On the Feasibility of Large-Scale Automated Highways *

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ABSTRACT
We investigate the use of large-scale Automated Highway Systems, also called one-dimensional vehicle platoons, as a practical traffic mitigation solution. We review recent theoretical results related to vehicle platooning and discuss their implications on the scalability and safety of large-scale platoons. We also quantify the performance of different platoon control strategies through simulations that use realistic models of vehicle dynamics and sensor accuracy.

1. INTRODUCTION
Traffic congestion is easily identified as a personal annoyance, however the problem accumulates on a global scale. In 2005, the United States population alone spent 42 billion hours in traffic, wasting 2.9 billion gallons of gas and $78.2 billion (USD) [12]. There are over 600 million cars in operation in the world today, and by 2030, that number is projected to double [14]. With this influx of vehicles and the potential for increase in traffic congestion, it is important to look to a variety of social and technological solutions to help mitigate the problem.

One unique solution to traffic management is the Automated Highway System (AHS). In an AHS, a string, or *platoon*, of driverless vehicles moves at high speeds along a dedicated lane of a highway. Each vehicle operates autonomously, accelerating or decelerating in response to local sensor measurements of velocity and position, as well as inter-vehicle communication, to maintain the platoon formation. Because the vehicles are automatically controlled, they can safely operate at higher speeds and closer inter-vehicle distances than human-driven vehicles, thus increasing the capacity and throughput of the highway.

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The AHS was first proposed in 1961 [4], and has been the subject of studies for decades, for example [7, 6, 11]. In the late 1990s, this work culminated in several on-road demonstrations, including the 1997 California PATH project San Diego demonstration [8]. In this demonstration, a completely autonomous platoon of 8 Buick LeSabres travelled along the high occupancy vehicle lane of Interstate 15 at highways speeds, while maintaining an inter-vehicle distance of 6.5 meters. If platoons like the one demonstrated in San Diego were deployed on a large scale, the throughput of the AHS would be double that of a standard highway. If the inter-vehicle distance could be reduced to 2.5 meters, the resulting reduction in drag force would yield a 20% improvement in fuel economy and emission reductions [8].

Despite these small-scale successes, the development of a large-scale vehicle platoon poses significant challenges, and the full blown AHS has not yet come to fruition. There has, however, been a flurry of recent work on the theoretical aspects of vehicular platoons and on automated formation control in general. Examples of this work include studies of necessary and sufficient conditions on the formation structure [5], theoretical bounds on the "tightness" of formations [1], and analysis of formation dynamics for different vehicle control strategies [10]. We also note that the problem of vehicle platooning and automated formation control is closely related to the distributed consensus problem, which has been widely studied in the contexts of sensor networks and parallel computing [13, 3, 2, 9, 15].

Vehicle platoons and the AHS are attractive in terms of both traffic reduction and environmental benefits. Development of a large-scale implementation of an AHS will require not only an understanding of the body of theoretical work, but also performance analysis using realistic models of vehicle dynamics and sensor accuracy. In this work, we first briefly review recent theoretical results related to vehicle platooning and discuss the implications on the feasibility large-scale

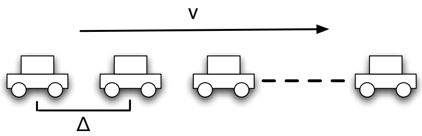

Figure 1: A vehicle platoon

platoons. We then quantify, through simulations, the scalability and safety of different platoon control strategies described in the literature. We aim to demonstrate how these strategies will perform in an actual physical implementation, and therefore, our simulations employ real-world values to model vehicle behavior, and our control strategies take into account the accuracy of existing sensor technologies.

The remainder of this paper is organized as follows. In Section 2, we review the vehicle platooning problem and some important theoretical results. In Section 3, we describe our simulation setup, and we give results of platoon simulations for various platoon control strategies. Finally, we conclude in Section 4, with a summary of insights for the design of large-scale platoons and pose topics for future work.

2. VEHICLE PLATOONING PROBLEM

A vehicle platoon is a string of N vehicles. The state of each vehicle is given by its absolute position x_k and its absolute velocity v_k. The objective is for each vehicle to follow the specified trajectory,

$$\bar{x}_k = vt + k\Delta,$$

where v is the velocity of the platoon and Δ is the space between vehicles [1]. In other words, all vehicles in the platoon should move at constant velocity v while maintaining an inter-vehicle distance of Δ. We assume that Δ is measured from the center of one vehicle to the center of the next vehicle in the platoon, as shown in Fig. 1.

Due to external forces such as wind gusts and irregularities in engine performance, vehicles cannot follow the trajectories exactly, but constantly must make adjustments to compensate for these disturbances. These adjustments are based on (potentially erroneous) sensor measurements of position and velocity and are applied as a change in vehicle acceleration. The vehicle behavior is therefore governed by the following double integrator dynamics,

$$\ddot{x_k} = u_k + w_k.$$

u_k is the control input, which uses the sensor readings and the defined control strategy. w is a mutually uncorrelated white stochastic process that models the effects of the external disturbances.

Control Strategies

We assume that each vehicle uses an identical control strategy that is based on local measurements of position and velocity. These measurements may be relative or absolute, and are defined as follows. Relative position is the distance between two consecutive vehicles in the platoon. To obtain the relative position information, each vehicle measures the distance between itself and its successor or predecessor in the platoon. Relative velocity is the difference between neighboring vehicles in the platoon, and can also be measured independently by each vehicle. Absolute position is the position relative to some fixed coordinate system, and absolute velocity is the road speed. Each measurement has some inaccuracy due to sensor error. These inaccuracies are modeled by uncorrelated white noise processes.

We consider the following four control strategies.

1. *Relative Position/Relative Velocity*: Each vehicle's control input is based on measurements of the position and velocity relative to the vehicles immediately in front and behind,

$$\begin{aligned} u_k &= \alpha\Big(\big((x_{k+1} - x_k) + m_k^F\big) - \Delta\Big) - \\ &\quad \Big(\big((x_k - x_{k-1}) + m_k^B\big) - \Delta\big)\Big) + \\ &\quad \beta\Big(\big((v_{k+1} - v_k) + q_k^F\big) - (v_k - v_{k-1}) + q_k^B)\Big). \end{aligned}$$

   The update due to the position term alone equalizes the inter-vehicle distance between vehicle and its successor and predecessor. m_k^F and m_k^B represent the error in measuring the distance to these two vehicles. Similarly, the velocity term specifies an update to match the average of the velocity of the vehicles in front and behind. q_k^F and q_k^B are the measurement errors for these relative velocities. α and β specify how heavily to weight each of the two terms in the update strategy.

   We note that the lead vehicle in the platoon does not use the same strategy as the other vehicles, Instead its strategy is based only on absolute velocity. It does not incorporate any position information.

2. *Relative Position/Absolute Velocity*: Each vehicle's control input uses the same relative position update rule as above, but updates its velocity based on absolute velocity measurement such as can be obtained from a speedometer,

$$\begin{aligned} u_k &= \alpha(\Big((x_{k+1} - x_k) + m_k^F) - \Delta) - \\ &\quad ((x_k - x_{k-1}) + m_k^B) - \Delta)\Big) + \\ &\quad \beta\left((v_k - v) + r_k\right). \end{aligned}$$

   r_k models the inaccuracy in the absolute velocity measurement.

3. *Absolute Position/Relative Velocity*: In this case, each vehicle uses absolute position measurements, such as GPS readings, in addition to relative velocity measurements,

$$\begin{aligned} u_k &= \alpha((x_k - \bar{x}_k) + s_k) + \\ &\quad \beta\Big(((v_k - v_k) + q_k^F) - ((v_k - v_{k-1}) + q_k^B)\Big). \end{aligned}$$

   The position term updates the vehicle position so as to move the vehicle closer to the position specified by the trajectory. s_k models the absolute position measurement error. As in the first strategy, the lead vehicle uses absolute velocity instead of relative velocity.

4. *Absolute Position/Absolute Velocity*: This update strategy relies on absolute measurements of position and

velocity,

$$u_k = \alpha((x_k - \bar{x}_k) + s_k) + \beta((v_k - v) + r_k).$$

We note that, while this is a valid platooning strategy, it is not considered a conventional strategy for formation control since there are no interactions between the vehicles.

The choice of measurements affects both the individual vehicle deviations from the trajectory, which has implications for safety, and the "tightness" of the formation as a whole, which can have drastic consequences on the throughput of automated highways. We summarize some of the theoretical results relating to these local and global properties in the next section, and we explore this topic in further detail in the simulations.

Theoretical Results

We are interested in studying performance of the control strategies on both the microscopic (inter-vehicle) and macroscopic level, and therefore we consider two different performance measures. The first is the variance of the inter-vehicle spacing, defined as

$$V_{sp} := \mathrm{var}(x_{k+1} - x_k - \Delta)$$

V_{sp} is directly related to the safety of vehicles, as a smaller variance enables us to choose a Δ for which the vehicles will not collide

The second performance measure we consider is the variance of the difference between the actual and desired platoon length.

$$V_{len} := \mathrm{var}(x_N - x_1 - \Delta(N-1)).$$

A small variance in the platoon length indicates that the formation is tight and occupies the expected amount of space on the road. A large variance indicates that the platoon may grow to lengths beyond those anticipated, thus reducing the capacity and the throughput of the AHS.

To provide insight into the perfomance of large-scale platoons, we outline some analytical results for these performance measures under the various control strategies.

Formation Guarantees: It has been shown that, for appropriate choices of α and β, all four control strategies yield inter-vehicle distances that have finite variances [1, 10], i.e. V_{sp} is finite. These results imply that the control strategies can each be used to implement a vehicle platoon for some choice of Δ. However, it is important to note that these results do not give any indication as to the size of V_{sp}. In fact, the magnitude of Δ necessary to avoid collisions varies greatly depending on the control strategy.

Microscopic Error: In [1], Bamieh et al. give analytical bounds for V_{sp} for various strategies. The authors show that, in a ring formation using a strategy that relies only on relative information, V_{sp} grows linearly with the number of vehicles N. Therefore, in practice, increasing the number of vehicles in the platoon necessitates an increase in Δ to prevent vehicle collisions. For the remaining three strategies, V_{sp} is bounded in N. With these strategies, it is possible to compute an a priori bound on the size of V_{sp} independent of the number of vehicles in the platoon and choose Δ accordingly.

Table 1: Simulation model parameters.

Parameter	Value
Vehicle Length	5 m
Platoon Size	100 vehicles
Platoon Velocity	100 km/hr
Inter-vehicle distance	6.5 m

Table 2: Measurement error parameters.

Parameter	Physical Device	Accuracy
Rel. Position	Laser Range Finder[1]	± 0.04m
Rel. Velocity	Laser Range Finder	± 0.89 m/s
Abs. Position	Augmented GPS[2]	±3 m
Abs. Velocity	Speedometer[3]	±3% of true velocity

Macroscopic Error: Bamieh at al. also give analytical bounds for the macroscopic error V_{len} in a ring formation [1]. They show that with a strategy that relies only on relative measurements, V_{len} grows as N^3, where N is the number of vehicles. This result suggests that the Relative Position/Relative Velocity strategy cannot be used to implement a tight, large-scale formation. However, for the Relative Position/Absolute Velocity and Absolute Velocity/Relative Position strategies, V_{len} grows as N, which will result in tighter formations.

The above performance bounds apply specifically to ring formations. In the next section, we explore how these results translate to vehicle platoons through realistic simulations of the four different control strategies.

3. SIMULATIONS

We begin with an explanation of the simulation setup and then present our simulation results.

Simulation Model

We analyze the performance of each strategy by simulating the dynamics of the deviations from the specified trajectory,

$$\tilde{x}_k := x_k - \bar{x}_k, \qquad \tilde{v}_k := \dot{x}_k - v.$$

Each strategy defined in the previous section is linear in $\tilde{x}$ and $\tilde{v}$, and therefore, the control input u can be written as a linear system of equations, $u := A\tilde{x} + B\tilde{v} + n$. A is an $N \times N$ matrix that depends on the type of position information (absolute or relative) and the choice of α, and B is an $N \times N$ matrix that depends on the type of velocity information and the choice of β. n is vector of random variables that model the errors in the sensor measurements. In order to model absolute position and velocity measurements, we use a fictitious lead vehicle that is not subject to external disturbances. Absolute measurements can be computed as

[1]See http://www.ascscientific.com/impulse.html.
[2]See http://www8.garmin.com/aboutGPS/waas.html.
[3]According to United States regulations, a speedometer must be accurate to within ± 5 % of actual velocity. We assume the speedometer is accurate to within ± 3 % of the trajectory-specified velocity of 28 m/s.

relative measurements to this vehicle. The evolution of $\tilde{x}$ and $\tilde{v}$ is then given by

$$\frac{d}{dt}\begin{bmatrix}\tilde{x}\\ \tilde{v}\end{bmatrix} = \begin{bmatrix}0 & I\\ A & B\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{v}\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix}n + \begin{bmatrix}0\\ I\end{bmatrix}w.$$

For each control strategy, we simulate a 100 vehicle platoon using this continuous time model. We set $\alpha = \beta = 1$ for all strategies. All simulations were implemented and run in Matlab and Simulink, using a sample time of 0.1 seconds.

To make our results as relevant as possible, we utilize real-world values for the simulation parameters. These values are given in Table 1. We have chosen the same inter-vehicle distance used in the California PATH project demonstration, 6.5 meters. With a vehicle length of 5 meters, Δ then equals 11.5 meters, and therefore, for a platoon of 100 vehicles, the target platoon length is 1150 meters. We use a target velocity of 100 km/hr.

Additionally, we model the position and velocity sensors using the accuracy of existing and emerging technologies. These technologies and their associated accuracies are described in Table 2. For relative position and velocity information, we model measurements from a laser range finder. For absolute position information, we use the accuracy of GPS, specifically augmented GPS[2]. Conventional GPS has an accuracy of approximately 12 meters, which would be too large for vehicle platoons, however higher accuracy can be achieved if the GPS service is augmented with correction information from ground base stations. We note that augmented GPS is only available in North America, and has not yet been widely adopted. For absolute velocity measurement, we anticipate that this information will be provided by the vehicle's speedometer, and therefore model this measurement accuracy using the specification for speedometer performance[3]. In order to model the external disturbances, we compute the change in acceleration of a 1500 kg vehicle mid-size vehicle that is caused by a wind gust of 15 km/hr, and we use this value as the variance to the white noise process w.

For all strategies, we weigh the position and velocity update equally, setting $\alpha = \beta = 1$. Each vehicle's state is initialized using the specified trajectory. We run the simulation until 25,000 samples are collected, and then analyze the data using the last 15,000 samples taken to allow the system to achieve a steady state before computing the statistics. The results of the simulations are given in the next section.

Simulation Results

Performance results for each control strategy are shown in Table 3. For the microscopic error V_{sp}, we use the variance of the distance between the 49th and 50th vehicles as a representative value. We note that there will be some variation in this value depending on which position in the platoon is selected, as illustrated in a later figure. In addition to the microscopic and macroscopic errors, we show the minimum inter-vehicle distance, which is an indicator of the smallest Δ that ensures that vehicles will not collide. We also show the maximum platoon length, which defines the maximum capacity of the AHS that can be achieved with the given control strategy.

On a high level, it is evident that the strategy that relies only on relative information performs significantly worse on both the microscopic and macroscopic levels than strategies that make use of any type of absolute information. In the Relative Position/Relative Velocity strategy, V_n is approximately an order of magnitude larger than that of the other strategies, and this difference is consistent with the theoretical results. The minimum inter-vehicle distance is actually negative, which means that the strategy would result in collisions for an inter-vehicle spacing of 6.5 meters. Additionally, the platoon length exhibits variation significantly larger than that of the other strategies. This result is also consistent with the theoretical bounds for ring formations. The maximum platoon length is over 1300 meters, meaning that, at some time during the simulation, the platoon expanded to nearly 200 meters longer than the goal platoon size of 1150 meters.

While all three strategies that use absolute measurements have good performance, absolute position information appears to contribute more to macroscopic error than absolute velocity information. However, the maximum platoon length is similar for the three strategies, and very close to the specified platoon length. Therefore, one can expect similar throughput for these strategies. At the microscopic level, the Relative Position/Absolute Velocity strategy and the Absolute Position/Relative Velocity strategy both perform better than a pure absolute information strategy, as indicated by a larger minimum inter-vehicle distance and a smaller inter-vehicle distance variance. Based on these results, it seems beneficial from a safety standpoint to use a strategy that incorporates at least some type of relative measurement.

In Fig. 2, we show the platoon length for each strategy as it varies over time. We display results for the first 1600 seconds of each simulation. The Relative Position/Relative Velocity strategy exhibits large variations in platoon length, consistently fluctuating between short and long lengths. Interesting observations can be made about the platoon that uses the Relative Position/Absolute Velocity strategy. The platoon appears to maintain a length that is actually smaller than the specified length. In addition, the platoon length does not change rapidly, as opposed to the Absolute Position/Relative Velocity strategy, which exhibits rapid, small fluctuations in platoon size.

Finally, in Fig. 3, we show the microscopic error V_{sp} for each pair of neighboring vehicles x_{k+1} and x_k in the platoon. The values along the horizontal axis indicate the position of the first vehicle in the pair. The results for the strategies that use relative position are somewhat counterintuitive. The variance of the inter-vehicle distance is largest near the front of the platoon. In the case of Relative Position/Relative Velocity strategy, the variance is very large near the front and steadily decreases as the position increases. We conjecture that this effect is due in part to the strategy used by the lead vehicle, which includes only absolute velocity, and no relative information. The fact that the lead vehicle uses a different strategy than the other vehicles may cause the lead vehicle to be "out of sync" with the rest of the platoon, and the performance effects of this difference may propagate backwards. For the Relative Position/Absolute Velocity and Absolute Position/Relative Velocity strategies, the lead ve-

Table 3: Errors for different strategies for a 100 vehicle platoon.

Strategy	Relative Position, Relative Velocity	Relative Position, Absolute Velocity	Absolute Position, Relative Velocity	Absolute Position, Absolute Velocity
Microscopic Error (${V_{sp}}^4$)	2.1149	0.1200	0.35489	0.5568
Min. Inter-vehicle Distance (in meters)	-1.3900	2.0869	2.3859	1.68160
Macroscopic Error (V_{len})	4255.98	57.65	7.5715	0.5669
Max. Platoon Length (in meters)	1333.18	1152.11	1157.22	1152.18

hicle also uses a different strategy than the other vehicles in the platoon. With the Relative Position/Absolute Velocity strategy, this difference may be the cause of the slightly higher variances near the front of the platoon. However, we do not observe such differences in the Absolute Position/Relative Velocity or Absolute Position/Absolute Velocity strategies.

Our performance results indicate that the strategy that uses relative position and absolute velocity is competitive with those that use absolute position in terms of throughput, and may offer benefits over a pure absolute information strategy in terms of safety. Therefore, the performance illustrated by these results can be obtained using existing, widely available sensor technologies.

4. CONCLUSION

From our review of theoretical results and our performance studies, we can glean several insights into the development of control strategies for large-scale Automated Highway Systems. First, we assert that some kind of absolute measurement is needed to implement safe and scalable platoons. This assertion is substantiated by the theoretical results on the macroscopic errors as well as simulations that demonstrate high microscopic and macroscopic errors for the strategy that uses only relative information. The second insight is that including at least some kind of relative information in the control strategy results in a decreased microscopic error, and therefore improves these safety of the platoon. We believe that it may be advantageous to design a strategy that utilizes all four types of measurements in order to obtain the benefits of each of the four strategies we studied.

Finally, we note that the control strategies presented in this work consider only one aspect of a successful AHS, maintaining platoon formation while traveling at highway speeds. There are many other issues that must be addressed to develop a complete AHS solution, including emergency actions such as obstacle avoidance and accommodation for vehicles that merge with and exit from the platoon. The investigation of strategies for these events is the subject of future work.

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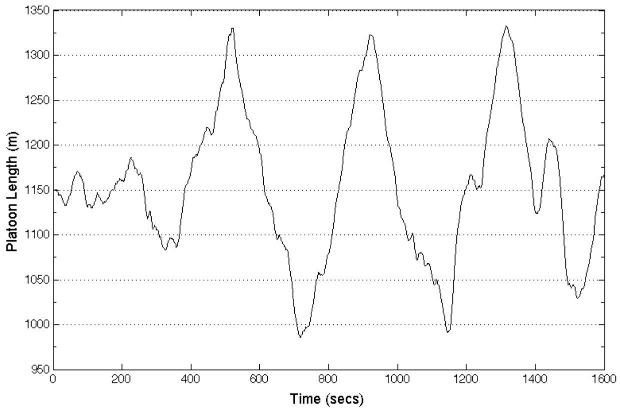
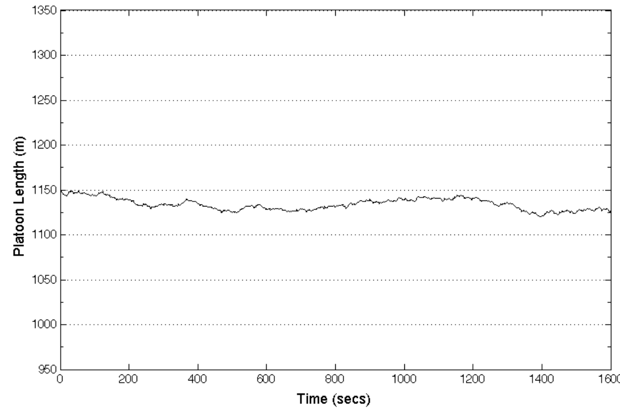
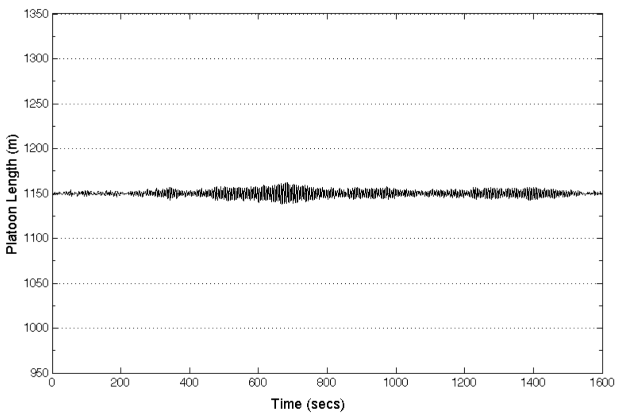


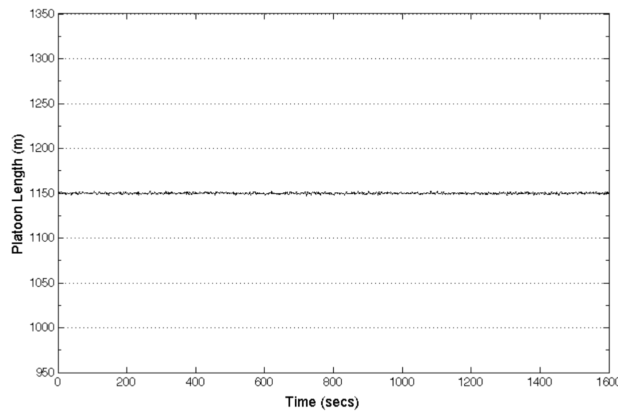
(a) Relative Position and Relative Velocity



(b) Relative Position and Absolute Velocity



(c) Absolute Position and Relative Velocity



(d) Absolute Position and Absolute Velocity

Figure 2: Variation in platoon length over time for different control strategies.

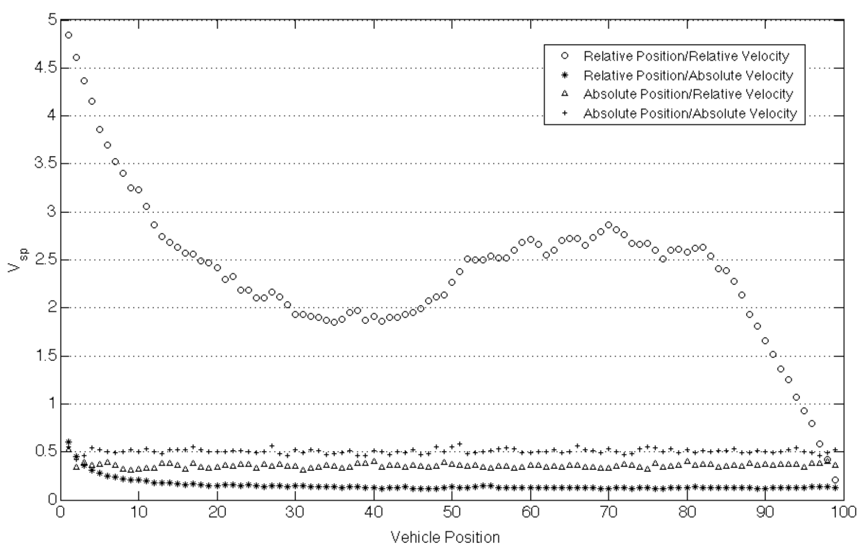


Figure 3: Variance in inter-vehicle distance for different vehicle positions.