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Architecture induced by distributed backstepping design

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***Abstract*— An important problem in the distributed control of large-scale and infinite dimensional systems is related to the choice of the appropriate controller architecture. We utilize backstepping as a tool for distributed control of nonlinear infinite dimensional systems on lattices, and provide the answer to the following question: *what is the worst case controller architecture induced by distributed backstepping design*? We demonstrate that distributed backstepping design yields controllers that are intrinsically decentralized, with a strong similarity between plant and controller architectures. In particular, we study the 'worst case' control design in which all interactions are cancelled at each step of backstepping. Any other backstepping strategy yields controllers with better information transmission properties. For this 'worst case' situation we *quantify* the number of *control induced interactions* necessary to guarantee desired dynamical behavior of the infinite dimensional system. We also provide an example of systems on lattices and show how the controllers with favorable architectures can be designed.**

***Index Terms*— Systems on Lattices; Distributed Backstepping Design; Controller Architecture.**

I. Introduction

System on lattices are ubiquitous in modern technological applications. These systems can range from the macroscopic−such as cross directional control in the process industry [1], [2], vehicular platoons [3]–[9], Unmanned Aerial Vehicles (UAVs) [10]–[12], and satellites in formation flight [13]–[15]−to the microscopic, such as arrays of micro-mirrors [16] or micro-cantilevers [17]. Systems on lattices are characterized by interactions between different subsystems which often results into intricate behavioral patterns, an example of which is the so-called *string instability* [18] (or, more generally, the *spatio-temporal instability* [19]). The complex dynamical responses of these systems are caused by the aggregate effects, and they cannot be predicted by analyzing the individual plant units.

System on lattices are characterized by a special structure: each subsystem is equipped with sensing and actuating capabilities. Thus, the key design issues in the control of these systems are architectural such as the choice of localized versus centralized control. This problem has attracted a lot of attention in the last 25–30 years. A large body of literature in the area that is usually referred to as 'decentralized control of large-scale systems' has been created [20]–[28]. We also refer the reader to [19], [29]–[34] and references therein for information about recent work on distributed control of systems on lattices.

In this paper, we study distributed control of nonlinear infinite dimensional systems on lattices. The motivation for studying infinite dimensional systems is twofold: a) our results can be used for control of discretized versions of PDEs with distributed controls and measurements, and b) infinite dimensional systems represent an insightful limit of large-scale systems: problems with, for example, stability of an infinite dimensional system indicate issues with performance of its large-scale equivalent. The latter point was recently illustrated in [35] where the theory for spatially invariant linear systems [19] was utilized to show that extending standard results from small to large-scale vehicular platoons has dangers.

In addition to showing how backstepping can be employed as a tool for distributed control design, we also provide the answer to the following question: *what is the worst case controller architecture induced by distributed backstepping design*? We show that distributed backstepping design produces controllers that are intrinsically decentralized, with a strong similarity between plant and controller architectures. In particular, we confine our attention to a situation in which all interactions are cancelled at each step of backstepping. We refer to this case as a 'worst case' design, because any other backstepping strategy yields controllers with better information passing properties. For this 'worst case' situation we *quantify* the number of *control induced interactions* necessary to provide desired dynamical behavior of the infinite dimensional system.

Our presentation is organized as follows: in section II, we introduce the notation used throughout the paper. In § III, we describe the classes of systems for which we design distributed backstepping controllers in § IV. In § V, we discuss the architecture of distributed controllers induced by a backstepping design. In § VI, we provide an example of systems on lattices, show how flexibility of backstepping can be utilized for obtaining controllers with less interactions, and illustrate performance of backstepping controllers by performing numerical simulations on a large-scale system. We end our presentation with some concluding remarks in § VII.

II. Notation

The sets of integers and natural numbers are denoted by $\mathbb{Z}$ and $\mathbb{N}$, respectively, $\mathbb{N}_0 := \{0\} \cup \mathbb{N}$, and $\mathbb{Z}_N := \{-N, \ldots, N\}$, $N \in \mathbb{N}_0$. The space of square summable sequences is denoted by l_2, and the space of bounded sequences is denoted by l_∞. The state and control of the n-th subsystem (cell) are respectively represented by $[\psi_{1n} \cdots \psi_{mn}]^T$ and u_n, $m \in \mathbb{N}$, $n \in \mathbb{Z}$. The capital letters denote infinite vectors defined, for example, as $\Psi_k := [\cdots\ \psi_{k,n-1}\ \psi_{k,n}\ \psi_{k,n+1}\ \cdots]^T =: \{\psi_{kn}\}_{n\in\mathbb{Z}}$, $k \in \{1, \ldots, m\}$. The n-th plant cell is denoted by G_n, and the n-th controller cell is denoted by K_n.

III. Classes of systems

In this section, we briefly summarize the classes of systems for which we design feedback controllers in § IV. We consider continuous time m-th order subsystems over discrete spatial lattice $\mathbb{Z}$ with *at most* $2N$ interactions per plant's cell (see Assumption 1)

$$\dot{\psi}_{1n} = f_{1n}(\Psi_1) + \psi_{2n}, \qquad n \in \mathbb{Z}, \tag{1a}$$
$$\dot{\psi}_{2n} = f_{2n}(\Psi_1, \Psi_2) + \psi_{3n}, \qquad n \in \mathbb{Z}, \tag{1b}$$
$$\vdots$$
$$\dot{\psi}_{mn} = f_{mn}(\Psi_1, \cdots, \Psi_m) + u_n, \qquad n \in \mathbb{Z}. \tag{1c}$$

We rewrite the dynamics of the entire system as

$$\dot{\Psi}_1 = \mathrm{F}_1(\Psi_1) + \Psi_2, \tag{2a}$$
$$\dot{\Psi}_2 = \mathrm{F}_2(\Psi_1, \Psi_2) + \Psi_3, \tag{2b}$$
$$\vdots$$
$$\dot{\Psi}_m = \mathrm{F}_m(\Psi_1, \cdots, \Psi_m) + \mathrm{U}. \tag{2c}$$

System (2) represents an abstract evolution equation in the *strict-feedback form* [36] defined on either a Hilbert space $\mathbb{H} := l_2^m$ or a Banach space $\mathbb{B} := l_\infty^m$.

We introduce the following assumptions about the system under study:

Assumption 1: There are at most $2N$ interactions per plant cell: n-th plant cell G_n interacts only with $\{G_{n-N}, \ldots, G_{n+N}\}$. In other words, functions f_{kn} depend on at most $2N + 1$ elements of $\Psi_1, \ldots, \Psi_k$, $k \in \{1, \ldots, m\}$, $n \in \mathbb{Z}$. For example, $f_{2n}(\Psi_1, \Psi_2) = f_{2n}(\{\psi_{1,n+j}\}_{j \in \mathbb{Z}_N}, \{\psi_{2,n+j}\}_{j \in \mathbb{Z}_N})$.

Assumption 2: Functions f_{kn} are known, continuously differentiable functions of their arguments, equal to zero at the origin of system (2). In addition to that, infinite vectors $\mathrm{F}_k := \{f_{kn}\}_{n \in \mathbb{Z}}$ for every $k \in \{1, \ldots, m\}$ satisfy:

$$\{\Psi_1 \in l_\infty, \cdots, \Psi_k \in l_\infty\} \;\Rightarrow\; \mathrm{F}_k(\Psi_1, \ldots, \Psi_k) \in l_\infty.$$

Under these assumptions the well-posedness of both open and closed-loop systems is readily established.

IV. Distributed backstepping control design

In this section, we design distributed backstepping controllers for systems described in § III. For notational convenience, the control design problem is solved for second order subsystems over discrete spatial lattice $\mathbb{Z}$, that is for $m = 2$. In this case, the dynamics of the n-th cell (1) and the entire infinite dimensional system (2) are respectively given by

$$\dot{\psi}_{1n} \;=\; f_{1n}(\Psi_1) \;+\; \psi_{2n}, \qquad n \in \mathbb{Z}, \qquad (3a)$$

$$\dot{\psi}_{2n} \;=\; f_{2n}(\Psi_1, \Psi_2) \;+\; u_n, \qquad n \in \mathbb{Z}, \qquad (3b)$$

and

$$\dot{\Psi}_1 \;=\; \mathrm{F}_1(\Psi_1) \;+\; \Psi_2, \qquad (4a)$$

$$\dot{\Psi}_2 \;=\; \mathrm{F}_2(\Psi_1, \Psi_2) \;+\; \mathrm{U}. \qquad (4b)$$

In § IV-A, we study a situation in which the desired dynamical properties of system (4) are accomplished by performing a global design. Unfortunately, this is not always possible. Because of this, in § IV-B, we also perform design on individual cells (3) to guarantee the desired behavior of system (4).

A. Global backstepping design

Before we illustrate the global distributed backstepping design we introduce the following assumption:

Assumption 3: The initial distributed state is such that both $\Psi_1(0) \in l_2$ and $\Psi_2(0) \in l_2$.

The design objective is to provide global asymptotic stability of the origin of system (4). This is accomplished using the distributed backstepping control design.

Step 1 The global recursive design starts with subsystem (4a) by considering Ψ_2 as control and proposing a radially unbounded CLF $V_1 : l_2 \to \mathbb{R}$ of the form

$$V_1(\Psi_1) \;=\; \frac{1}{2} \langle \Psi_1, \Psi_1 \rangle \;:=\; \frac{1}{2} \sum_{n \in \mathbb{Z}} \psi_{1n}^2.$$

The derivative of $V_1(\Psi_1)$ along the solutions of (4a) is given by

$$\dot{V}_1 \;=\; \left\langle \Psi_1, \dot{\Psi}_1 \right\rangle \;=\; \langle \Psi_1, \mathrm{F}_1(\Psi_1) + \Psi_2 \rangle. \qquad (5)$$

Assumption 4: There exist a continuously differentiable 'stabilizing function' $\Psi_{2d} := \Lambda(\Psi_1)$, $\Lambda(0) = 0$, such that

$$\Psi_1 \in l_2 \;\Rightarrow\; \Lambda(\Psi_1) \in l_2,$$

and

$$W_1(\Psi_1) \;:=\; -\langle \Psi_1, \mathrm{F}_1(\Psi_1) + \Lambda(\Psi_1) \rangle \;>\; 0,$$

for every $\Psi_1 \in l_2 \setminus \{0\}$.

Since Ψ_2 is not actually a control, but rather, a state variable, we introduce the change of variables

$$\mathrm{Z}_2 \;:=\; \Psi_2 \;-\; \Psi_{2d} \;=\; \Psi_2 \;-\; \Lambda(\Psi_1), \qquad (6)$$

which adds an additional term on the right-hand side of (5)

$$\dot{V}_1 \;=\; -W_1(\Psi_1) \;+\; \langle \Psi_1, \mathrm{Z}_2 \rangle. \qquad (7)$$

The sign indefinite term in (7) will be taken care of at the second step of backstepping.

Step 2 Coordinate transformation (6) renders (4b) into a form suitable for the remainder of backstepping design

$$\dot{\mathrm{Z}}_2 \;=\; \mathrm{F}_2(\Psi_1, \Psi_2) - \frac{\partial \Lambda(\Psi_1)}{\partial \Psi_1} (\mathrm{F}_1(\Psi_1) + \Psi_2) + \mathrm{U}.$$

Augmentation of the CLF from Step 1 by a term which penalizes the error between Ψ_2 and Ψ_{2d} yields a function

$$V_2(\Psi_1, \mathrm{Z}_2) \;:=\; V_1(\Psi_1) \;+\; \frac{1}{2} \langle \mathrm{Z}_2, \mathrm{Z}_2 \rangle,$$

whose derivative along the solutions of

$$\dot{\Psi}_1 \;=\; \mathrm{F}_1(\Psi_1) + \Lambda(\Psi_1) + \mathrm{Z}_2,$$

$$\dot{\mathrm{Z}}_2 \;=\; \mathrm{F}_2(\Psi_1, \Psi_2) - \frac{\partial \Lambda(\Psi_1)}{\partial \Psi_1} (\mathrm{F}_1(\Psi_1) + \Psi_2) + \mathrm{U},$$

is determined by

$$\dot{V}_2 \;=\; \dot{V}_1 \;+\; \left\langle \mathrm{Z}_2, \dot{\mathrm{Z}}_2 \right\rangle \;=\; -W_1(\Psi_1) \;+\; \left\langle \mathrm{Z}_2, \Psi_1 + \mathrm{F}_2 - \frac{\partial \Lambda(\Psi_1)}{\partial \Psi_1} (\mathrm{F}_1 + \Psi_2) + \mathrm{U} \right\rangle.$$

In particular, the following choice of control law

$$\mathrm{U} \;=\; -\Big(\Psi_1 \;+\; \mathrm{F}_2 \;-\; \frac{\partial \Lambda(\Psi_1)}{\partial \Psi_1} (\mathrm{F}_1 + \Psi_2) \;+\; k_2 \mathrm{Z}_2\Big), \qquad (8)$$

with $k_2 > 0$ yields

$$\dot{V}_2(\Psi_1, \mathrm{Z}_2) \;=\; -W_1(\Psi_1) \;-\; k_2 \langle \mathrm{Z}_2, \mathrm{Z}_2 \rangle \;<\; 0,$$

for every $\Psi_1 \in l_2 \setminus \{0\}$, $\mathrm{Z}_2 \in l_2 \setminus \{0\}$. Therefore, control law (8) guarantees global asymptotic stability of the origin of system (4).

Results of this subsection are summarized in the following theorem.

Theorem 1: Suppose that system (4) satisfies Assumptions 1–4. Then there exists a state-feedback control law $\mathrm{U} = \Upsilon(\Psi_1, \Psi_2)$ which guarantees global asymptotic stability of the origin of system (4). One such control law is given by

$$\mathrm{U} \;=\; -\Big(\Psi_1 \;+\; \mathrm{F}_2(\Psi_1, \Psi_2) \;+\; k_2(\Psi_2 - \Lambda(\Psi_1)) - \frac{\partial \Lambda(\Psi_1)}{\partial \Psi_1} (\mathrm{F}_1(\Psi_1) + \Psi_2)\Big), \quad k_2 > 0.$$

B. Individual cell backstepping design

As already mentioned, the distributed backstepping control design on the space of square summable sequences cannot always be performed. For example, if either Assumption 3 or Assumption 4 is not satisfied the construction of a quadratic CLF for system (4) is not possible. In this subsection, we show that global asymptotic stability of the origin of (4) can be achieved by performing design on each individual cell (3) rather than on the entire system (4). For a moment, let the control objective be the regulation of $\psi_{1n}(t)$ and boundedness of $\psi_{2n}(t)$, that is

$$\{\psi_{1n}(t) \to 0 \text{ as } t \to \infty;\; |\psi_{2n}(t)| < \infty,\; \forall\, t \geq 0\},$$

for every $n \in \mathbb{Z}$, and for all $\psi_{1n}(0) \in \mathbb{R}$, $\psi_{2n}(0) \in \mathbb{R}$. We will achieve this objective by providing the global asymptotic tracking of the following trajectory: $(\psi_{1n}, \psi_{2n}) = (0, -f_{1n}(\Psi_1)|_{\psi_{1n}=0})$. If this is accomplished for each individual cell G_n (i.e., for every $n \in \mathbb{Z}$), then by virtue of the fact that Ψ_1 is driven to zero and that $f_{1n}(\Psi_1)$ vanishes at $\Psi_1 = 0$ for every $n \in \mathbb{Z}$ (see Assumption 2), we conclude global asymptotic stability of the origin of system (4).

Step 1 The individual cell backstepping design starts with subsystem (3a) by considering ψ_{2n} as control and proposing a quadratic radially unbounded CLF $V_{1n} : \mathbb{R} \to \mathbb{R}$

$$V_{1n}(\psi_{1n}) \;=\; \frac{1}{2}\psi_{1n}^2. \tag{9}$$

The derivative of $V_{1n}(\psi_{1n})$ along the solutions of (3a) is determined by

$$\dot{V}_{1n} \;=\; \psi_{1n}\dot{\psi}_{1n} \;=\; \psi_{1n}(f_{1n}(\Psi_1) + \psi_{2n}). \tag{10}$$

Clearly, if ψ_{2n} were a control, subsystem (3a) could be stabilized by cancelling nonlinearity $f_{1n}(\Psi_1)$ and adding an additional term to ensure stability

$$\psi_{2nd} \;:=\; \lambda_n(\Psi_1) \;=\; -\,(f_{1n}(\Psi_1) \,+\, k_1\psi_{1n}),$$

with $k_1 > 0$. Let ζ_{2n} denote the difference between ψ_{2n} and its desired value $\lambda_n(\Psi_1)$

$$\zeta_{2n} \;:=\; \psi_{2n} \,-\, \lambda_n(\Psi_1) \;=\; \psi_{2n} \,+\, (f_{1n}(\Psi_1) \,+\, k_1\psi_{1n}).$$

This transforms (10) into

$$\dot{V}_{1n} \;=\; -\,k_1\psi_{1n}^2 \,+\, \psi_{1n}\zeta_{2n}.$$

The sign indefinite term in the last equation will be accounted for at the second step of backstepping.

Step 2 We rewrite system (3b) into a form suitable for the remainder of individual cell backstepping design

$$\dot{\zeta}_{2n} \;=\; f_{2n}(\Psi_1, \Psi_2) - \frac{\partial \lambda_n(\Psi_1)}{\partial \Psi_1}(\mathrm{F}_1(\Psi_1) + \Psi_2) + u_n.$$

Augmentation of (9) by a term which penalizes the deviation of ψ_{2n} from ψ_{2nd} yields a quadratic CLF

$$V_{2n}(\psi_{1n}, \zeta_{2n}) \;:=\; V_{1n}(\psi_{1n}) \,+\, \frac{1}{2}\zeta_{2n}^2,$$

whose derivative along the solutions of

$$\begin{aligned}\dot{\psi}_{1n} \;&=\; -k_1\psi_{1n} + \zeta_{2n},\\ \dot{\zeta}_{2n} \;&=\; f_{2n}(\Psi_1, \Psi_2) - \frac{\partial \lambda_n(\Psi_1)}{\partial \Psi_1}(\mathrm{F}_1(\Psi_1) + \Psi_2) + u_n,\end{aligned}$$

is determined by

$$\begin{aligned}\dot{V}_{2n} \;&=\; \dot{V}_{1n} \,+\, \zeta_{2n}\dot{\zeta}_{2n} \;=\; -k_1\psi_{1n}^2 \,+\\ &\qquad \zeta_{2n}\big(\psi_{1n} + f_{2n} - \frac{\partial \lambda_n(\Psi_1)}{\partial \Psi_1}(\mathrm{F}_1 + \Psi_2) + u_n\big).\end{aligned}$$

The simplest choice of controller that provides negative definiteness of $\dot{V}_{2n}$ is given by

$$u_n \;=\; -\,(\psi_{1n} + f_{2n} - \frac{\partial \lambda_n(\Psi_1)}{\partial \Psi_1}(\mathrm{F}_1 + \Psi_2) + k_2\zeta_{2n}), \tag{11}$$

where k_2 represents a positive design parameter. This choice of control gives

$$\dot{V}_{2n}(\psi_{1n}, \zeta_{2n}) \;=\; -\,k_1\psi_{1n}^2 \,-\, k_2\zeta_{2n} \;<\; 0,$$

for every $(\psi_{1n}, \zeta_{2n}) \in \mathbb{R}^2 \setminus \{0\}$, and every $n \in \mathbb{Z}$. Thus, control law (11) warrants global asymptotic tracking of $(\psi_{1n}, \psi_{2n}) = (0, -f_{1n}(\Psi_1)|_{\psi_{1n}=0})$ of system (3) for every $n \in \mathbb{Z}$, which in turn implies (see Assumption 2) global asymptotic stability of the origin of system (4).

Results of this subsection are summarized in the following theorem.

Theorem 2: Suppose that system (4) satisfies Assumptions 1–2. Then, for every $n \in \mathbb{Z}$, there exists a state-feedback control law $u_n = \gamma_n(\Psi_1, \Psi_2)$ which guarantees global asymptotic stability of the origin of system (4). One such control law is given by

$$\begin{aligned}u_n \;&=\; -\,\big(\psi_{1n} \,+\, f_{2n}(\Psi_1, \Psi_2) \,+\, k_2(\psi_{2n} - \lambda_n(\Psi_1)) \,-\\ &\qquad \frac{\partial \lambda_n(\Psi_1)}{\partial \Psi_1}(\mathrm{F}_1(\Psi_1) + \Psi_2)\big), \quad k_2 > 0,\end{aligned}$$

where

$$\lambda_n(\Psi_1) \;:=\; -\,(f_{1n}(\Psi_1) \,+\, k_1\psi_{1n}), \quad k_1 > 0.$$

V. Architecture induced by backstepping controllers

In this section, we analyze the architecture of distributed controllers induced by a backstepping design. In particular, we study a 'worst case' situation in which all interactions are cancelled at each step of backstepping. Any other backstepping design will result into controllers with more favorable architectures (i.e., less interactions). We show that backstepping design yields distributed controllers that are inherently decentralized, and that there is a strong similarity between plant and controller architectures. More precisely, the controller architecture is determined by two factors: the plant architecture and the largest number of integrators that separate control from certain interactions. For example, since there are $m-1$ integrators between interactions $f_{1n}(\Psi_1)$ in (1a) and location at which control u_n enters, this largest number of integrators in system (1) is equal to $m-1$.

The 'worst case' (i.e., the cancellation) backstepping controller for system (4) is given by

$$\begin{aligned}\mathrm{U} \;&=\; -\,\big((1 + k_1k_2)\Psi_1 \,+\, (k_1 + k_2)(\Psi_2 + \mathrm{F}_1(\Psi_1)) \,+\\ &\qquad \mathrm{F}_2(\Psi_1, \Psi_2) \,+\, \mathrm{P}(\Psi_1) \,+\, \mathrm{Q}(\Psi_1, \Psi_2)\big),\end{aligned}$$

where

$$\begin{aligned}\mathrm{P}(\Psi_1) \;&:=\; \frac{\partial \mathrm{F}_1(\Psi_1)}{\partial \Psi_1}\mathrm{F}_1(\Psi_1),\\ \mathrm{Q}(\Psi_1, \Psi_2) \;&:=\; \frac{\partial \mathrm{F}_1(\Psi_1)}{\partial \Psi_1}\Psi_2.\end{aligned}$$

Equivalently, the n-th cell controller is given by

$$\begin{aligned}u_n \;&=\; -\,\big((1 + k_1k_2)\psi_{1n} \,+\, (k_1 + k_2)(\psi_{2n} + f_{1n}(\Psi_1)) \,+\\ &\qquad f_{2n}(\Psi_1, \Psi_2) \,+\, p_n(\Psi_1) \,+\, q_n(\Psi_1, \Psi_2)\big), \quad \forall\, n \in \mathbb{Z},\end{aligned}$$

where $p_n(\Psi_1)$ and $q_n(\Psi_1, \Psi_2)$ respectively denote the n-th components of infinite vectors $\mathrm{P}(\Psi_1)$ and $\mathrm{Q}(\Psi_1, \Psi_2)$. Based on Assumption 1 and definitions of $\mathrm{P}(\Psi_1)$ and $\mathrm{Q}(\Psi_1, \Psi_2)$, these two quantities are determined by

$$\begin{aligned}p_n(\Psi_1) \;&=\; \frac{\partial f_{1n}(\Psi_1)}{\partial \Psi_1}\mathrm{F}_1(\Psi_1)\\ &=\; \sum_{j \in \mathbb{Z}_N} \frac{\partial f_{1n}(\Psi_1)}{\partial \psi_{1,n+j}} f_{1,n+j}(\{\psi_{1,n+j+i}\}_{i \in \mathbb{Z}_N}),\end{aligned}$$

$$\begin{aligned}q_n(\Psi_1, \Psi_2) \;&=\; \frac{\partial f_{1n}(\Psi_1)}{\partial \Psi_1}\Psi_2\\ &=\; \sum_{j \in \mathbb{Z}_N} \frac{\partial f_{1n}(\Psi_1)}{\partial \psi_{1,n+j}}\psi_{2,n+j}.\end{aligned}$$

The case in which no integrators separate interactions and location at which control enters is referred to as the 'matched' case (or equivalently, we say that the 'matching condition' is satisfied). If system (4) satisfies the matching condition then $f_{1n} = 0$ for every $n \in \mathbb{Z}$ (i.e., $F_1 \equiv 0$). Clearly, in this case both $p_n \equiv 0$ and $q_n \equiv 0$ which implies that the 'worst case' backstepping controller simplifies to

$$u_n = -\left((1 + k_1 k_2)\psi_{1n} + (k_1 + k_2)\psi_{2n} + f_{2n}(\Psi_1, \Psi_2)\right),$$

for every $n \in \mathbb{Z}$. Thus, when (interactions are) matched (by control) the 'worst case' distributed backstepping controller inherits the plant architecture. On the other hand, if the matching condition is not satisfied the additional interactions are induced by the 'worst case' backstepping design. This is because of cancellation of the interactions at the first step of backstepping, their propagation through an integrator, and subsequent cancellation at the second step of our recursive design. Information about these additional interactions is contained in function $p_n(\Psi_1)$. Based on the expression for $p_n(\Psi_1)$ we are able to explicitly quantify the number of interactions induced by a 'worst case' distributed backstepping design: for system (4) with at most $2N$ interactions per plant cell, the 'worst case' distributed backstepping design induces at most $4N$ interactions per controller cell.

This statement can be generalized for system (2): if the n-th plant cell G_n of system (2) interacts with $\{G_{n-N}, \ldots, G_{n+N}\}$ and if $f_{1n}(\psi_{1,n-N}, \ldots, \psi_{1,n+N}) \neq 0$ for every $n \in \mathbb{Z}$, then the n-th cell K_n of the 'worst case' backstepping controller interacts with $\{K_{n-mN}, \ldots, K_{n+mN}\}$. In other words, for system (2) with at most $2N$ interactions per plant cell, the 'worst case' distributed backstepping design induces at most $2mN$ interactions per controller cell.

VI. Example

We consider the following, purely academic, example

$$\dot{\psi}_{1n} \;=\; \psi_{1,n-1}^2 \;+\; \psi_{1n}^2 \;+\; \psi_{1,n+1}^2 \;+\; \psi_{2n}, \qquad (12a)$$

$$\dot{\psi}_{2n} \;=\; u_n, \qquad (12b)$$

where $n \in \mathbb{Z}$. Clearly, system (12) is in form (3) with $f_{1n}(\Psi_1) := \psi_{1,n-1}^2 + \psi_{1n}^2 + \psi_{1,n+1}^2$, and $f_{2n} \equiv 0$. The architecture of the 'worst case' distributed backstepping controller for this system is illustrated in Fig. 1. Thus, to provide global asymptotic stability of system (12) whose n-th cell has only the nearest neighbor interactions, the n-th cell K_n of the cancellation backstepping controller has to interact with $\{K_{n-2}, K_{n-1}, K_{n+1}, K_{n+2}\}$. In § VI-A, we show that domination of harmful interactions, rather than their cancellation, provides less controller interactions.

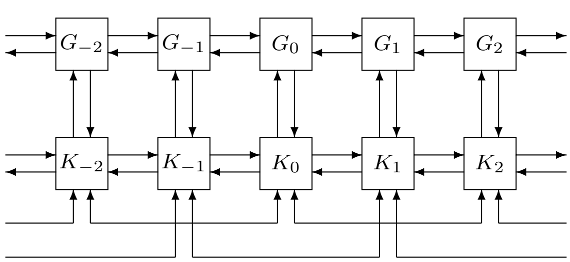

Fig. 1. The architecture of the 'worst case' distributed backstepping controller for system (12).

In applications, we clearly have to work with large-scale systems on lattices. All considerations related to infinite dimensional systems are applicable here, but with minor modifications. For example, if we consider system (12) with $M \in \mathbb{N}$ cells ($n = 1, \ldots, M$) results of § IV are still valid with the appropriate 'boundary conditions': $\psi_{1j} = \psi_{2j} = u_j = f_{1j} \equiv 0,\ \forall\, j \in \mathbb{Z} \setminus \{1, \ldots, M\}$.

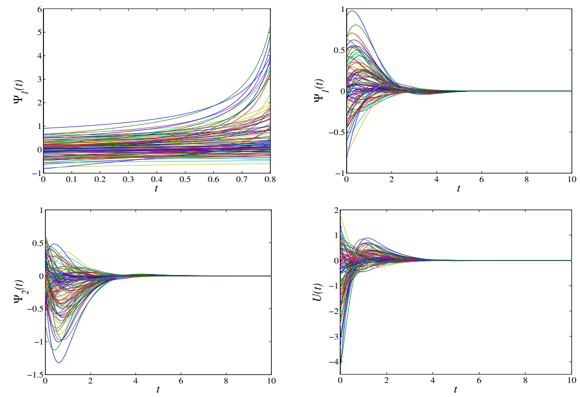

Fig. 2. Control of system (12) with $M = 100$ cells using the 'worst case' backstepping controller with $k_1 = k_2 = 1$.

Figure 2 shows simulation results of uncontrolled (upper left) and controlled system (12) with $M = 100$ cells using the 'worst case' backstepping controller with $k_1 = k_2 = 1$. The initial state of the system is randomly selected. Clearly, the desired control objective is achieved with a reasonable quality of the transient response. This transient response can be further improved with a different choice of design parameters k_1 and k_2 at the expense of increasing the control effort.

A. Design of controllers with less interactions

In this subsection, we demonstrate how global backstepping design can be utilized to obtain controllers with less interactions. In particular, for system (12), whose initial state satisfies Assumption 3, we design a distributed controller with the nearest neighbor interactions and a fully decentralized controller. This is accomplished by a careful analysis of the interactions in system (12), and feedback domination rather than feedback cancellation of harmful interactions. The procedure presented here can be applied to systems in which interactions are bounded by polynomial functions of their arguments.

1) Nearest neighbor interaction controller: **Step 1** As in § IV-A, the global design starts with subsystem (12a) by considering Ψ_2 as control and proposing a quadratic radially unbounded CLF $V_1 : l_2 \to \mathbb{R}$

$$V_1(\Psi_1) \;=\; \frac{1}{2} \sum_{n \,\in\, \mathbb{Z}} \psi_{1n}^2, \qquad (13)$$

whose derivative along the solutions of (12a) is given by

$$\dot{V}_1 \;=\; \sum_{n \,\in\, \mathbb{Z}} \psi_{1n}(\psi_{1,n-1}^2 + \psi_{1n}^2 + \psi_{1,n+1}^2 + \psi_{2n}).$$

We now use Young's Inequality (see [36], expression (2.254)) to bound the interactions between G_n and its immediate neighbors G_{n-1} and G_{n+1}, for every $n \in \mathbb{Z}$

$$\psi_{1n}\psi_{1i}^2 \;\le\; \kappa\psi_{1n}^2 \;+\; \frac{1}{4\kappa}\psi_{1i}^4, \quad \kappa \,>\, 0, \quad i = \{n-1, n+1\}.$$

Hence, $\dot{V}_1$ is upper-bounded by

$$\dot{V}_1 \;\le\; \sum_{n \,\in\, \mathbb{Z}} \psi_{1n}(2\kappa\psi_{1n} + \psi_{1n}^2 + \frac{1}{2\kappa}\psi_{1n}^3 + \psi_{2n}). \qquad (14)$$

Clearly, the following choice of $\psi_{2nd} := \lambda_n(\psi_{1n})$, with $k_1 > 0$,

$$\lambda_n(\psi_{1n}) \;=\; -\left((k_1 \,+\, 2\kappa)\psi_{1n} \;+\; \psi_{1n}^2 \;+\; \frac{1}{2\kappa}\psi_{1n}^3\right), \quad (15)$$

and a coordinate transformation

$$\zeta_{2n} \;:=\; \psi_{2n} \;-\; \lambda_n(\psi_{1n}), \tag{16}$$

yield

$$\dot{V}_1 \;\le\; -\,k_1 \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}^2 \;+\; \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}\zeta_{2n}.$$

The sign indefinite term in the last equation will be accounted for at the second step of backstepping.

Step 2 CLF from Step 1 is augmented by a term which penalizes the deviation of ψ_{2n} from ψ_{2nd}

$$V_2(\Psi_1, \mathrm{Z}_2) \;:=\; V_1(\Psi_1) \;+\; \frac{1}{2}\sum_{n\,\in\,\mathbb{Z}} \zeta_{2n}^2.$$

The derivative of V_2 along the solutions of

$$\begin{aligned} \dot{\psi}_{1n} \;&=\; -\,k_1\psi_{1n} + \zeta_{2n}, && n \in \mathbb{Z}, \\ \dot{\zeta}_{2n} \;&=\; -\,\frac{\partial\lambda_n(\psi_{1n})}{\partial\psi_{1n}}(f_{1n}(\Psi_1) + \psi_{2n}) + u_n, && n \in \mathbb{Z}, \end{aligned}$$

is determined by

$$\begin{aligned} \dot{V}_2 \;\le\; & -k_1 \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}^2 \;+ \\ & \sum_{n\,\in\,\mathbb{Z}} \zeta_{2n}(\psi_{1n} - \frac{\partial\lambda_n(\psi_{1n})}{\partial\psi_{1n}}(f_{1n}(\Psi_1) + \psi_{2n}) + u_n). \end{aligned}$$

We choose a control law of the form

$$u_n \;=\; -\,(\psi_{1n} - \frac{\partial\lambda_n(\psi_{1n})}{\partial\psi_{1n}}(f_{1n}(\Psi_1) + \psi_{2n}) + k_2\zeta_{2n}), \tag{17}$$

with $k_2 > 0$, to obtain

$$\dot{V}_2 \;\le\; -\,k_1 \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}^2 \;-\; k_2 \sum_{n\,\in\,\mathbb{Z}} \zeta_{2n}^2.$$

Hence, controller (17) guarantees global exponential stability of the origin of the infinite dimensional system (12). This controller has the very same architecture as the original plant: the n-th controller cell K_n interacts only with its nearest neighbors K_{n-1} and K_{n+1}.

Figure 3 shows simulation results of uncontrolled (upper left) and controlled system (12) with $M = 100$ cells using the nearest neighbor interaction backstepping controller (15,16,17) with $k_1 = k_2 = 1$ and $\kappa = 0.5$. The initial state of the system is randomly selected. The desired control objective is achieved with a good quality of the transient response and a reasonable amount of control effort.

2) Fully decentralized controller: **Step 1** We start the recursive design with subsystem (12a) by proposing a CLF (13). The derivative of $V_1(\Psi_1)$ along the solutions of (12a) is determined by (14). However, we now choose a 'stabilizing function' $\psi_{2nd} := \lambda_n(\psi_{1n})$ of the form

$$\lambda_n(\psi_{1n}) \;=\; -\left((k_1 \,+\, 2\kappa)\psi_{1n} \;+\; \psi_{1n}^2 \;+\; (k_0 \,+\, \frac{1}{2\kappa})\psi_{1n}^3\right), \tag{18}$$

with $k_0, k_1 > 0$, which clearly renders $\dot{V}_1$ negative definite. Coordinate transformation $\zeta_{2n} := \psi_{2n} - \lambda_n(\psi_{1n})$ yields

$$\dot{V}_1 \;\le\; -\,k_1 \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}^2 \;-\; k_0 \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}^4 \;+\; \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}\zeta_{2n}.$$

The sign indefinite term in the last equation will be taken care of at the second step of backstepping.

Step 2 The second step of our design closely follows the procedure outlined in § VI-A.1. The only difference is that we employ the Young's inequality to upper-bound

$$\begin{gathered} \zeta_{2n}\frac{\partial\lambda_n(\psi_{1n})}{\partial\psi_{1n}}\psi_{1i}^2 \;\le\; \kappa\left(\zeta_{2n}\frac{\partial\lambda_n(\psi_{1n})}{\partial\psi_{1n}}\right)^2 \;+\; \frac{1}{4\kappa}\psi_{1i}^4, \\ \kappa \,>\, 0, \;\;\; \forall\, n \in \mathbb{Z}, \;\;\; \forall\, i = \{n-1, n+1\}, \end{gathered}$$

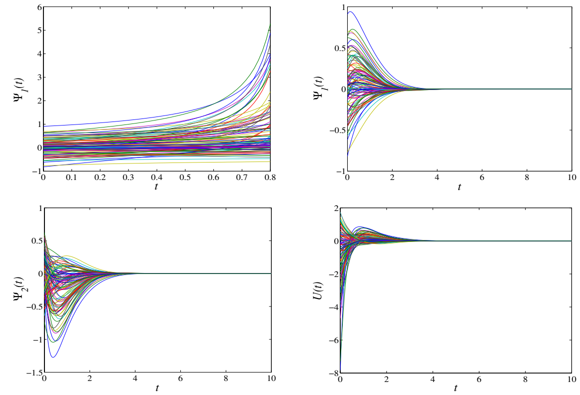

Fig. 3. Control of system (12) with $M = 100$ cells using the nearest neighbor interaction backstepping controller (15,16,17) with $k_1 = k_2 = 1$ and $\kappa = 0.5$.

in the expression for the temporal derivative of $V_2(\Psi_1, \mathrm{Z}_2) := V_1(\Psi_1) + \frac{1}{2}\sum_{n\,\in\,\mathbb{Z}} \zeta_{2n}^2$. This allows us to choose a fully decentralized controller of the form

$$\begin{aligned} u_n \;=\; -\,\big(&\psi_{1n} - \frac{\partial\lambda_n(\psi_{1n})}{\partial\psi_{1n}}(\psi_{1n}^2 + \psi_{2n}) \;+ \\ &(\psi_{2n} - \lambda_n(\psi_{1n}))(k_2 + 2\kappa(\frac{\partial\lambda_n(\psi_{1n})}{\partial\psi_{1n}})^2)\big), \end{aligned} \tag{19}$$

with $k_2 > 0$, to obtain

$$\dot{V}_2 \;\le\; -\,k_1 \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}^2 \;-\; (k_0 \,-\, \frac{1}{2\kappa}) \sum_{n\,\in\,\mathbb{Z}} \psi_{1n}^4 \;-\; k_2 \sum_{n\,\in\,\mathbb{Z}} \zeta_{2n}^2.$$

Thus, controller (19) with $\kappa > 0$, $k_0 \ge \frac{1}{2\kappa}$, $k_1 > 0$, and $k_2 > 0$ guarantees global asymptotic stability of the origin of the infinite dimensional system (12). This controller is fully decentralized: the n-th controller cell K_n interacts only with the plant cell on which it acts G_n.

Figure 4 shows simulation results of uncontrolled (upper left) and controlled system (12) with $M = 100$ cells using the fully decentralized backstepping controller (18,19) with $k_0 = k_1 = k_2 = 1$ and $\kappa = 0.5$. The initial state of the system is randomly selected. Clearly, the fully decentralized controller requires big amount of initial effort to account for the lack of information about interactions between different subsystems. We remark that there is some room for improvement of these large initial excursions of control signals by the different choice of design parameters k_0, k_1, k_2, and κ. However, the obtained results seem to be in agreement with our intuition: *higher gain is required to achieve the desired control objective when controller cells do not communicate with each other.*

Remark 1: We note that neither a distributed controller with the nearest neighbor interactions nor a fully decentralized controller for system (12) can be obtained using the individual cell backstepping procedure of § IV-B. This is because the harmful interactions–that are dominated by feedback in the global design–are treated as the exogenous signals in the individual cell design. Thus, the 'worst case' backstepping controller in which K_n interacts with $\{K_{n-2}, K_{n-1}, K_{n+1}, K_{n+2}\}$ is pretty much the only controller that can come out of the individual cell backstepping design.

VII. Concluding remarks

This paper deals with architectural questions in distributed control of nonlinear infinite dimensional systems on lattices. We show that distributed backstepping design yields decentralized controllers whose architecture can be significantly altered by different choices of stabilizing functions during the

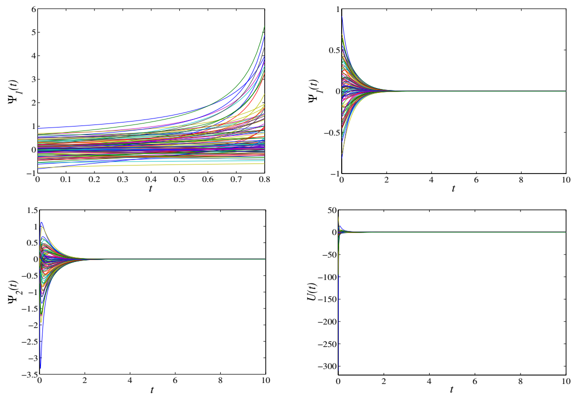

Fig. 4. Control of system (12) with $M = 100$ cells using the fully decentralized backstepping controller (18,19) with $k_0 = k_1 = k_2 = 1$ and $\kappa = 0.5$.

recursive design. For the 'worst case' situation in which all interactions are cancelled at each step of backstepping we quantify the number of control induced interactions necessary to achieve the desired design objective. Our results are also valid for output-feedback design of systems in which nonlinearities depend only on the measured variables.

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