a well-studied design tool [7, 8] for finite dimensional systems. In the infinite dimensional setting, a backstepping-like approach can be used to obtain stabilizing boundary feedback control laws for a class of parabolic systems (see [9, 10] for details). Backstepping boundary control can also be used as a tool for vibration suppression in flexible-link gantry robots [11]. However, backstepping has not been applied to distributed control of systems on lattices to the best of our knowledge.

Our presentation is organized as follows: in section 2, we give an example of systems on lattices and describe the classes of systems for which we design state-feedback controllers in § 3. In § 4, we discuss application of controllers developed in § 3, analyze their structure, and validate their performance using computer simulations of systems containing a large number of units. We conclude by summarizing major contributions and future research directions in § 5.

2 Systems on lattices

In this section an example of systems on lattices is given. In particular, we consider a mass-spring system on a line. This system is chosen because it represent a simple non-trivial example of an unstable system where the interactions between different plant units are caused by the physical connections between them. Another example of systems with this property is given by an array of microcantilevers [3]. The interactions between different plant units may also arise because of a specific control objective that we want to meet. Examples of systems on lattices with this property include: a system of cars in an infinite string, aerial vehicles and spacecrafts in formation flights. We also describe the classes of systems for which we design state-feedback controllers in § 3.

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2.1 An example of systems on lattices

A system consisting of an infinite number of masses and springs on a line is shown in Figure 1. The dynamics

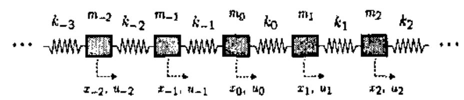

Figure 1: Mass-spring system.

of the n-th mass are given by

$$m_n \ddot{x}_n \;=\; F_{n-1} \;+\; F_n \;+\; u_n, \quad n \in \mathbb{Z}, \tag{1}$$

where x_n represents the displacement from a reference position of the n-th mass, F_n represent the restoring force of the n-th spring, and u_n is the control applied on the n-th mass. For relatively small displacements, restoring forces can be considered as linear functions of displacements $F_n = k_n(x_{n+1} - x_n)$, $F_{n-1} = k_{n-1}(x_{n-1} - x_n)$, $n \in \mathbb{Z}$, where k_n is the n-th spring constant. We also consider a situation in which the spring restoring forces depend nonlinearly on displacement. One such model is given by the so-called *hardening spring* (see, for example [7]) where, beyond a certain displacement, large force increments are obtained for small displacement increments

$$\begin{aligned} F_n &= k_n\left\{(x_{n+1} - x_n) + c_n^2(x_{n+1} - x_n)^3\right\} \\ &=: k_n(x_{n+1} - x_n) + q_n(x_{n+1} - x_n)^3, \\ F_{n-1} &= k_{n-1}\left\{(x_{n-1} - x_n) + c_{n-1}^2(x_{n-1} - x_n)^3\right\} \\ &=: k_{n-1}(x_{n-1} - x_n) + q_{n-1}(x_{n-1} - x_n)^3. \end{aligned}$$

For both cases (1) can be rewritten in terms of its state-space representation $\forall\, n \in \mathbb{Z}$ as

$$\begin{aligned} \dot{\psi}_{1n} &= \psi_{2n}, \\ \dot{\psi}_{2n} &= f_n(\psi_{1,n-1}, \psi_{1n}, \psi_{1,n+1}) \;+\; \kappa_n u_n, \end{aligned} \tag{2}$$

where $\psi_{1n} := x_n$ and $\psi_{2n} := \dot{x}_n$.

In the particular situation in which the restoring forces are linear functions of displacements and all masses and springs are homogeneous, that is, $m_n = m =$ const., $k_n = k =$ const., $\forall\, n \in \mathbb{Z}$, (2) represents a linear *spatially invariant* system. This implies that it can be analyzed using the tools of [12, 13]. The other mathematical representations of a mass-spring system are either nonlinear or spatially-varying. The main purpose of the present study is to design state-feedback controllers for this broader class of systems.

2.2 Classes of systems

In this subsection, we summarize the classes of systems for which we design state-feedback controllers in § 3. In particular, we consider second order systems parameterized by $n \in \mathbb{F}$ with finite number of interconnections with other plant units. Clearly, the models presented in § 2.1 belong to this class of systems, as well as the model of an array of microcantilevers [3]. Furthermore, our results can be readily extended to a class of fully actuated two and three-dimensional mechanical systems.

We consider state-feedback design for nominal systems of the form

$$\dot{\psi}_{1n} \;=\; \psi_{2n}, \qquad n \in \mathbb{F}, \tag{3a}$$

$$\dot{\psi}_{2n} \;=\; f_n(\psi_1, \psi_2) \;+\; \kappa_n u_n, \qquad n \in \mathbb{F}, \tag{3b}$$

and systems with parametric uncertainties of the form

$$\dot{\psi}_{1n} = \psi_{2n}, \qquad n \in \mathbb{F}, \tag{4a}$$

$$\dot{\psi}_{2n} = \tau_n(\psi_1, \psi_2) + h_n^*(\psi_1, \psi_2)\theta_n + \kappa_n u_n, \quad n \in \mathbb{F}, \tag{4b}$$

where $\psi_1 := \{\psi_{1n}\}_{n \in \mathbb{F}}$, $\psi_2 := \{\psi_{2n}\}_{n \in \mathbb{F}}$, and κ_n's are the so-called *control coefficients* [8]. In the latter case, the unknown parameters θ_n and κ_n are assumed to be temporally constant, but are allowed to be spatially varying.

We introduce the following assumptions about the systems under study:

Assumption 1 *The number of interconnections between different plant units is uniformly bounded. In other words, there exist $M \in \mathbb{N}$, $M \neq M(n)$, such that f_n, h_n, and τ_n depend on at most M elements of ψ_1 and ψ_2.*

Assumption 2 *f_n, h_n, and τ_n are known, continuously differentiable functions of their arguments.*

Assumption 3 *The signs of κ_n, $\forall\, n \in \mathbb{F}$, in (4b) are known.*

These assumptions are used in the sections related to the distributed control design. Furthermore, under these assumptions the well-posedness of both open and closed-loop systems can be easily established.

3 Lyapunov-based distributed control design

In this section, we address the problem of designing controllers that provide stability/regulation/asymptotic tracking of systems described in § 2.2. Assuming that the full state information is available and that every unit is equipped with sensors and an actuator, we use the Lyapunov-based approach to solve this problem. The Lyapunov design is very suitable because it leads to distributed controllers that are not centralized. This feature is of paramount importance for practical implementation. It is also remarkable that the basic ideas of finite dimensional Lyapunov-based adaptive design [8] are easily extendable to this infinite dimensional setting.

3.1 Nominal state-feedback design

We first consider Lyapunov design for systems without any parametric uncertainties. We observe that system (3) is amenable to be analyzed by the backstepping design methodology. Even though a stabilizing controller can be designed using various tools, we choose backstepping because it gives both a stabilizing feedback law and a Control Lyapunov Function (CLF) for a system under consideration. Once CLF is constructed its derivative can be made negative definite using a variety of control laws.

In the first step of backstepping, equation (3a) is stabilized $\forall\, n \in \mathbb{F}$ by considering $\psi_2 = \{\psi_{2n}\}_{n \in \mathbb{F}}$ as its control. Since ψ_2 is not actually a control, but rather,

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a state variable, the error between ψ_2 and the value which stabilizes (3a) must be penalized in the augmented Lyapunov function at the next step. In this way, a stabilizing control law is designed for the overall system.

Before we illustrate the distributed Lyapunov-based design we introduce the following assumption:

Assumption 4 *The initial distributed state is such that both $\psi_1(0) \in l_2$ and $\psi_2(0) \in l_2$.*

Step 1 The recursive design starts with subsystem (3a) by proposing a CLF of the form

$$V_1(\psi_1) \;=\; \frac{1}{2}\langle \psi_1, \psi_1 \rangle \;:=\; \frac{1}{2}\sum_{n \in \mathbb{F}} \psi_{1n}^2. \qquad (5)$$

The derivative of $V_1(\psi_1)$ along the solutions of (3a) is given by

$$\dot{V}_1 \;=\; \left\langle \psi_1, \dot{\psi}_1 \right\rangle \;=\; \langle \psi_1, \psi_2 \rangle \;=\; \sum_{n \in \mathbb{F}} \psi_{1n}\psi_{2n}. \qquad (6)$$

In particular, the choice of a 'stabilizing function' ψ_{2nd} of the form $\psi_{2nd} = -a_n\psi_{1n},\ a_n > 0,\ \forall n \in \mathbb{F}$, clearly renders $\dot{V}_1(\psi_1)$ negative definite. Since ψ_2 is not actually a control, but rather, a state variable, we introduce the change of variables

$$\zeta_{2n} \;:=\; \psi_{2n} - \psi_{2nd} \;=\; \psi_{2n} + a_n\psi_{1n}, \quad \forall n \in \mathbb{F}, \quad (7)$$

which adds an additional term on the right-hand side of (6)

$$\dot{V}_1 \;=\; -\sum_{n \in \mathbb{F}} a_n\psi_{1n}^2 \;+\; \sum_{n \in \mathbb{F}} \psi_{1n}\zeta_{2n}. \qquad (8)$$

The sign indefinite term in (8) will be taken care of at the second step of backstepping.

Step 2 Coordinate transformation (7) renders (3b) into a form suitable for the remainder of our design

$$\dot{\zeta}_{2n} = a_n\dot{\psi}_{1n} + \dot{\psi}_{2n} = a_n\psi_{2n} + f_n(\psi_1,\psi_2) + \kappa_n u_n,\ n \in \mathbb{F}.$$

Augmentation of the CLF from Step 1 by a term which penalizes the error between ψ_2 and ψ_{2d} yields a function

$$V_2(\psi_1,\zeta_2) \;:=\; V_1(\psi_1) \;+\; \frac{1}{2}\langle \zeta_2, \zeta_2 \rangle, \qquad (9)$$

whose derivative along the solutions of

$$\dot{\psi}_{1n} \;=\; -a_n\psi_{1n} + \zeta_{2n}, \qquad n \in \mathbb{F}, \quad (10a)$$

$$\dot{\zeta}_{2n} \;=\; a_n\psi_{2n} + f_n(\psi_1,\psi_2) + \kappa_n u_n, \quad n \in \mathbb{F}, \quad (10b)$$

is determined by

$$\dot{V}_2 = \dot{V}_1 + \sum_{n \in \mathbb{F}} \zeta_{2n}\dot{\zeta}_{2n} = -\sum_{n \in \mathbb{F}} a_n\psi_{1n}^2 \;+$$
$$\sum_{n \in \mathbb{F}} \zeta_{2n}\left\{\psi_{1n} + a_n\psi_{2n} + f_n(\psi_1,\psi_2) + \kappa_n u_n\right\}.$$

With a control law of the form

$$u_n \;=\; -\frac{1}{\kappa_n}\left\{\psi_{1n} + a_n\psi_{2n} + f_n + b_n\zeta_{2n}\right\} \qquad (11)$$
$$\;=\; -\frac{1}{\kappa_n}\left\{(1 + a_nb_n)\psi_{1n} + (a_n + b_n)\psi_{2n} + f_n\right\},$$

where b_n's are positive design parameters $\forall n \in \mathbb{F}$, $\dot{V}_2$ becomes a negative definite function, that is

$$\dot{V}_2 \;=\; -\sum_{n \in \mathbb{F}} a_n\psi_{1n}^2 \;-\; \sum_{n \in \mathbb{F}} b_n\zeta_{2n}^2 \;<\; 0.$$

Remark 1 *Control law (11) achieves global asymptotic stability of the origin of system (3) on $\mathbb{H} := l_2 \times l_2$ by completely altering its dynamics through the process of cancellation at the second step of backstepping. As a result of feedback design, a closed-loop system that contains infinite number of* decoupled *second order linear subsystems is obtained. The designed controller is essentially a 'feedback linearization type' controller [14]. However, as a design tool, backstepping is less restrictive than feedback linearization [8]. Furthermore, the results presented here are easily generalizable to the adaptive setting, as we show in § 3.2. We also remark that, by a careful analysis of the dynamical properties of a particular system, the problem of unnecessary cancellations can be circumvented.*

Remark 2 *If the initial state of system (3) does not satisfy Assumption 4, controller (11) can still be used to attain global asymptotic stability of its origin, but on a Banach space $\mathbb{B} := l_\infty \times l_\infty$, rather than on a Hilbert space $\mathbb{H} := l_2 \times l_2$. This follows from the fact that for every* fixed *$n \in \mathbb{F}$, the derivative of $V_{2n}(\psi_{1n},\psi_{2n}) := \frac{1}{2}\psi_{1n}^2 + \frac{1}{2}(\psi_{2n} + a_n\psi_{1n})^2 > 0$, $\forall \psi_n := \begin{bmatrix} \psi_{1n} & \psi_{2n} \end{bmatrix}^* \in \mathbb{R}^2 \setminus \{0\}$, along the solutions of the 'n-th subsystem' of (3,11) is determined by $\dot{V}_{2n} = -a_n\psi_{1n}^2 - b_n(\psi_{2n} + a_n\psi_{1n})^2 < 0$, $\forall \psi_n \in \mathbb{R}^2 \setminus \{0\}$. Therefore, we conclude global asymptotic stability of the origin of the 'n-th subsystem' of (3,11) for every $n \in \mathbb{F}$, which in turn guarantees global asymptotic stability of the origin of (3,11) on a Banach space $\mathbb{B} := l_\infty \times l_\infty$. Furthermore, Assumptions 1–2 imply boundedness of u_n for every $n \in \mathbb{F}$. However, we note that $V_2(\psi_1,\psi_2)$ defined by (9) no longer represents a Lyapunov function for system (3,11) if Assumption 4 is violated.*

Remark 3 *If a control objective is to asymptotically track a reference output $r_n(t)$, with output of system (3) being defined as $y_n := \psi_{1n}$, $\forall n \in \mathbb{F}$, then it can be readily shown that the following control law*

$$u_n \;=\; -\frac{1}{\kappa_n}\{(1 + a_nb_n)(\psi_{1n} - r_n(t))$$
$$+\;(a_n + b_n)(\psi_{2n} - \dot{r}_n(t)) + f_n - \ddot{r}_n(t)\},\ n \in \mathbb{F},$$

fulfills this objective. We assume that, for every $n \in \mathbb{F}$, r_n, $\dot{r}_n$, and $\ddot{r}_n$ are known and uniformly bounded, and that $\ddot{r}_n$ is piecewise continuous.

3.2 Adaptive state-feedback design

In this subsection we consider adaptive state-feedback design. We study systems with temporally constant unknown parameters that are allowed to be spatially varying. The dynamic controllers that guarantee boundedness of all signals in the closed-loop and achieve 'regulation' of the plant state are obtained using Lyapunov-based approach.

The first step of the backstepping design is the same as in § 3.1. However, in the second step, we have to construct an Adaptive CLF to account for the error between a virtual control and its desired value, and for our lack of knowledge of parameters in (4b). In other words, we need to estimate the values of unknown parameters θ_n and reciprocals of unknown control coefficients κ_n, $\varrho_n := \frac{1}{\kappa_n}$, in order to avoid the division with an estimate of κ_n which can occasionally assume zero value.

Step 2 We start the second step of our design by augmenting CLF (9) by two terms that account for the errors between unknown parameters θ_n and ϱ_n and their estimates $\hat{\theta}_n$ and $\hat{\varrho}_n$

$$V_{a2}(\psi_1, \zeta_2, \tilde{\theta}, \tilde{\varrho}) := V_2(\psi_1, \zeta_2) + \frac{1}{2}\sum_{n\in\mathbb{F}} \tilde{\theta}_n^* \Gamma_n^{-1} \tilde{\theta}_n + \frac{1}{2}\sum_{n\in\mathbb{F}} \frac{|\kappa_n|}{\beta_n} \tilde{\varrho}_n^2, \qquad (12)$$

where $\tilde{\theta}_n(t) := \theta_n - \hat{\theta}_n(t)$, $\tilde{\varrho}_n(t) := \varrho_n - \hat{\varrho}_n(t)$, Γ_n is a positive definite matrix, and β_n is a positive constant. We assume that $V_{a2}(\psi_1(0), \zeta_2(0), \tilde{\theta}(0), \tilde{\varrho}(0))$ is finite (this assumption will be relaxed in Remark 4). The derivative of V_{a2} along the solutions of

$$\dot{\psi}_{1n} = -a_n \psi_{1n} + \zeta_{2n}, \qquad n \in \mathbb{F}, \quad (13a)$$

$$\dot{\zeta}_{2n} = a_n \psi_{2n} + \tau_n + h_n^*(\hat{\theta}_n + \tilde{\theta}_n) + \kappa_n u_n, \; n \in \mathbb{F}, \quad (13b)$$

is determined by

$$\dot{V}_{a2} = -\sum_{n\in\mathbb{F}} a_n \psi_{1n}^2 + \sum_{n\in\mathbb{F}} \zeta_{2n}\{\psi_{1n} + a_n\psi_{2n} + \tau_n + h_n^*\hat{\theta}_n + \kappa_n u_n\} + \sum_{n\in\mathbb{F}} \tilde{\theta}_n^*\{\Gamma_n^{-1}\dot{\tilde{\theta}}_n + \zeta_{2n} h_n\} + \sum_{n\in\mathbb{F}} \frac{|\kappa_n|}{\beta_n}\tilde{\varrho}_n\dot{\tilde{\varrho}}_n. \quad (14)$$

We eliminate $\tilde{\theta}_n$ from (14) with

$$\dot{\hat{\theta}}_n = \zeta_{2n}\Gamma_n h_n(\psi_1, \psi_2), \quad n \in \mathbb{F}. \quad (15)$$

A choice of control law of the form

$$u_n = -\hat{\varrho}_n\{\psi_{1n} + a_n\psi_{2n} + \tau_n + h_n^*\hat{\theta}_n + b_n\zeta_{2n}\}, \quad (16)$$

$\forall n \in \mathbb{F}$, together with (15) and the relationship

$$\kappa_n \hat{\varrho}_n = \kappa_n(\varrho_n - \tilde{\varrho}_n) = 1 - \kappa_n \tilde{\varrho}_n,$$

renders (14) into

$$\dot{V}_{a2} = -\sum_{n\in\mathbb{F}} a_n\psi_{1n}^2 - \sum_{n\in\mathbb{F}} b_n\zeta_{2n}^2 + \sum_{n\in\mathbb{F}} \frac{|\kappa_n|}{\beta_n}\tilde{\varrho}_n\left\{\dot{\tilde{\varrho}}_n + \beta_n \operatorname{sign}(\kappa_n)\zeta_{2n}s_n\right\},$$

where $s_n := \psi_{1n} + a_n\psi_{2n} + \tau_n + h_n^*\hat{\theta}_n + b_n\zeta_{2n}$. With a choice of update law for the estimate $\hat{\varrho}_n$

$$\dot{\hat{\varrho}}_n = \beta_n \operatorname{sign}(\kappa_n)\zeta_{2n}s_n, \quad n \in \mathbb{F}, \quad (17)$$

$\dot{V}_{a2}$ simplifies to

$$\dot{V}_{a2} = -\sum_{n\in\mathbb{F}} a_n\psi_{1n}^2 - \sum_{n\in\mathbb{F}} b_n\zeta_{2n}^2 =: W(\psi_1, \zeta_2) \le 0.$$

Using the definition of ζ_{2n} given by (7), we can rewrite (16), (15), and (17) for every $n \in \mathbb{F}$ as

$$\begin{aligned} u_n &= -\hat{\varrho}_n s_n, \\ \dot{\hat{\theta}}_n &= (\psi_{2n} + a_n\psi_{1n})\Gamma_n h_n, \\ \dot{\hat{\varrho}}_n &= \beta_n \operatorname{sign}(\kappa_n)(\psi_{2n} + a_n\psi_{1n})s_n, \\ s_n &= (1 + a_n b_n)\psi_{1n} + (a_n + b_n)\psi_{2n} + \tau_n + h_n^*\hat{\theta}_n. \end{aligned} \quad (18)$$

Since $\dot{V}_{a2} \le 0$, we conclude that V_{a2} is a non-increasing function of time. Thus, using (12) and the original assumption that $V_{a2}(\psi_1(0), \zeta_2(0), \tilde{\theta}(0), \tilde{\varrho}(0)) < \infty$, we conclude global uniform boundedness of ψ_1, ψ_2, $\hat{\theta}_n$, and $\hat{\varrho}_n$, that is, $\{\|\psi_1(t)\|^2 := \sum_{n\in\mathbb{F}}\psi_{1n}^2(t) < \infty, \; \|\psi_2(t)\|^2 := \sum_{n\in\mathbb{F}}\psi_{2n}^2(t) < \infty, \; \|\hat{\theta}_n(t)\|^2 := \hat{\theta}_n^*(t)\hat{\theta}_n(t) < \infty, \; \|\hat{\varrho}_n(t)\|^2 := \hat{\varrho}_n^2(t) < \infty, \; \forall t \ge 0\}$. Using the last set of expressions, properties of h_n and τ_n (see Assumptions 1–2), and (16), it follows that $\{\hat{\theta}_n \in \mathcal{L}_\infty, \hat{\varrho}_n \in \mathcal{L}_\infty, \psi_{1n} \in \mathcal{L}_\infty, \psi_{2n} \in \mathcal{L}_\infty, u_n \in \mathcal{L}_\infty, \forall n \in \mathbb{F}\}$, which in turn implies $\{\dot{\hat{\theta}}_n \in \mathcal{L}_\infty, \dot{\hat{\varrho}}_n \in \mathcal{L}_\infty, \dot{\psi}_{1n} \in \mathcal{L}_\infty, \dot{\psi}_{2n} \in \mathcal{L}_\infty, \forall n \in \mathbb{F}\}$. Moreover, since $V_{a2}(\psi_1(t), \zeta_2(t), \tilde{\theta}(t), \tilde{\varrho}(t))$ is a non-increasing function of time bounded from below by zero, it has a limit $V_{a2\infty} := \lim_{t\to\infty} V_{a2}(\psi_1(t), \zeta_2(t), \tilde{\theta}_n(t), \tilde{\varrho}_n(t))$. Hence, integration of $\dot{V}_{a2}$ gives

$$\int_0^\infty W(\psi_1(t), \zeta_2(t))\, dt = \sum_{n\in\mathbb{F}}\left\{a_n\int_0^\infty \psi_{1n}^2(t)\, dt + b_n\int_0^\infty \zeta_{2n}^2(t)\, dt\right\} \le V_{a2}(\psi_1(0), \zeta_2(0), \tilde{\theta}(0), \tilde{\varrho}(0)) - V_{a2\infty} < \infty,$$

which in combination with the definition of ζ_2 implies $\psi_{1n} \in \mathcal{L}_2$, $\psi_{2n} \in \mathcal{L}_2$, $\forall n \in \mathbb{F}$. Therefore, we have shown that $\psi_{1n}, \psi_{2n} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\dot{\psi}_{1n}, \dot{\psi}_{2n} \in \mathcal{L}_\infty$, $\forall n \in \mathbb{F}$. Application of the Barbălat lemma [7, 15] implies that both $\psi_{1n}(t)$ and $\psi_{2n}(t)$ asymptotically go to zero for all $n \in \mathbb{F}$. In other words, the state of (4) converges to zero as $t \to \infty$. Therefore, the dynamical controller obtained as a result of the backstepping design guarantees boundedness of all signals in the closed-loop system (4,18) and asymptotic convergence of the state of (4) to zero.

Remark 4 *The assumption on finiteness of $V_{a2}(t = 0)$ is not natural. Namely, it is reasonable to assume that the initial distributed state belongs to the underlying state-space (in this case $l_2 \times l_2$), but, since we want to consider systems with infinite number of unknown parameters it is somewhat artificial to assume that we know most of them. If we have some* a priori *information about values that the unknown parameters can assume we can choose sequences $\{\Gamma_n\}_{n\in\mathbb{F}}$ and $\{\beta_n\}_{n\in\mathbb{F}}$ such that $V_{a2}(t = 0)$ is finite. However, this would lead to parameter update laws with very large gains since elements of these two sequences have to increase their values as $n \to \infty$. Clearly, this is not desirable from a practical point of view. Here we show that controller (18) guarantees boundedness of all signals in the closed loop, and convergence of the state of (4) to zero as $t \to \infty$ even when $V_{a2}(t = 0) = \infty$. This follows from the fact that for every fixed $n \in \mathbb{F}$ the derivative of*

$$V_{a2n} := \frac{1}{2}\psi_{1n}^2 + \frac{1}{2}\zeta_{2n}^2 + \frac{1}{2}\tilde{\theta}_n^*\Gamma_n^{-1}\tilde{\theta}_n + \frac{1}{2}\frac{|\kappa_n|}{\beta_n}\tilde{\varrho}_n^2,$$

along the solutions of the 'n-th subsystem' of (13,18) is determined by

$$\dot{V}_{a2n} = -a_n\psi_{1n}^2 - b_n\zeta_{2n}^2 =: W_n(\psi_{1n}, \zeta_{2n}) \le 0.$$

Using similar argument as before we conclude that $\{\hat{\theta}_n \in \mathcal{L}_\infty, \hat{\varrho}_n \in \mathcal{L}_\infty, \dot{\hat{\theta}}_n \in \mathcal{L}_\infty, \dot{\hat{\varrho}}_n \in \mathcal{L}_\infty, \psi_{1n} \in \mathcal{L}_2 \cap \mathcal{L}_\infty, \psi_{2n} \in \mathcal{L}_2 \cap \mathcal{L}_\infty, u_n \in \mathcal{L}_\infty, \forall n \in \mathbb{F}\}$

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which in turn implies boundedness of all signals in the closed-loop system (4,18) and asymptotic convergence of both $\psi_{1n}(t)$ and $\psi_{2n}(t)$ to zero, for all $n \in \mathbb{F}$. It is noteworthy that when $\sum_{n \in \mathbb{F}} V_{a2n}(t=0) < \infty$ it might be advantageous to use $\sum_{n \in \mathbb{F}} V_{a2n}$ as a CLF for the entire infinite dimensional system rather than to consider V_{a2n} as a CLF for the n-th plant unit. By performing analysis of this type both beneficial interactions between different subsystems and beneficial nonlinearities can be identified and unnecessary cancellations can be avoided in the process of control design.

Remark 5 *The results of this subsection can also be applied for the control of systems with temporally and spatially constant parametric uncertainties. It can be shown that, for this class of systems, centralized distributed controllers are obtained if we start our design with one parameter estimate per unknown parameter. On the other hand, controller (18) has a different parameter update law in every single control unit K_n (K_n denotes a controller that acts on the n-th plant unit G_n, $\forall n \in \mathbb{F}$), even when all parameters are both spatially and temporally constant. This 'over-parameterization' is advantageous in applications because it allows for implementation of decentralized adaptive distributed controllers, as illustrated in § 4. This useful property cannot be achieved with controllers that have one estimate per unknown parameter since they require information about the entire distributed state to estimate unknown parameters.*

Remark 6 *One can show that the dynamical controller of the form*

$$
\begin{aligned}
u_n &= -\hat{\varrho}_n s_n, \\
\dot{\hat{\theta}}_n &= \{(\psi_{2n} - \dot{r}_n(t)) + a_n(\psi_{1n} - r_n(t))\}\Gamma_n h_n, \\
\dot{\hat{\varrho}}_n &= \beta_n \operatorname{sign}(\kappa_n)\{(\psi_{2n} - \dot{r}_n(t)) + a_n(\psi_{1n} - r_n(t))\} s_n,
\end{aligned} \tag{19}
$$

guarantees boundedness of all signals in the closed-loop system (4,19) and asymptotic convergence of $\psi_{1n}(t)$ to $r_n(t)$, for all $n \in \mathbb{F}$, where

$$
\begin{aligned}
s_n := {} & (1 + a_n b_n)(\psi_{1n} - r_n(t)) + (a_n + b_n)(\psi_{2n} - \dot{r}_n(t)) \\
& + \tau_n + h_n^* \hat{\theta}_n - \ddot{r}_n(t).
\end{aligned}
$$

It is assumed that, for every $n \in \mathbb{F}$, the reference signal r_n, and its first two derivatives $\dot{r}_n$, and $\ddot{r}_n$ are known and uniformly bounded, and that $\ddot{r}_n$ is piecewise continuous.

4 Examples

In this section, we discuss application of controllers developed in § 3 to the mass-spring system. Furthermore, we analyze the structure of these controllers and validate their performance using computer simulations of systems containing a large number of units.

Figure 2 illustrates controller architecture of mass-spring system with the controllers of § 3. Remarkably, in all cases, Lyapunov-based design yields decentralized controllers K_n, $\forall n \in \mathbb{Z}$, that require only measurements from the n-th plant unit G_n and its immediate neighbors G_{n-1} and G_{n+1}, to achieve desired objective. In applications, we clearly have to work with systems on lattices that contain large but

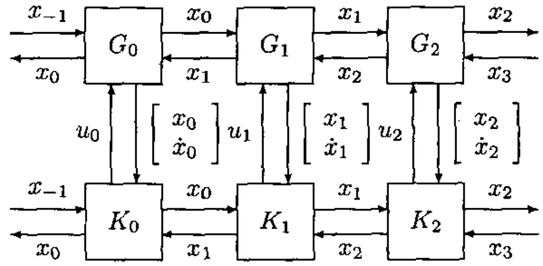

Figure 2: Controller architecture of mass-spring system.

finite number of units. All considerations related to infinite dimensional systems are applicable here, but with minor modifications. For example, if we consider the mass-spring system shown in Figure 3 with N masses ($n = 1, 2, \ldots, N$) both the equations presented in § 2.1 and the control laws of § 3 are still valid with appropriate 'boundary conditions' of the form: $x_j = \dot{x}_j = u_j \equiv 0$, $\forall j \in \mathbb{Z} \setminus \{1, 2, \ldots, N\}$.

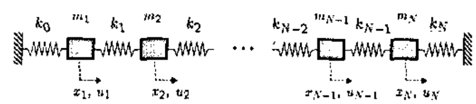

Figure 3: Finite dimensional mass-spring system.

4.1 Nominal state-feedback design

Figure 4 shows simulation results of uncontrolled (upper left) and controlled nonlinear mass-spring system with $N = 100$ masses and $m = k = q = 1$. The initial state of the system is randomly selected. The nominal controller (11) with $a_n = b_n = 1$, $\forall n = 1, 2, \ldots, N$ is used. Clearly, the desired control objective is met with a reasonable transient response. This transient response can be further improved with a different choice of design parameters a_n and b_n.

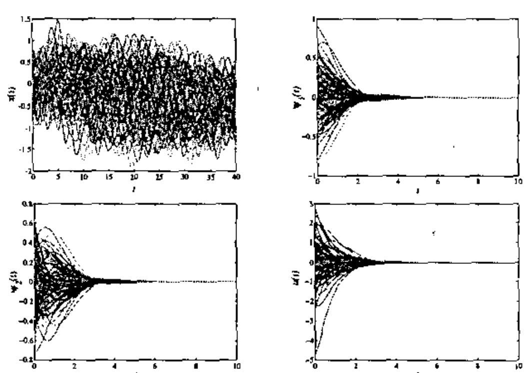

Figure 4: Nominal control of nonlinear mass-spring system.

4.2 Adaptive state-feedback design

Simulation results of a linear mass-spring system with unknown parameters $\theta_{n1} = \frac{k_{n-1}}{m_n}$, $\theta_{n2} = \frac{k_n}{m_n}$, and $\kappa_n = \frac{1}{m_n} > 0$ are shown in Figure 5. The simulation is done with $N = 100$ masses for $m_n = k_n = 1$,

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$\forall n = 1, 2, \ldots, N$ using adaptive controller (18) with $\gamma_n = \beta_n = a_n = b_n = 1$, and $\hat{\theta}_n(0) = \hat{\rho}_n(0) = 0.5$, $\forall n = 1, 2, \ldots, N$. The initial state of the mass-spring system is randomly selected. We observe that distributed adaptive controller (18) provides boundedness of all parameter estimates and convergence of the state of the error system to zero.

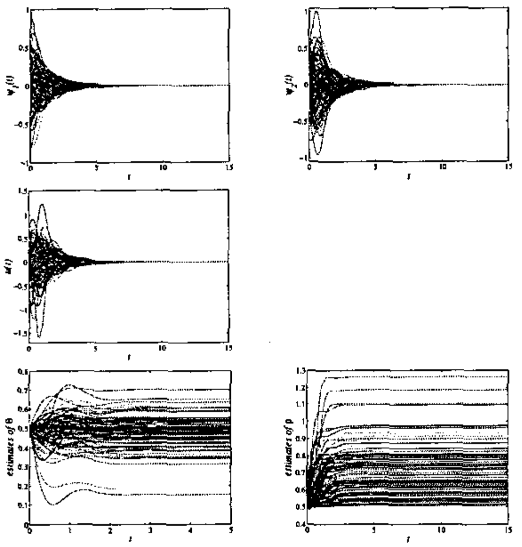

Figure 5: Adaptive control of linear mass-spring system.

5 Concluding remarks

This paper deals with the distributed control of spatially discrete infinite dimensional systems. It has been illustrated that Lyapunov-based approach can be successfully used to obtain state-feedback controllers for both nominal systems and systems with parametric uncertainties. It has been also shown that the control problem can be posed in such a way to yield controllers of the same structure as the original plant. Therefore, as a result of Lyapunov-based design control systems with an intrinsic degree of decentralization are obtained. Furthermore, within Lyapunov framework desired control objective can be achieved irrespective of whether the plant dynamics is linear or nonlinear.

Our current efforts are directed towards development of output-feedback controllers and modular adaptive schemes in which parameter update laws and controllers are designed separately. The major advantage of using this approach rather than the Lyapunov-based design is the versatility that it offers. Namely, adaptive controllers of this paper are limited to Lyapunov-based estimators. From a practical point of view it might be advantageous to use the appropriately modified standard gradient or least-squares type identifiers.

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