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Lyapunov-based output-feedback distributed control of systems on lattices

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Abstract

We study output-feedback control of systems on lattices with spatially distributed sensing and actuating capabilities. These systems are encountered in a wide range of modern applications such as: platoons of vehicles, arrays of microcantilevers, unmanned aerial vehicles in formation, and satellites in synchronous orbit. We use a Lyapunov-based framework as a tool for stabilization/regulation of systems in which nonlinearities depend only on the distributed output variable. We first present results for nominal design and then describe the design of adaptive output-feedback controllers in the presence of parametric uncertainties. These uncertainties are assumed to be temporally constant, but are allowed to be spatially varying. We show that our design yields the distributed controllers that inherit the information passing structure from the original plant. We also provide several examples of systems on lattices and validate derived results using computer simulations of systems containing a large number of units.

1 Introduction

Systems on lattices arise in a variety of modern technical applications. Typical examples of such systems include: platoons of vehicles ([1, 2, 3]), arrays of microcantilevers [4], unmanned aerial vehicles in formation [5], and satellites in synchronous orbit ([6, 7]). These systems are characterized by the interactions between different subsystems which often results in surprisingly complex behavior. A distinctive feature of this class of systems is that every single unit is equipped with sensors and actuators. The controller design problem is thus dominated by architectural questions such as localized versus centralized control, and the information passing structure in both the plant and the controller. This is in contrast with 'spatially lumped' control design problems, where the dominant issues are optimal and reduced order controller design.

A framework for considering spatially distributed systems is that of a spatio-temporal system [8]. In the specific case of systems on discrete spatial domains, signals of interest are functions of time and a spatial variable $n \in \mathbb{F}$, where $\mathbb{F}$ is a discrete spatial lattice (e.g. $\mathbb{Z}$ or $\mathbb{N}$).

In this paper, we extend results of [9] to the case where only distributed output is available for measurement, rather than the entire state of the system. We consider models in which nonlinearities depend only on the measured signals and use a Lyapunov-based approach to provide stability/regulation of nominal systems and systems with parametric uncertainties. In the latter case, we assume that the unknown parameters are temporally constant, but are allowed to be spatially varying. As a result of our adaptive design, boundedness of all signals in the closed-loop in the presence of unknown parameters is guaranteed. In addition to that, the adaptive controllers provide convergence of the states of the original system to their desired values. We also show that the distributed design results in controllers whose information passing structure is similar to that of the original plant. This means, for example, that if the plant has only nearest neighbor interactions, then the distributed controller also has only nearest neighbor interactions.

Our presentation is organized as follows: in section 2, we give an example of systems on lattices and describe the classes of systems for which we design output-feedback controllers in § 3. In § 4, we discuss application of controllers developed in § 3, analyze their structure, and validate their performance using computer simulations of systems containing a large number of units. We conclude by summarizing major contributions and future research directions in § 5.

2 Systems on lattices

In this section an example of systems on lattices is given. In particular, we consider a mass-spring system on a line. This system is chosen because it represent a simple non-trivial example of an unstable system where the interactions between different plant units are caused by the physical connections between them. Another example of systems with this property is given by an array of microcantilevers [4]. We remark that the interactions between different plant units may also arise because of a specific control objective that we want to meet. Examples of systems on lattices with this property include: a system of cars in an infinite string, aerial vehicles and spacecrafts in formation flights. We also describe the classes of systems for which we design output-feedback controllers in § 3.

2.1 An example of systems on lattices

A system consisting of an infinite number of masses and springs on a line is shown in Figure 1. The dynamics of the n-th mass are given by

$$m_n \ddot{x}_n \;=\; F_{n-1} \;+\; F_n \;+\; u_n, \quad n \in \mathbb{Z}, \qquad (1)$$

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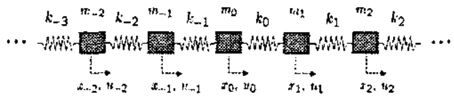

Figure 1: Mass-spring system.

where x_n represents the displacement from a reference position of the n-th mass, F_n represents the restoring force of the n-th spring, and u_n is the control applied on the n-th mass. For relatively small displacements, restoring forces can be considered as linear functions of displacements $F_n = k_n(x_{n+1} - x_n)$, $F_{n-1} = k_{n-1}(x_{n-1} - x_n)$, $n \in \mathbb{Z}$, where k_n is the n-th spring constant. We also consider a situation in which the spring restoring forces depend nonlinearly on displacement. One such model is given by the so-called *hardening spring* (see, for example [10]) where, beyond a certain displacement, large force increments are obtained for small displacement increments

$$\begin{aligned} F_n &= k_n\left\{(x_{n+1} - x_n) + c_n^2(x_{n+1} - x_n)^3\right\} \\ &=: k_n(x_{n+1} - x_n) + q_n(x_{n+1} - x_n)^3, \\ F_{n-1} &= k_{n-1}\left\{(x_{n-1} - x_n) + c_{n-1}^2(x_{n-1} - x_n)^3\right\} \\ &=: k_{n-1}(x_{n-1} - x_n) + q_{n-1}(x_{n-1} - x_n)^3. \end{aligned}$$

For both cases (1) can be rewritten in terms of its state-space representation $\forall\, n \in \mathbb{Z}$ as

$$\begin{aligned} \dot{\psi}_{1n} &= \psi_{2n}, \\ \dot{\psi}_{2n} &= f_n(\psi_{1,n-1}, \psi_{1n}, \psi_{1,n+1}) + \kappa_n u_n, \\ y_n &= \psi_{1n}, \end{aligned} \tag{2}$$

where $\psi_{1n} := x_n$ and $\psi_{2n} := \dot{x}_n$, provided that the positions of all masses are available for measurement.

In the particular situation in which the restoring forces are linear functions of displacements and all masses and springs are homogeneous, that is, $m_n = m =$ const., $k_n = k =$ const., $\forall\, n \in \mathbb{Z}$, (2) represents a linear *spatially invariant* system. This implies that it can be analyzed using the tools of [11, 12]. The other mathematical representations of a mass-spring system are either nonlinear or spatially-varying. The main purpose of the present study is to design output-feedback controllers for this broader class of systems.

2.2 Classes of systems

In this subsection, we briefly summarize the classes of systems for which we design output-feedback controllers in § 3. In particular, we consider m-th order subsystems over discrete spatial lattice $\mathbf{F}$ with finite number of interconnections with other plant units and nonlinearities that do not depend on the unmeasured signals. We assume that all subsystems satisfy the *matching condition* [13]. Clearly, the models presented in § 2.1 belong to this class of systems, as well as the model of an array of microcantilevers [4], provided that we can measure the positions of all masses (respectively microcantilevers). Furthermore, we remark that our results can be also used for control of fully actuated systems in two and three spatial dimensions with nonlinearities that depend only on the distributed output.

We consider output-feedback design for nominal systems of the form

$$\begin{aligned} \dot{\psi}_{1n} &= \psi_{2n}, & n &\in \mathbf{F}, & \text{(3a)} \\ \dot{\psi}_{2n} &= \psi_{3n}, & n &\in \mathbf{F}, & \text{(3b)} \\ &\vdots \\ \dot{\psi}_{mn} &= f_n(\psi_1) + \kappa_n u_n, & n &\in \mathbf{F}, & \text{(3c)} \\ y_n &= \psi_{1n}, & n &\in \mathbf{F}, & \text{(3d)} \end{aligned}$$

and systems with parametric uncertainties of the form

$$\begin{aligned} \dot{\psi}_{1n} &= \psi_{2n}, & n &\in \mathbf{F}, & \text{(4a)} \\ \dot{\psi}_{2n} &= \psi_{3n}, & n &\in \mathbf{F}, & \text{(4b)} \\ &\vdots \\ \dot{\psi}_{mn} &= \tau_n(\psi_1) + h_n^*(\psi_1)\theta_n + \kappa_n u_n, & n &\in \mathbf{F}, & \text{(4c)} \\ y_n &= \psi_{1n}, & n &\in \mathbf{F}, & \text{(4d)} \end{aligned}$$

where $\psi_k := \{\psi_{kn}\}_{n\in\mathbf{F}}$, $k \in \{1, \ldots, m\}$, and κ_n's are the so-called *control coefficients* [13]. The distributed output is denoted by $y := \{y_n\}_{n\in\mathbf{F}} = \{\psi_{1n}\}_{n\in\mathbf{F}}$, θ_n represents a vector of unknown parameters, and '$*$' denotes the transpose of vector h_n.

We introduce the following assumptions about the systems under study:

Assumption 1 *The number of interconnections between different plant units is uniformly bounded. In other words, there exist $M \in \mathbb{N}$, $M \neq M(n)$, such that f_n, h_n, and τ_n depend on at most M elements of ψ_1.*

Assumption 2 *f_n, h_n, and τ_n are known, continuously differentiable functions of their arguments.*

Assumption 3 *The signs of κ_n, $\forall\, n \in \mathbf{F}$, in (4c) are known.*

These assumptions are used in the sections related to the distributed control design. We remark that under these assumptions the well-posedness of both open and closed-loop systems can be easily established.

Remark 1 *For notational convenience, the control design problems are solved for second order systems over discrete spatial lattice $\mathbf{F}$, that is for $m = 2$.*

3 Lyapunov-based distributed control design

In this section, we address the problem of designing output-feedback controllers that provide stability/regulation of systems described in § 2.2. Assuming that every unit is equipped with sensors and an actuator, we use the Lyapunov-based approach to solve this problem. The Lyapunov design is very suitable because it leads to distributed controllers with the same localization as the original plant. This feature is of paramount importance for practical implementation.

3.1 Nominal output-feedback design

The controllers of [9] provide stability/regulation/asymptotic tracking of the closed-loop systems on lattices under the assumption that the full state information is available. Here, we study a more realistic situation in which only a distributed output variable is measured. We show that, as in the case of finite dimensional systems [13], the observer backstepping can be used as a tool for fulfilling the desired control objective for systems on lattices in which nonlinearities depend only on the measured signals. The starting point of the output-feedback approach is a design of an observer which guarantees the exponential convergence of the state estimates to their real values. Once this is accomplished, the combination of backstepping and nonlinear damping is used to account for the observation errors and provide closed-loop stability.

We rewrite (3), for $m = 2$, in a form suitable for observer design

$$\dot{\psi}_n = A\psi_n + \varphi_n(y) + \kappa_n e_2 u_n, \quad n \in \mathbf{F}, \quad (5a)$$

$$y_n = C\psi_n, \quad n \in \mathbf{F}, \quad (5b)$$

where

$$\psi_n := \begin{bmatrix} \psi_{1n} \\ \psi_{2n} \end{bmatrix}, \; \varphi_n(y) := \begin{bmatrix} 0 \\ f_n(y) \end{bmatrix}, \; e_2 := \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$A := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; C := \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

We proceed by designing an equivalent of *Krener-Isidori observer* (see, for example, [14, 13]) for (5)

$$\dot{\hat{\psi}}_n = A\hat{\psi}_n + L_n(y_n - \hat{y}_n) + \varphi_n(y) + \kappa_n e_2 u_n, \quad (6a)$$

$$\hat{y}_n = C\hat{\psi}_n, \quad (6b)$$

where $L_n := \begin{bmatrix} l_{1n} & l_{2n} \end{bmatrix}^*$ is chosen such that $A_{0n} := A - L_n C$ is a Hurwitz matrix for every $n \in \mathbf{F}$. Clearly, this is going to be satisfied if and only if $l_{in} > 0, \forall i = \{1,2\}, \forall n \in \mathbf{F}$. In this case, an exponentially stable system of the form

$$\dot{\tilde{\psi}}_n = A_{0n}\tilde{\psi}_n, \quad n \in \mathbf{F}, \quad (7)$$

is obtained by subtracting (6) from (5). The properties of A_{0n} imply the exponential convergence of $\tilde{\psi}_n := \psi_n - \hat{\psi}_n$ to zero and the existence of the positive definite matrix P_{0n} that satisfies

$$A_{0n}^* P_{0n} + P_{0n} A_{0n} = -I, \quad \forall n \in \mathbf{F}. \quad (8)$$

We are now ready to design an output-feedback controller that guarantees stability of (3).

Step 1 The observer-backstepping design starts with subsystem (3a) by rewriting it as

$$\dot{\psi}_{1n} = \hat{\psi}_{2n} + \tilde{\psi}_{2n}, \quad n \in \mathbf{F}, \quad (9)$$

and considering $\hat{\psi}_{2n}$ as a virtual control and $\tilde{\psi}_{2n}$ as a disturbance generated by (7). We propose a CLF for the 'n-th subsystem' of (9)

$$V_{1n}(\psi_{1n}, \tilde{\psi}_n) := \frac{1}{2}\psi_{1n}^2 + \frac{1}{d_{1n}}\tilde{\psi}_n^* P_{0n}\tilde{\psi}_n,$$

where P_{0n} is a positive definite matrix that satisfies (8), and $d_{1n} > 0$ is a design parameter. The derivative of V_{1n} along the solutions of (9,7) for every $n \in \mathbf{F}$ is determined by

$$\begin{aligned} \dot{V}_{1n} &= \psi_{1n}(\hat{\psi}_{2n} + \tilde{\psi}_{2n}) - \frac{1}{d_{1n}}\|\tilde{\psi}_n\|_2^2 \\ &\le \psi_{1n}(\hat{\psi}_{2n} + d_{1n}\psi_{1n}) + \frac{1}{4d_{1n}}\tilde{\psi}_{2n}^2 - \frac{1}{d_{1n}}\|\tilde{\psi}_n\|_2^2 \\ &\le \psi_{1n}(\hat{\psi}_{2n} + d_{1n}\psi_{1n}) - \frac{3}{4d_{1n}}\|\tilde{\psi}_n\|_2^2, \end{aligned} \quad (10)$$

where Young's Inequality (see [13], expression (2.254)) is used to upper bound $\psi_{1n}\tilde{\psi}_{2n}$. In particular, the choice of a 'stabilizing function' $\hat{\psi}_{2nd}$ of the form

$$\hat{\psi}_{2nd} = -(a_n + d_{1n})\psi_{1n}, \quad a_n > 0, \quad \forall n \in \mathbf{F},$$

clearly renders $\dot{V}_{1n}(\psi_{1n}, \tilde{\psi}_n)$ negative definite. Since $\hat{\psi}_{2n}$ is not actually a control, but rather, an estimate of a state variable, we introduce the change of variables

$$\zeta_{2n} := \hat{\psi}_{2n} - \hat{\psi}_{2nd} = \hat{\psi}_{2n} + (a_n + d_{1n})\psi_{1n}, \quad (11)$$

for every $n \in \mathbf{F}$, which adds an additional term on the right-hand side of (10)

$$\dot{V}_{1n} \le -a_n\psi_{1n}^2 - \frac{3}{4d_{1n}}\|\tilde{\psi}_n\|_2^2 + \psi_{1n}\zeta_{2n}. \quad (12)$$

The sign indefinite term in (12) will be taken care of at the second step of backstepping.

Step 2 We express the 'n-th subsystem' of our system into new coordinates as

$$\dot{\psi}_{1n} = -(a_n + d_{1n})\psi_{1n} + \zeta_{2n} + \tilde{\psi}_{2n}, \quad (13a)$$

$$\begin{aligned} \dot{\zeta}_{2n} &= (a_n + d_{1n})(\hat{\psi}_{2n} + \tilde{\psi}_{2n}) + l_{2n}(\psi_{1n} - \hat{\psi}_{1n}) \\ &\quad + f_n(y) + \kappa_n u_n, \end{aligned} \quad (13b)$$

$$\dot{\tilde{\psi}}_n = A_{0n}\tilde{\psi}_n, \quad (13c)$$

and propose the following CLF for it

$$V_{2n}(\psi_{1n}, \zeta_{2n}, \tilde{\psi}_n) := V_{1n} + \frac{1}{2}\zeta_{2n}^2 + \frac{1}{d_{2n}}\tilde{\psi}_n^* P_{0n}\tilde{\psi}_n,$$

with $d_{2n} > 0$. The derivative of V_{2n} along the solutions of (13) for every $n \in \mathbf{F}$ is determined by

$$\begin{aligned} \dot{V}_{2n} &= \dot{V}_{1n} + \zeta_{2n}\dot{\zeta}_{2n} - \frac{1}{d_{2n}}\|\tilde{\psi}_n\|_2^2 \\ &\le -a_n\psi_{1n}^2 - \frac{3}{4}\left(\frac{1}{d_{1n}} + \frac{1}{d_{2n}}\right)\|\tilde{\psi}_n\|_2^2 + \\ &\quad \zeta_{2n}\{\kappa_n u_n + \psi_{1n} + (a_n + d_{1n})\hat{\psi}_{2n} + f_n(y) + \\ &\quad l_{2n}(\psi_{1n} - \hat{\psi}_{1n}) + d_{2n}(a_n + d_{1n})^2\zeta_{2n}\}. \end{aligned}$$

A control law of the form

$$\begin{aligned} u_n = -\frac{1}{\kappa_n}\{&\psi_{1n} + (a_n + d_{1n})\hat{\psi}_{2n} + l_{2n}(\psi_{1n} - \hat{\psi}_{1n}) + \\ &f_n(y) + d_{2n}(a_n + d_{1n})^2\zeta_{2n} + b_n\zeta_{2n}\}, \end{aligned} \quad (14)$$

with $b_n > 0$ for every $n \in \mathbf{F}$, guarantees negative definiteness of $\dot{V}_{2n}$, that is

$$\dot{V}_{2n} \le -a_n\psi_{1n}^2 - b_n\zeta_{2n}^2 - \frac{3}{4}\left(\frac{1}{d_{1n}} + \frac{1}{d_{2n}}\right)\|\tilde{\psi}_n\|_2^2.$$

Therefore, we conclude that our design guarantees global asymptotic stability of the origin of the closed-loop system (3,7,14), for $m = 2$, on the Banach space $\mathbb{B} := l_\infty \times l_\infty \times l_\infty \times l_\infty$.

3.2 Adaptive output-feedback design

We rewrite (4), for $m = 2$, in a form suitable for adaptive output-feedback design

$$\dot{\psi}_n = A\psi_n + \eta_n(y) + \sum_{j=1}^{r} \theta_{jn}\varphi_{jn}(y) + \kappa_n e_2 u_n, \quad (15a)$$

$$y_n = C\psi_n, \quad (15b)$$

where, for every $n \in \mathbf{F}$, $\psi_n := [\ \psi_{1n} \quad \psi_{2n}\ ]^*$ and

$$\eta_n(y) := \begin{bmatrix} 0 \\ \tau_n(y) \end{bmatrix}, \ \varphi_{jn}(y) := \begin{bmatrix} 0 \\ h_{jn}(y) \end{bmatrix},$$

$$A := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ e_2 := \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C := [\ 1 \quad 0\ ].$$

We proceed by designing filters which provide 'virtual estimates' of unmeasured state variables (see [13], § 7.3)

$$\dot{\xi}_n^{(0)} = A_{0n}\xi_n^{(0)} + L_n y_n + \eta_n(y), \quad (16a)$$

$$\dot{\xi}_n^{(j)} = A_{0n}\xi_n^{(j)} + \varphi_{jn}(y), \quad 1 \le j \le r, \quad (16b)$$

$$\dot{v}_n = A_{0n}v_n + e_2 u_n, \quad (16c)$$

where $L_n := [\ l_{1n} \quad l_{2n}\ ]^*$ is chosen such that $A_{0n} := A - L_n C$ is Hurwitz for every $n \in \mathbf{F}$. Clearly, A_{0n} is going to be Hurwitz if and only if $l_{in} > 0$, $\forall\, i = \{1, 2\}$, $\forall\, n \in \mathbf{F}$. In this case, an exponentially stable system of the form

$$\dot{\varepsilon}_n = A_{0n}\varepsilon_n, \quad (17)$$

is obtained by combining (15) and (16) for every $n \in \mathbf{F}$, with $\varepsilon_n := \psi_n - \{\xi_n^{(0)} + \sum_{j=1}^{r} \theta_{jn}\xi_n^{(j)} + \kappa_n v_n\}$. The properties of A_{0n} imply the exponential convergence of ε_n to zero and the existence of the positive definite matrix P_{0n} that satisfies (8).

We are now ready to design an adaptive output-feedback controller for (4) using backstepping.

Step 1 The adaptive observer-backstepping design starts with subsystem (4a) by rewriting it as

$$\dot{\psi}_{1n} = \xi_{2n}^{(0)} + \sum_{j=1}^{r} \theta_{jn}\xi_{2n}^{(j)} + \kappa_n v_{2n} + \varepsilon_{2n}, \quad (18)$$

and considering v_{2n} as a virtual control and ε_{2n} as a disturbance generated by (17). If v_{2n} were control, and all parameters were known, then (18) could be stabilized by

$$v_{2n} = -\frac{1}{\kappa_n}\{\xi_{2n}^{(0)} + (a_n + d_{1n})\psi_{1n}\} - \sum_{j=1}^{r} \frac{\theta_{jn}}{\kappa_n}\xi_{2n}^{(j)}, \quad (19)$$

where a_n and d_{1n} are positive design parameters. To account for parametric uncertainties we add and subtract the right-hand side of (19) to v_{2n} in (18) to obtain

$$\dot{\psi}_{1n} = -(a_n + d_{1n})\psi_{1n} + \kappa_n\{v_{2n} + \omega_n^{(1)^*}\hat{\vartheta}_n^{(1)}\} + \kappa_n\omega_n^{(1)^*}\tilde{\vartheta}_n^{(1)} + \varepsilon_{2n}, \quad (20)$$

where

$$\omega_n^{(1)} := \begin{bmatrix} \xi_{2n}^{(0)} + (a_n + d_{1n})\psi_{1n} \\ \xi_{2n}^{(1)} \\ \vdots \\ \xi_{2n}^{(r)} \end{bmatrix}, \ \vartheta_n^{(1)} := \begin{bmatrix} \frac{1}{\kappa_n} \\ \frac{\theta_{1n}}{\kappa_n} \\ \vdots \\ \frac{\theta_{rn}}{\kappa_n} \end{bmatrix},$$

and $\tilde{\vartheta}_n^{(1)} := \vartheta_n^{(1)} - \hat{\vartheta}_n^{(1)}$, with $\vartheta_n^{(1)}$ denoting the vector of unknown parameters.

We propose a CLF of the form

$$V_{a1n}(\psi_{1n}, \tilde{\vartheta}_n^{(1)}, \varepsilon_n) := \frac{1}{2}\psi_{1n}^2 + \frac{|\kappa_n|}{2}\tilde{\vartheta}_n^{(1)^*}\Gamma_n^{-1}\tilde{\vartheta}_n^{(1)} + \frac{1}{d_{1n}}\varepsilon_n^* P_{0n}\varepsilon_n,$$

where P_{0n} is a positive definite matrix that satisfies (8), $d_{1n} > 0$ is a design parameter, and $\Gamma_n = \Gamma_n^* > 0$. The derivative of V_{a1n} along the solutions of (20,17) for every $n \in \mathbf{F}$ is determined by

$$\dot{V}_{a1n} \le -a_n\psi_{1n}^2 + \kappa_n\psi_{1n}(v_{2n} + \omega_n^{(1)^*}\hat{\vartheta}_n^{(1)}) - \frac{3}{4d_{1n}}\|\varepsilon_n\|_2^2 + |\kappa_n|\tilde{\vartheta}_n^{(1)^*}\{\Gamma_n^{-1}\dot{\tilde{\vartheta}}_n^{(1)} + \operatorname{sign}(\kappa_n)\psi_{1n}\omega_n^{(1)}\},$$

where we used Young's Inequality (see [13], expression (2.254)) to upper bound $\psi_{1n}\varepsilon_{2n}$. In particular, the following choices of a 'stabilizing function' v_{2nd} and update law for the estimate $\hat{\vartheta}_n^{(1)}$

$$v_{2nd} = -\omega_n^{(1)^*}\hat{\vartheta}_n^{(1)}, \qquad \forall\, n \in \mathbf{F},$$

$$\dot{\hat{\vartheta}}_n^{(1)} = \operatorname{sign}(\kappa_n)\psi_{1n}\Gamma_n\omega_n^{(1)}, \qquad \forall\, n \in \mathbf{F},$$

clearly render $\dot{V}_{a1n}(\psi_{1n}, \tilde{\vartheta}_n^{(1)}, \varepsilon_n)$ negative semi-definite. Since v_{2n} is not actually a control, we introduce the second error variable as

$$\zeta_{2n} := v_{2n} - v_{2nd} = v_{2n} + \omega_n^{(1)^*}\hat{\vartheta}_n^{(1)}, \quad \forall\, n \in \mathbf{F},$$

which adds an additional term on the right-hand side of $\dot{V}_{a1n}$

$$\dot{V}_{a1n} \le -a_n\psi_{1n}^2 - \frac{3}{4d_{1n}}\|\varepsilon_n\|_2^2 + \kappa_n\psi_{1n}\zeta_{2n}.$$

The sign indefinite term in the last equation will be taken care of at the second step of backstepping.

Step 2 The differentiation of ζ_{2n} with respect to time for $m = 2$ yields

$$\dot{\zeta}_{2n} = \dot{v}_{2n} + \dot{\omega}_n^{(1)^*}\hat{\vartheta}_n^{(1)} + \omega_n^{(1)^*}\dot{\hat{\vartheta}}_n^{(1)}$$

$$= -l_{2n}v_{1n} + u_n + \dot{\omega}_n^{(1)^*}\hat{\vartheta}_n^{(1)} + \omega_n^{(1)^*}\dot{\hat{\vartheta}}_n^{(1)}.$$

We now use the definition of $\omega_n^{(1)}$ to rewrite $\dot{\omega}_n^{(1)^*}\hat{\vartheta}_n^{(1)}$ as

$$\dot{\omega}_n^{(1)^*}\hat{\vartheta}_n^{(1)} = \mu_n^*\hat{\vartheta}_n^{(1)} + (a_n + d_{1n})\hat{\vartheta}_{1n}^{(1)}\dot{\psi}_{1n}$$

$$= \mu_n^*\hat{\vartheta}_n^{(1)} + (a_n + d_{1n})\hat{\vartheta}_{1n}^{(1)}(\xi_{2n}^{(0)} + \varepsilon_{2n}) + \hat{\vartheta}_{1n}^{(1)}\omega_n^{(2)^*}\vartheta_n^{(2)},$$

where $\hat{\vartheta}_{1n}^{(1)}$ represents the first element of vector $\hat{\vartheta}_n^{(1)}$ and

$$\mu_n := \left[\begin{array}{cccc} \dot{\xi}_{2n}^{(0)} & \xi_{2n}^{(1)} & \cdots & \xi_{2n}^{(r)} \end{array} \right]^*,$$

$$\omega_n^{(2)} := (a_n + d_{1n}) \left[\begin{array}{cccc} \xi_{2n}^{(1)} & \cdots & \xi_{2n}^{(r)} & v_{2n} \end{array} \right]^*,$$

$$\vartheta_n^{(2)} := \left[\begin{array}{cccc} \theta_{1n} & \cdots & \theta_{rn} & \kappa_n \end{array} \right]^*.$$

Hence, $\dot{\zeta}_{2n}$ can be expressed as

$$\dot{\zeta}_{2n} = \sigma_n + \hat{\vartheta}_{1n}^{(1)} \omega_n^{(2)^*} \tilde{\vartheta}_n^{(2)} + (a_n + d_{1n}) \hat{\vartheta}_{1n}^{(1)} \varepsilon_{2n} + u_n,$$

where

$$\begin{aligned} \sigma_n &:= -l_{2n} v_{1n} + \omega_n^{(1)^*} \dot{\hat{\vartheta}}_n^{(1)} + \mu_n^* \hat{\vartheta}_n^{(1)} + \\ &\quad \hat{\vartheta}_{1n}^{(1)} \{ (a_n + d_{1n}) \xi_{2n}^{(0)} + \omega_n^{(2)^*} \hat{\vartheta}_n^{(2)} \}, \\ \tilde{\vartheta}_n^{(2)} &:= \vartheta_n^{(2)} - \hat{\vartheta}_n^{(2)}. \end{aligned}$$

The CLF from Step 1 is augmented by the three terms that penalize ζ_{2n}, $\tilde{\vartheta}_n^{(2)}$, and ε_n, respectively, to obtain

$$V_{a2n}(\psi_{1n}, \zeta_{2n}, \tilde{\vartheta}_n^{(1)}, \tilde{\vartheta}_n^{(2)}, \varepsilon_n) := V_{a1n}(\psi_{1n}, \tilde{\vartheta}_n^{(1)}, \varepsilon_n) + \frac{1}{2}\zeta_{2n}^2 + \frac{1}{2}\tilde{\vartheta}_n^{(2)^*} \Delta_n^{-1} \tilde{\vartheta}_n^{(2)} + \frac{1}{d_{2n}} \varepsilon_n^* P_{0n} \varepsilon_n,$$

where $d_{2n} > 0$ and $\Delta_n = \Delta_n^* > 0$. The derivative of V_{a2n} is determined by

$$\begin{aligned} \dot{V}_{a2n} &= \dot{V}_{a1n} + \zeta_{2n}\dot{\zeta}_{2n} + \tilde{\vartheta}_n^{(2)^*} \Delta_n^{-1} \dot{\tilde{\vartheta}}_n^{(2)} - \frac{1}{d_{2n}} \|\varepsilon_n\|_2^2 \\ &\leq -a_n \psi_{1n}^2 - \frac{3}{4}\left(\frac{1}{d_{1n}} + \frac{1}{d_{2n}}\right) \|\varepsilon_n\|_2^2 + \\ &\quad \tilde{\vartheta}_n^{(2)^*} \{ \Delta_n^{-1} \dot{\tilde{\vartheta}}_n^{(2)} + \zeta_{2n}(\psi_{1n} e_{r+1} + \hat{\vartheta}_{1n}^{(1)} \omega_n^{(2)}) \} + \\ &\quad \zeta_{2n} \{ u_n + \sigma_n + \psi_{1n} e_{r+1}^* \hat{\vartheta}_n^{(2)} + \\ &\quad d_{2n}((a_n + d_{1n}) \hat{\vartheta}_{1n}^{(1)})^2 \zeta_{2n} \}, \end{aligned}$$

with e_{r+1} being the $(r+1)$-st coordinate vector in $\mathbb{R}^{r+1}$. In particular, the following choices of a control law u_n and update law for the estimate $\hat{\vartheta}_n^{(2)}$

$$\begin{aligned} u_n &= -\{ \sigma_n + \psi_{1n} e_{r+1}^* \hat{\vartheta}_n^{(2)} + b_n \zeta_{2n} + \\ &\quad d_{2n}((a_n + d_{1n}) \hat{\vartheta}_{1n}^{(1)})^2 \zeta_{2n} \}, \\ \dot{\hat{\vartheta}}_n^{(2)} &= \zeta_{2n} \Delta_n \left\{ \psi_{1n} e_{r+1} + \hat{\vartheta}_{1n}^{(1)} \omega_n^{(2)} \right\}, \end{aligned}$$

with $b_n > 0$, for every $n \in \mathbf{F}$, transform $\dot{V}_{a2n}(\psi_{1n}, \zeta_{2n}, \tilde{\vartheta}_n^{(1)}, \tilde{\vartheta}_n^{(2)}, \varepsilon_n)$ into a negative semi-definite function of the form

$$\begin{aligned} \dot{V}_{a2n} &\leq -a_n \psi_{1n}^2 - b_n \zeta_{2n}^2 - \frac{3}{4}\left(\frac{1}{d_{1n}} + \frac{1}{d_{2n}}\right) \|\varepsilon_n\|_2^2 \\ &\leq 0, \quad \forall n \in \mathbf{F}. \end{aligned}$$

One can establish boundedness of all signals in the closed-loop adaptive system and asymptotic convergence of ψ_{1n}, ζ_{2n}, and ε_n to zero for every $n \in \mathbf{F}$, using similar proof technique to the one presented in [9].

4 Examples

In this section, we discuss application of controllers developed in § 3 to the systems described in § 2.1. Furthermore, we analyze the structure of these controllers and validate their performance using computer simulations of systems containing a large number of units.

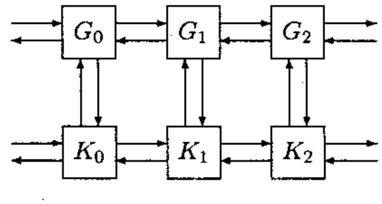

Figure 2: Controller architecture of mass-spring system with controllers of § 3.1 and § 3.2.

Figure 2 illustrates controller architecture of mass-spring system with the aforementioned controllers. We observe that nominal and adaptive output-feedback designs result in the closed-loop systems with the same passage of information. Remarkably, in both these cases, Lyapunov-based design yields decentralized controllers K_n, $\forall n \in \mathbb{Z}$, that require only measurements from the n-th plant unit G_n and its immediate neighbors G_{n-1} and G_{n+1}, to achieve desired objective.

In applications, we clearly have to work with systems on lattices that contain large but finite number of units. All considerations related to infinite dimensional systems are applicable here, but with minor modifications. For example, if we consider the mass-spring system shown in Figure 3 with N masses ($n = 1, 2, \ldots, N$) both the equations presented in § 2.1 and the control laws of § 3 are still valid with appropriate 'boundary conditions' of the form: $x_j = \dot{x}_j = u_j \equiv 0$, $\forall j \in \mathbb{Z} \setminus \{1, 2, \ldots, N\}$.

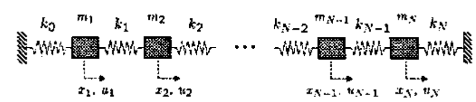

Figure 3: Finite dimensional mass-spring system.

4.1 Nominal output-feedback design

The nominal Lyapunov-based output-feedback design for mass-spring system leads to decentralized dynamic distributed controllers of the form

$$\begin{aligned} \dot{\hat{\psi}}_{1n} &= \hat{\psi}_{2n} + l_{1n}(\psi_{1n} - \hat{\psi}_{1n}) \\ \dot{\hat{\psi}}_{2n} &= l_{2n}(\psi_{1n} - \hat{\psi}_{1n}) + f_n + \kappa_n u_n \\ u_n &= -\frac{1}{\kappa_n}\{ \psi_{1n} + (a_n + d_{1n})\hat{\psi}_{2n} + f_n + \\ &\quad l_{2n}(\psi_{1n} - \hat{\psi}_{1n}) + (d_{2n}(a_n + d_{1n})^2 + b_n) \times \\ &\quad (\hat{\psi}_{2n} + (a_n + d_{1n})\psi_{1n}) \} \end{aligned} \tag{21}$$

where, for example, for a nonlinear mass-spring system with a hardening spring and $\{m_n = m, k_n = k, q_n = q, \forall n \in \mathbb{Z}\}$, f_n is determined by

$$\begin{aligned} f_n &= \frac{k}{m}\{ \psi_{1,n-1} - 2\psi_{1n} + \psi_{1,n+1} \} + \\ &\quad \frac{q}{m}\{ (\psi_{1,n-1} - \psi_{1n})^3 + (\psi_{1,n+1} - \psi_{1n})^3 \}. \end{aligned}$$

Figure 4 illustrates simulation results of nominal nonlinear mass-spring system with $N = 100$ masses and $m_n = k_n = q_n = 1$. Output-feedback control law (21) is used with $a_n = b_n = 1$, $d_{1n} = d_{2n} = 0.2$, $l_{1n} = 5$, $l_{2n} = 6$, and $\hat{\psi}_{1n}(0) = \hat{\psi}_{2n}(0) = 0$, $\forall n = 1, 2, \ldots, N$. The initial state of the system is randomly selected. Clearly, numerical results show that the nominal output-feedback distributed controller (21) achieves desired control objective in an effective manner with a reasonable amount of control effort.

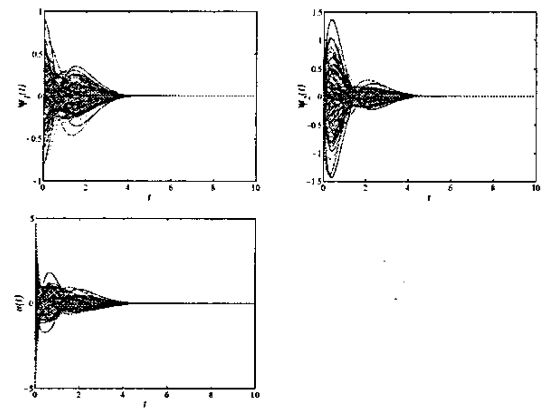

Figure 4: Nominal output-feedback control of nonlinear mass-spring system.

4.2 Adaptive output-feedback design

Formulae for adaptive output-feedback controllers for mass-spring systems can be readily obtained combining results of § 2.1 and § 3.2. We remark that as a result of our design we obtain decentralized distributed dynamic controllers whose architecture is shown in Figure 2.

5 Concluding remarks

This paper has dealt with the output-feedback distributed control of spatially discrete infinite dimensional systems in which nonlinearities depend only on the distributed output variable. It has been illustrated that Lyapunov-based approach can be successfully used to obtain output-feedback controllers for both nominal systems and systems with parametric uncertainties. It has been also shown that the design procedure yields dynamical controllers that inherit the passage of information from the original plant. Therefore, as a result of Lyapunov-based design control systems with an intrinsic degree of decentralization are obtained.

Our current efforts are directed towards development of modular adaptive schemes in which parameter update laws and controllers are designed separately. The major advantage of using this approach rather than the Lyapunov-based design is the versatility that it offers. Namely, adaptive controllers of this paper are limited to Lyapunov-based estimators. From a practical point of view it might be advantageous to use the appropriately modified standard gradient or least-squares type identifiers.

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