A frequency domain analysis of compressible linearized Navier-Stokes equations in a hypersonic compression ramp flow

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Abstract—We utilize the frequency response analysis of the linearized Navier-Stokes equations to quantify amplification of exogenous disturbances in a hypersonic flow over a compression ramp. Using the spatial structure of the dominant response to time-periodic inputs, we explain the origin of steady reattachment streaks. Our analysis of the laminar shock/boundary layer interaction reveals that the streaks arise from a preferential amplification of upstream counter-rotating vortical perturbations with a specific spanwise wavelength. These streaks are associated with heat flux striations at the wall near flow reattachment and they can trigger transition to turbulence. The streak wavelength predicted by our analysis compares favorably with observations from two different hypersonic compression ramp experiments. Furthermore, we utilize the dominant response to analyze the physical effects in the linearized dynamical system responsible for amplification of disturbances. We show that flow compressibility that arises from base flow deceleration contributes to the amplification of streamwise velocity and that the baroclinic effects are responsible for the production of streamwise vorticity. Both these effects contribute to the appearance of temperature streaks observed in experiments and are critically important for the development of control-orientated models for transition to turbulence in hypersonic flows.

I. Introduction

Hypersonic flows over complex surfaces are governed by the compressible Navier-Stokes (NS) equations. At these conditions, where the fluid flows at least five times faster than the speed of sound, aerodynamic heating of the vehicle surface becomes a major concern: it is about five times larger in turbulent than in laminar flow [1] and can lead to structural failure. The dynamics at such extreme flow conditions are poorly understood and development of control-oriented models is critically important for vehicle performance and survivability. In this paper, we analyze dynamical properties of the compressible NS equations in the presence of spatially distributed and temporally varying external disturbances. These disturbances are considered as inputs, while various combinations of velocities, temperature, and pressure are considered as outputs. In principle, input-output analysis using the non-linear models based on the compressible NS equations can be carried out. However, the implementation involves large number of degrees of freedom and prohibitively expensive direct numerical simulations. Furthermore, it is difficult to obtain useful results in such a generality. Thus, in the present work, we conduct input-output analysis of the compressible linearized NS equations.

To demonstrate the usefulness of input-output framework, we consider the hypersonic flow over a compression ramp as a canonical flow configuration over a control surface. At such high speeds, this inevitably results in shock/boundary layer interaction (SBLI) [2] that involves separation and reattachment of flow close to the surface (the boundary layer) and discontinuity associated with a shock system. Even though the compression ramp geometry is homogeneous in the spanwise direction, experiments [3] show that the flow over it exhibits three-dimensionality in the form of streamwise streaks near reattachment. The streaks are associated with persistent large local peaks of heat transfer; they can destabilize the boundary layer and cause transition to turbulence [2], [3].

Recently, there has been an increased interest in experimental investigation of hypersonic compression ramp flows [3], [4]. Techniques such as temperature sensitive paint (TSP) and infrared (IR) imaging were employed to study the formation of streamwise streaks and reattachment heat flux patterns. Previous studies [2], [4], [5] indicated that these streaks might originate from upstream perturbations to the SBLI setup. However, developing physical insights responsible for the development of these three-dimensional structures by relying solely on the experimental and numerical observations can be misleading [6]. In this context, we demonstrate how physical mechanisms responsible for the development of these streaks can be discovered by examining the input-output dynamics of the linearized NS equations.

The linearized NS equations have been widely used for carrying out the modal and non-modal stability analysis of idealized low-speed setups involving parallel (spatially inhomogeneous in one direction) and non-parallel (spatially inhomogeneous in multiple direction) flows [7]. In particular, an input-output framework that evaluates the response (outputs) of a the linearized NS equations to external perturbation sources (inputs) has been successfully employed to quantify the amplification...
and study transition mechanisms in low speed channels [8], [9], boundary layers [10]–[12], and jets [13]. However, applications to the high-speed flows in absence of any simplifying geometrical and physical assumptions is still an open challenge. In this paper, we utilize the I/O analysis to demonstrate that the hypersonic shock/boundary layer interaction over a compression ramp strongly amplifies low-frequency upstream disturbances with a specific spanwise length scale. The dominant I/O pair resulting from our analysis is used to explain the emergence of reattachment streaks and to compare our results with experiments.

Our presentation is organized as follows. In Section II, we present the linearized model and provide a brief summary of the I/O formulation. Section III provides details about the flow geometry and computational setup. In Section IV, we evaluate the frequency response of 2D laminar hypersonic base flow on a compression ramp to 3D upstream disturbances and illustrate that the dominant output field appears in the form of steady streamwise streaks near reattachment. In Section V, we examine the physical mechanisms responsible for streak amplification by analyzing the spatial structure of the dominant response. This is used to identify amplification mechanisms active in the different regions of the flow field and demonstrate the utility of the input-output analysis in analyzing complex flow phenomenon. We conclude our presentation in Section VI.

II. Linearized model for compressible flows

The compressible NS equations for perfect gas in conservative form are given by

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x_j} = 0,
\]

where \( \mathbf{F}(\mathbf{U}) \) is the flux vector and \( \mathbf{U} = (\rho, \rho \mathbf{u}, E) \) is the vector of conserved variables representing mass, three components of momentum, and total energy per unit volume of the gas [14]. We decompose the state vector \( \mathbf{U}(x, t) \) into a steady base component \( \mathbf{U}(x) \) and a time-varying perturbation component \( \mathbf{U}'(x, t) \), \( \mathbf{U}(x, t) = \mathbf{U}(x) + \mathbf{U}'(x, t) \). The evolution of small perturbations is then governed by the linearized flow equations,

\[
\frac{\partial}{\partial t} \mathbf{U}'(x, t) = \mathbf{A}(\mathbf{U})\mathbf{U}'(x, t),
\]

where \( \mathbf{A}(\mathbf{U}) \) represents the compressible NS operator resulting from linearization of (1) around the base flow \( \mathbf{U} \). We assume that \( (\rho \mathbf{u})' \) satisfies homogeneous Dirichlet boundary conditions while \( \rho' \) and \( E' \) satisfy homogeneous Neumann boundary conditions at the wall. Along the rest of the boundaries, all the perturbations are assumed to satisfy homogeneous Dirichlet boundary conditions. A second order central finite volume discretization (as described in [15]) is used to obtain the finite dimensional approximation of Eq. (2),

\[
\frac{d}{dt} \mathbf{q} = \mathbf{Aq},
\]

which describes the dynamics of the spatially discretized perturbation vector \( \mathbf{q} \). In general, for a finite volume discretization with \( \text{nel} \) cells, the discrete state \( \mathbf{q} \in \mathbb{R}^{5\text{nel} \times 1} \) and system matrix \( \mathbf{A} \in \mathbb{R}^{5\text{nel} \times 5\text{nel}} \).

In this paper, we are interested in quantifying the amplification of exogenous disturbances in boundary layer flows [7], [9]. To accomplish this objective, we augment the evolution model (3) with external excitation

\[
\frac{d}{dt} \mathbf{q} = \mathbf{Aq} + \mathbf{Bd},
\]

where \( \mathbf{d} \) is a spatially distributed and temporally varying disturbance source (input) and \( \phi = (\rho', \mathbf{u}', T') \) is the quantity of interest (output), where \( T' \) denotes temperature perturbations. In Eq. (4) the matrix \( \mathbf{B} \) specifies how the input enters into the state equation, while the matrix \( \mathbf{C} \) extracts the output from the state \( \mathbf{q} \).

In boundary layer flows, the linearized flow system is globally stable. Thus, for a time-periodic input with frequency \( \omega \), \( \mathbf{d}(t) = \mathbf{d}(\omega)e^{i\omega t} \), the steady-state output of a stable system (4) is given by \( \phi(t) = \phi(\omega)e^{i\omega t} \), where \( \phi(\omega) = \mathbf{H}(\omega)\mathbf{d}(\omega) \) and \( \mathbf{H}(\omega) \) is the frequency response

\[
\mathbf{H}(\omega) = \mathbf{C}(\mathbf{i} \omega \mathbf{I} - \mathbf{A})^{-1}\mathbf{B}.
\]

At any \( \omega \), the singular value decomposition of \( \mathbf{H}(\omega) \) can be used to quantify amplification of time-periodic inputs [7], [8],

\[
\mathbf{H}(\omega) = \mathbf{\Phi}(\omega)\Sigma(\omega)\mathbf{D}^*(\omega).
\]

Here, \((\cdot)^*\) denotes the complex-conjugate transpose, \( \mathbf{\Phi} \) and \( \mathbf{D} \) are unitary matrices, and \( \Sigma \) is the rectangular diagonal matrix of the singular values \( \sigma_i(\omega) \). The columns \( \mathbf{d}_i \) of the matrix \( \mathbf{D} \) represent the input forcing directions that are mapped through the frequency response \( \mathbf{H} \) to the corresponding columns \( \phi_i \) of the matrix \( \mathbf{\Phi} \); for \( \mathbf{d} = \mathbf{d}_i \), the output \( \phi_i \) is in the direction \( \phi_i \) and the amplification is determined by the corresponding singular value \( \sigma_i \). For a given temporal frequency \( \omega \),

\[
G(\omega) := \sigma_1(\omega) = \frac{\|\mathbf{H}(\omega)\mathbf{d}_1(\omega)\|_E}{\|\mathbf{d}_1(\omega)\|_E} = \frac{\|\phi_1(\omega)\|_E}{\|\mathbf{d}_1(\omega)\|_E}
\]

denotes the largest induced gain with respect to Chu’s compressible energy norm [16] which is defined as,

\[
\| \cdot \|_E^2 = \int_\Omega \frac{\hat{\rho}}{2} u'^2 + \frac{\hat{\rho}}{2\hat{\rho}_0} \rho'^2 + \frac{\hat{\rho}C_v}{2\hat{T}} T'^2 \, d\Omega,
\]

where \( (\hat{\rho}, \hat{\rho}_0, \hat{T}) \) denote the base flow pressure, density and temperature, and \( C_v \) is the specific heat at constant volume in the domain \( \Omega \). Consistent with this energy norm, we evaluate the eigenvalue decomposition of the self-adjoint system \( \mathbf{H}^* \mathbf{H} \) using a parallel implementation of the power method [17] to obtain the spatial structure of the dominant I/O pair \( (\mathbf{d}_1(\omega), \phi_1(\omega)) \) and gain \( G \).
TABLE I

Free-stream conditions for experiments [4] and [3]

<table>
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<tr>
<th>Re&lt;sub&gt;L&lt;/sub&gt;</th>
<th>p&lt;sub&gt;∞&lt;/sub&gt; (Pa)</th>
<th>T&lt;sub&gt;∞&lt;/sub&gt; (K)</th>
<th>U&lt;sub&gt;∞&lt;/sub&gt; (m/s)</th>
<th>ρ&lt;sub&gt;∞&lt;/sub&gt; (Kg/m&lt;sup&gt;3&lt;/sup&gt;)</th>
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<td>55 K</td>
<td>1190 m/s</td>
<td>0.022 Kg/m&lt;sup&gt;3&lt;/sup&gt;</td>
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<tr>
<td>2.0 × 10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>164 Pa</td>
<td>55 K</td>
<td>1188 m/s</td>
<td>0.010 Kg/m&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
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III. Flow over a hypersonic compression ramp

Streamwise streaks in wall temperature are often observed in compression ramp experiments. Although their appearance is typically attributed to amplification that arises near reattachment from centrifugal [4] effects, quantifying amplification in the presence of a recirculation bubble is an open challenge. Herein, we employ the I/O framework to study the amplification of infinitesimal spanwise periodic upstream disturbances in hypersonic compression ramp flow and explain origin of the heat streaks at reattachment.

Recently, [3] and [4] reported multiple hypersonic compression ramp experiments in two different facilities with matched free-stream Mach and Reynolds numbers. Temperature sensitive paint (TSP) and infrared thermography measurements of reattachment heat-flux wall patterns revealed quantitatively similar streaks. The effects of free-stream Reynolds number Re and leading edge radius on the spanwise wavelength \(\lambda_z\) of the streaks were also reported. Our objective is to identify the spanwise wavelength \(\lambda_z\) that is selected by the linearized compressible NS equations in the shock/boundary layer interaction.

We consider the experiments performed in the UT-1M Ludwing tube [4] at Mach 8. As illustrated in figure 1(a), the geometry consists of an \(L = 50\) mm isothermal flat plate with a sharp leading edge and wall temperature \(T_w = 293\) K, followed by an inclined ramp at 15°. The streamwise domain extends from \(x/L = 0\) to \(x/L = 1.65\). Table I summarizes the two free-stream conditions which determine the 2D base flows considered in our study. For the first free-stream condition, figure 1(b) provides comparison of the experimental schlieren image with the 2D base flow density gradient magnitude field that we computed using the finite volume compressible flow solver US3D [14]. Our 2D simulations correctly capture the presence of both the separation and reattachment shocks.

The Stanton number \(St\) is a non-dimensional parameter that determines the wall heat-transfer coefficient [4]. In experiments, the Stanton number can be inferred from TSP and infrared thermography measurements. Figure 1(c) compares experimental values of \(St\) to those predicted by our 2D simulations at different grid resolutions. We see that the computed flow captures the heat flux trends correctly except near the separation and the post-reattachment regions. In experiments, these regions display significant spanwise variation in \(St\) and they are marked by the grey band in figure 1(c).

Since the flow is globally stable with respect to 3D perturbations [15], we conjecture that spanwise variations arise from non-modal amplification of 3D perturbations around the 2D base flow. To verify our hypothesis, we employ global I/O analysis to quantify the amplification of exogenous disturbances and uncover mechanisms that can trigger the early stages of transition in a hypersonic compression ramp flow.

IV. Frequency response analysis

We utilize frequency response analysis to investigate the amplification of infinitesimal upstream perturbations in a hypersonic compression ramp flow. This choice is motivated by the experimental studies [3], [4] where variation in the properties of the incoming boundary layer were found to have profound effects on the downstream streaks. We model these upstream disturbances by restricting the inputs to the domain prior to separation (i.e., \(x_s/L < 0.5\)). This is implemented by selecting the matrix \(B := I_{ny \times ny} \otimes A_{nx \times nx}\), where \(A\) is a diagonal matrix with \(\text{tan}(x - x_s)/L\) along the diagonal (\(\epsilon = 10^{-6}\)). Furthermore, we choose the perturbation field in the entire domain as the output, \(\phi = q\), by setting \(C = I\). The I/O analysis is conducted on a grid with 412 cells in the streamwise and 249 cells in the wall-normal direction (labeled as G3 in figure 1(c)). Numerical sponge boundary conditions [15] (to no reflection from the inflow and outflow) are applied near the leading edge \((x/L < 0.02)\) and the outflow \((x/L > 1.6)\).

The left plot in figure 2 shows the input-output amplification \(G(\omega)\), defined in Eq. (7), in a flow with high Reynolds number \((Re_L = 3.7 \times 10^4)\) for different spanwise wavelengths \(\lambda_z\). Here, \(\lambda_z := \lambda_z^\ast/\delta_{sep}\) and \(\omega := \omega^\ast/\delta_{sep}/U_\infty\) denote the non-dimensional spanwise wavelength and temporal frequency, respectively, \(\lambda_z^\ast\) and \(\omega^\ast\) are the corresponding quantities in physical units, whereas \(\delta_{sep}\) represents the displacement boundary layer thickness at separation. We observe the low-pass feature of the amplification curve: \(G\) achieves its largest value at \(\omega = 0\), it decreases slowly for low frequencies, and it experiences a rapid decay after the roll-off frequency \((\omega \approx 0.01)\). The visualization of the dominant input-output directions \(d_1\) and \(\phi_1\) in figures 2(a) and 2(b) reveals that the flat region of the amplification curve corresponds to incoming streamwise vortical disturbances (as inputs) that generate streak-like downstream perturbations (as outputs). In contrast, figure 2(c) demonstrates that, at high temporal frequency \((\omega = 0.1)\), dominant input-output pairs exhibit streamwise periodicity and take the form of oblique waves. It should be noted that the low-pass frequency response features as well as the resulting changes in the response shape (from streaks to oblique waves) were also observed in canonical channel and boundary layer flows [8], [12].

The impact of the spanwise wavelengths \(\lambda_z\) on the amplification \(G\) for steady perturbations (i.e., at \(\omega = 0\)) is shown in figure 3(a). For both Reynolds numbers, the
amplification curve achieves its maximum for a particular value of $\lambda_z$. This indicates that SBLI preferentially amplifies upstream perturbations with a specific spanwise wavelength. The experimental estimates of $\lambda_z$ resulting from the observed spanwise modulations in the TSP images in figure 3(b) agree well with the predictions of our I/O analysis. This shows that the compression ramp flow strongly amplifies steady upstream disturbances with a preferential spanwise length scale.

V. Amplification of steady reattachment streaks: physical mechanism

As demonstrated in the previous section, the hyper-sonic flow over a compression ramp selectively amplifies small upstream perturbations of a specific spanwise wavelength. In order to gain insights into the structure of the most amplified perturbations we analyze the velocity and vorticity components ($u'_s, \omega'_s$). These components of the dominant output $\phi_1$ quantify the kinetic energy of the spanwise periodic streamwise vortices along the base flow streamlines [6]. We utilize the $(s, n, z)$ coordinate system which is locally aligned with the streamlines of the base flow ($\bar{u}_s, 0, 0$) to simplify our analysis. Here, $s$ denotes direction along streamlines and $n$ denotes direction normal to it. This is shown in figure 4, where we also show the wall-aligned coordinate system, with $\xi$ and $\eta$ denoting the directions parallel and normal to the wall, respectively.

In the flow with $Re_L = 3.7 \times 10^5$, figure 4(b) illustrates the output components corresponding to $\lambda_z = 3$ near reattachment in the $(n, z)$ plane. We note that the most amplified perturbations are given by alternating regions of high and low velocities with counter rotating vortices between them and that $u'_s$ and $\omega'_s$ are 90° out of phase in the spanwise direction.

To investigate the physical mechanisms responsible for the amplification of 3D reattachment streaks we analyze the dominant terms in the system matrix $A$. In particular, we examine the spatial structure of the streamwise vorticity, velocity, and temperature components associated with the dominant output $\phi_1$ and identify amplification mechanisms that result from the interactions of 3D flow perturbations with 2D base flow gradients.

A. Spatial evolution of streamwise vorticity

In order to quantify the development of streaks, we consider the spatial evolution of streamwise vorticity $\omega'_s$. After comparing relative magnitude of various terms and dropping terms (see [6] for details) with small magnitude we obtain,

$$\bar{u}_s \partial_s \omega'_s \approx \frac{\partial n \rho}{\rho^2} \beta p'$$  

Fig. 1. (a) Flow geometry and 2D steady streamwise velocity at $Re_L = 3.7 \times 10^5$; (b) comparison to experimental schlieren; and (c) variation of Stanton number ($St$) with $x/L$. Curves G1-G4 denote computational grids at varying resolution ($n_x \times n_y$), with G1 (577 $\times$ 349), G2 (495 $\times$ 300), G3 (412 $\times$ 249), and G4 (330 $\times$ 200) where $n_x$ and $n_y$ denote the number of streamwise and wall normal grid cells, respectively. The shaded grey region denotes envelope of spanwise variation of $St$ measured in experiments.

Fig. 2. Left: the $\omega$-dependence of the largest induced gain with respect to the compressible energy norm, $G(\omega)$, for unsteady inputs with the spanwise wavelengths $\lambda_z = \{1, 3, 6, 10\}$. Right: isosurfaces of streamwise vorticity corresponding to the input $d_1$ and temperature corresponding to the output $\phi_1$ for (a) $\omega = 0$; (b) $\omega = 0.02$; and (c) $\omega = 0.1$ for $\lambda_z = 3$. 

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Fig. 3. (a) The $\lambda_z$-dependence of the amplification map $G$ for steady inputs (i.e., $\omega = 0$); and (b) comparison of experiments and dominant output $\phi_1$ at reattachment. The vertical dashed line in (b) denotes the approximate reattachment line in experiments and 2D simulations.

Fig. 4. (a) Schematic of the different coordinate systems for analyzing the perturbation evolution; and (b) color plots of streamwise velocity $u'_s$ and contour lines of streamwise vorticity $\omega'_s$ at streamwise location $x/L = 1.4$ (post-reattachment) corresponding to the dominant output $\phi_1$ at $\lambda_z = 3$. Solid lines denote positive values and dashed lines denote negative values of $\omega'_s$.

Fig. 5. (a) Illustration of the baroclinic term $\nabla\tilde{\rho} \times \nabla p'$ for the dominant output with $\lambda_z = 3$; and (b) corresponding quantities near the reattachment plane at $x/L = 1.36$.

The term on the left quantifies evolution of $\omega'_s$ as we go along the base streamlines. The right-hand side appears due to the baroclinic effect, which arises from misalignment of pressure and density gradients and accounts for differential acceleration caused by variable inertia [18].

We illustrate the linear baroclinic mechanism in figure 5(a) by showing three quantities: (i) the base flow density $\tilde{\rho}$ in the $(x, y)$ plane using an orange colormap; (ii) the spanwise gradient of the pressure perturbations $p'$ in the $(y, z)$ plane near reattachment using the red-white-blue colormap; and (iii) the iso-surfaces of streamwise vorticity $\omega'_s$ using a grey-black colormap. Since the linearized baroclinic torque that is active in the steady response is associated with $\nabla\tilde{\rho} \times \nabla p'$, we focus on examining the gradients of $\tilde{\rho}$ and $p'$ shown in figure 5(b). Near reattachment, the density gradient is aligned with the wall-normal direction $\eta$. This is because the $\tilde{\rho}$ colormap becomes darker as we move away from the wall in the direction of increasing $\eta$. At the same $x$ location, the gradient of $p'$ is orthogonal to the $(x, y)$ plane; it achieves its largest value midway between the blue and the red lobes and it points in the direction from the center of the blue to the center of red lobes. As illustrated in figures 5(a) and (b), the resulting linearized baroclinic torque $\nabla\tilde{\rho} \times \nabla p'$ aligns with the streamwise vorticity $\omega'_s$, thereby leading to its production.

B. Spatial evolution of streamwise velocity and temperature

Recent experimental [19] and numerical studies [20] demonstrated that streamwise velocity perturbations contribute most to the kinetic energy in SBLI. The spatial evolution of streamwise velocity $u'_s$ is governed by,

\[
\bar{u}_s \partial_s u'_s \approx - \partial_s \bar{u}_s u'_s
\]  

This equation demonstrates that the growth of $u'_s$ appears because of the streamwise deceleration of the base flow (where $\partial_s \bar{u}_s < 0$). This is a unique feature of compressible flows and demonstrates the utility of the I/O approach in identifying new physical phenomenon present in complex fluid flows.
C. Spatial evolution of temperature perturbations

To understand the formation of heat streaks near reattachment, we consider the spatial amplification of temperature perturbations $T'$ as they are transported by the base flow. For the most amplified output perturbations, we retain the terms with significant contribution to the inviscid transport equation for $T'$,

$$
\bar{u}_s \partial_s (T'^2 / 2) \approx - \partial_s \bar{T} (u'_s T') - \partial_n \bar{T} (u'_n T') - (\gamma - 1) (\nabla \cdot \bar{u}) T'^2.
$$

The first term on the right-hand-side leads to production of temperature perturbations due to streamwise gradient $\partial_s \bar{T}$ near reattachment, whereas the second term accounts for the transport from $\omega'_s$ due to the wall-normal thermal base flow gradients in the boundary layer. Therefore, both $u'_s$ and $\omega'_s$ contribute to production of $T'$ at reattachment. The third term quantifies the base flow dilatation in the reattachment shock where $\nabla \cdot \bar{u}$ takes large negative values. All of these three physical effects significantly contribute to the amplification of $T'$ near reattachment.

VI. Concluding remarks

We have employed an input-output analysis to investigate the dynamics of compressible linearized NS equations and to study amplification of disturbances in compressible boundary layer flows. Our approach utilizes global linearized dynamics to study the growth of flow perturbations and identify the spatial structure of the dominant response.

In an effort to explain the heat streaks observed in the hypersonic flows, we have examined the experimentally observed reattachment streaks in shock/boundary layer interaction on Mach 8 flow over 15° compression ramp. The I/O analysis predicts large amplification of incoming steady streamwise vortical disturbances with a specific spanwise length scale. The dominant output takes the form of steady streamwise streaks near reattachment.

We have also uncovered physical mechanisms responsible for amplification of steady reattachment streaks by evaluating the dominant terms in the system matrix. Physically, this quantifies the contribution of the base flow gradients to the production of the perturbation quantities. Using our approach we demonstrate that the appearance of the temperature streaks near reattachment is triggered by the compressible flow effects associated with streamwise deceleration in the recirculation bubble and the baroclinic effects near reattachment.

The I/O approach provides a useful computational framework to quantify the growth of external perturbations in complex high-speed flows. Improved understanding of amplification mechanisms can provide important physical insights about transition to turbulence. We expect that our work will motivate additional studies that explore nonlinear aspects of transition and pave the way for the development of predictive transition models and effective flow control strategies.

References