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On the peaking phenomenon in the control of vehicular platoons

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Abstract

We investigate the peaking phenomenon in the control of large-scale vehicular platoons. These systems are of considerable practical importance as they represent an example of systems on lattices in which different subsystems are dynamically coupled only through feedback controls. We demonstrate that imposing a uniform rate of convergence for all vehicles towards their desired trajectories may generate large transient peaks in both velocity and control. We further derive explicit constraints on feedback gains – for any given set of initial conditions – to achieve desired position transients without magnitude and rate peaking. These constraints are used to generate the trajectories around which the states of the platoon system are driven towards their desired values without the excessive use of control effort. All results are illustrated using computer simulations of platoons containing a large number of vehicles.

Keywords: Distributed control; Inter-connected systems; Peaking phenomenon; Vehicular platoons

1. Introduction

Control of vehicular platoons has been an intensive area of research for almost four decades [24,25,10,30,36,26,33,34]. These systems belong to the class of systems on lattices in which the interactions between different subsystems originate because of a specific control objective that the designer wants to accomplish [19]. Additional examples of systems on lattices with this property include unmanned aerial vehicles in formation [9,14,15] and satellites in synchronous orbit [22,7,38]. These interactions often generate surprisingly complex responses that cannot be inferred by analyzing the individual plant units. Rather, intricate behavioral patterns, an instance of which is the so-called *string instability* [32] (or, more generally, the *spatio-temporal instability* [6]), arise because of the aggregate effects. Another particularity of this class of systems is that every subsystem is equipped with sensing and actuating capabilities. The controller design problem is thus dominated by architectural questions such as the choice of localized vs centralized control.

Jovanović and Bamieh [20] and Seiler et al. [29] recently addressed some fundamental design limitations in vehicular platoons. Jovanović and Bamieh [20] exhibited shortcomings of several widely cited solutions [24,25] to the Linear Quadratic Regulator (LQR) problem for large-scale vehicular platoons. By considering infinite platoons as the limit of the large-but-finite platoons, spatially invariant theory was employed to show *analytically* how these formulations lack stabilizability or detectability. On the other hand, Seiler et al. [29] showed that string stability of a finite platoon with linear dynamics cannot be achieved with any linear controller that uses only information about relative distance between the vehicle on which it acts and its immediate predecessor. A similar result was previously established for a spatially invariant infinite string of vehicles with static feedback controllers having the same architecture [10]. This necessitates the use of distributed strategies for control of platoons and underscores the importance of developing distributed schemes with favorable architectures. We refer the reader to the

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Fig. 1. Platoon of M vehicles.

references above for a fuller discussion of various algorithms that can be used for platoon control. Additional information about recent work on distributed control of spatially interconnected systems can be found in [6,4,27,12,16,13,19,8] and the references therein.

In this paper, we study some additional fundamental limitations and tradeoffs in the control of large-scale vehicular platoons. In particular, we investigate *the peaking phenomenon*, and show that imposing a uniform rate of convergence for all vehicles towards their desired trajectories may generate large transient peaks in both velocity and control. This indicates that in very long platoons one needs to account explicitly for the initial distances of vehicles from their desired trajectories to avoid large position and velocity deviations and the excessive use of control effort. We derive an initial condition dependent set of requirements that the control gains need to satisfy to guarantee the desired quality of position transient response, and rule out peaking in both velocity and control. These requirements are used to generate the trajectories around which the states of the platoon are driven to their desired values without the excessive use of control effort.

Our presentation is organized as follows: in Section 2, we formulate a control problem and propose a distributed control strategy that solves it. In Section 3, we illustrate that commanding a uniform rate of convergence for all vehicles towards their desired trajectories may require large control efforts. In Section 4, we remark on some basic design limitations and tradeoffs in vehicular platoons and determine the conditions that control gains need to satisfy to provide operation within the imposed saturation limits. We also redesign the controller of Section 2 to provide the desired quality of transient response and avoid large control excursions. We summarize the major contributions and the ongoing research directions in Section 5.

2. Localized distributed inversely optimal control of vehicular platoons

A system of M identical unit mass vehicles is shown in Fig. 1. The dynamics of this system can be captured by representing each vehicle as a moving mass with the second order dynamics

$$\ddot{x}_n = u_n, \quad n \in \{1, \ldots, M\}, \tag{1}$$

where x_n represents the position of the nth vehicle, and u_n is the control applied on the nth vehicle.

A control objective is to provide a desired constant cruising velocity v_d and to keep the inter-vehicular distance at a constant pre-specified level δ. For each vehicle, we introduce the position and velocity error variables with respect to the (absolute) desired trajectories

$$\xi_n(t) := x_n(t) - v_d t + n\delta,$$

$$\zeta_n(t) := \dot{x}_n(t) - v_d,$$

and rewrite (1) as

$$\begin{aligned} \begin{bmatrix} \dot{\xi} \\ \dot{\zeta} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \zeta \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \\ &=: A\psi + Bu, \end{aligned} \tag{2}$$

where $\xi := \text{col}\{\xi_n\}$, $\zeta := \text{col}\{\zeta_n\}$, and $u := \text{col}\{u_n\}$. The relative position errors between neighboring vehicles are determined by

$$\begin{aligned} \eta_n(t) &:= x_n(t) - x_{n-1}(t) + \delta \\ &= \xi_n(t) - \xi_{n-1}(t), \quad n \in \{2, \ldots, M\}. \end{aligned}$$

We propose the following static distributed controller

$$u = K\psi = -\begin{bmatrix} aI + bL & cI \end{bmatrix} \psi, \tag{3}$$

where a, b, and c denote positive design parameters, and $L \in \mathbb{R}^{M \times M}$ is a matrix describing information exchange between different vehicles. Clearly, controller (3) contains two parts: a) a local feedback, $a\xi + c\zeta$, that utilizes information about absolute position and velocity of each vehicle; b) a term, $bL\xi$, that dynamically couples vehicles through feedback control. For simplicity, we have selected a control architecture that exchanges only information about vehicle's positions; information about vehicle's velocities can be utilized in a similar manner.

Fig. 2 illustrates control architectures that can be used for distributed control of vehicular platoons. If L is a diagonal matrix then there is no information exchange between the vehicles and control strategy (3) is *fully decentralized*. This approach ignores the fact that a vehicle is a part of the platoon and as such is not safe for implementation. If L is a full matrix then there is communication between all the vehicles and controller (3) is *centralized*. In this case, every vehicle utilizes information from all other vehicles for achieving the desired control objective which usually results in best performance, but it requires excessive communication. If L is a banded matrix then there is a communication between few neighboring vehicles, and (3) represents a *localized distributed* controller. For example, if L is given by

$$L := \begin{bmatrix} 1 & -1 & 0 & & 0 & 0 & 0 \\ -1 & 2 & -1 & & 0 & 0 & 0 \\ 0 & -1 & 2 & & 0 & 0 & 0 \\ & & & \ddots & & & \\ 0 & 0 & 0 & & 2 & -1 & 0 \\ 0 & 0 & 0 & & -1 & 2 & -1 \\ 0 & 0 & 0 & & 0 & -1 & 1 \end{bmatrix}, \tag{4}$$

then the controller for vehicle n utilizes information about absolute position and velocity of vehicle n, and information about the distances between vehicle n and neighboring vehicles $n-1$ and $n+1$. The architecture of this localized controller with nearest neighbor interactions is shown in the middle plot of Fig. 2.

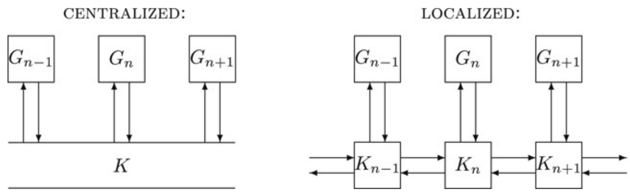

Fig. 2. Distributed controller architectures for centralized, localized (with nearest neighbor interactions), and fully decentralized strategies.

A spectral decomposition of L

$$L = V \Lambda V^*, \quad V V^* = V^* V = I,$$
$$\Lambda = \mathrm{diag}\{\lambda_1(L), \ldots, \lambda_M(L)\},$$

can be used to establish the stability of (2)–(4) for any choice of positive design parameters a, b, and c. This follows directly from the fact that the eigenvalues of L are determined by (see, for example, [17]):

$$\lambda_n(L) = \begin{cases} 2\left(1 - \cos\dfrac{n\pi}{M}\right) & n \in \{1, \ldots, M-1\}, \\ 0 & n = M. \end{cases}$$

Next, we address the question of whether it is possible to select the weights in the LQR problem

$$J = \frac{1}{2}\int_0^\infty \left(\xi^* Q_\xi \xi + \zeta^* Q_\zeta \zeta + u^* R u\right) \mathrm{d}t, \tag{5}$$

with $Q_\xi = Q_\xi^* \geq 0$, $Q_\zeta = Q_\zeta^* \geq 0$, and $R = R^* > 0$, to obtain a *localized distributed controller* for (2). Any quadratic cost functionals for system (2) that does not penalize products between positions and velocities, can be represented by (5). The design of optimal distributed controllers with *a priori* assigned localization constraints is, in general, a difficult problem (we refer the reader to [4,27,37,5,23,8] and the references therein for recent efforts in this area). Instead, we ask the following question:

- *Given a stabilizing localized distributed controller* (3) *and* (4) *for* (2), *is it inversely optimal with respect to cost functional* (5)?

This problem is inverse because we start with a stabilizing state-feedback for (2) and search for performance indices (5) for which this state-feedback is optimal. In other words, state and control weights Q_ξ, Q_ζ, and R in (5) are not *a priori* assigned; rather, they are determined *a posteriori* by stabilizing control law (3) and (4). Optimality of the closed-loop system is desirable because it guarantees, among other properties, favorable gain and phase margins [21,3]. These margins provide robustness to different types of uncertainty.

System (2) with a state-feedback control law $u = K\psi$ can be equivalently represented by a feedback arrangement shown in Fig. 3.

The so-called *return difference* of the system whose block diagram is shown in Fig. 3 is defined by [21,3]

$$H(s) := I - K(sI - A)^{-1}B =: I - KG(s)B.$$

It is well known that $H(j\omega)$ for every $\omega \in \mathbb{R}$ satisfies the *return difference equality* [21,3]

$$R + B^* G^*(j\omega) Q G(j\omega) B = H^*(j\omega) R H(j\omega), \tag{6}$$

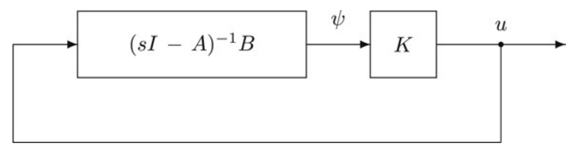

Fig. 3. Block diagram of (2) with $u = K\psi$.

where, for example, $G(j\omega) := (j\omega I - A)^{-1}$ and $G^*(j\omega) := -(j\omega I + A^*)^{-1}$. A direct consequence of this equality is

$$H^*(j\omega) R H(j\omega) \geq R. \tag{7}$$

Relationships (6) and (7) can be used to obtain a frequency domain condition for inverse optimality [21,3]:

$$\sigma_{\min}\left(R^{1/2} H(j\omega) R^{-1/2}\right) \geq 1, \quad \forall \omega \in \mathbb{R}, \tag{8}$$

where $\sigma_{\min}$ denotes the minimal singular value. By selecting $R = rI$ in (5) with $r > 0$, a spectral decomposition of L can be used to show that controller (3) and (4) represents an inversely optimal controller for the LQR problem (2) and (5) if and only if

$$c^2 \geq 2(a + b\lambda_1(L)).$$

If this condition is not satisfied than (3) and (4) fail to be optimal in the LQR sense. If this condition is satisfied, the state penalty

$$Q := \begin{bmatrix} Q_\xi & 0 \\ 0 & Q_\zeta \end{bmatrix},$$

can be determined from (6), and it is given by

$$Q_\xi = r\,(aI + bL)^2, \qquad Q_\zeta = r\left((c^2 - 2a)I - 2bL\right).$$

These penalties on ξ and ζ represent unique solutions to (6) provided that the products between positions and velocities are not penalized in J. However, for given $R := rI > 0$, there are many other matrices

$$Q = \begin{bmatrix} Q_\xi & Q_{\xi\zeta} \\ Q_{\xi\zeta}^* & Q_\zeta \end{bmatrix} \geq 0,$$

with non-zero off-diagonal elements (that is, $Q_{\xi\zeta} \neq 0$) that satisfy (6) and give controller (3) and (4) as a solution to the corresponding LQR problem.

The main result of this section – which is summarized in Theorem 1 – is a direct consequence of the above derivations.

Theorem 1. *A localized distributed controller*

$$u = -((aI + bL)\xi + c\zeta),$$

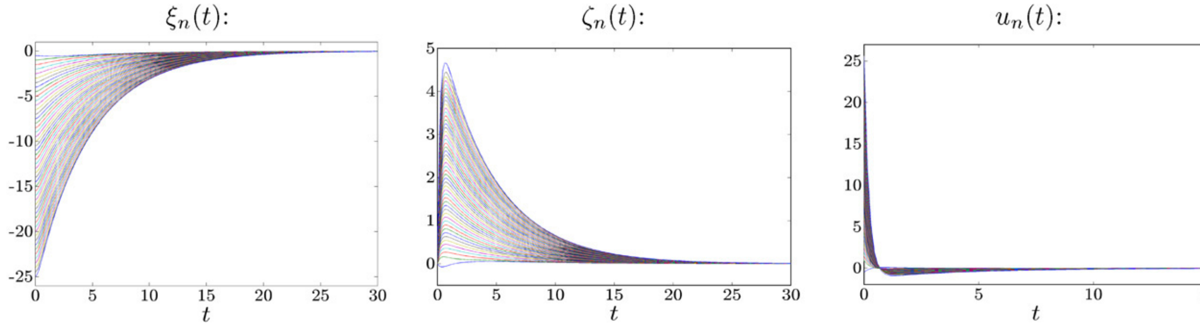

Fig. 4. Simulation results of system (2) using control law (3) and (4) with $a = 1, b = 2$, and $c = 5$. The initial state of the platoon system is given by (10) with $M = 50$ and $\mu = 0.5$.

with $\{a > 0, b > 0, c \geq \sqrt{2(a + b\lambda_1(L))}\}$ *and* L *given by* (4), *represents a stabilizing solution to the following LQR problem:*

$$\begin{bmatrix}\dot{\xi}\\ \dot{\zeta}\end{bmatrix} = \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\begin{bmatrix}\xi\\ \zeta\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix}u,$$

$$J = \frac{1}{2}\int_0^{\infty} \left(\xi^* Q_\xi \xi + \zeta^* Q_\zeta \zeta + r u^* u\right) \mathrm{d}t, \quad r > 0,$$

$$Q_\xi = r\,(aI + bL)^2, \qquad Q_\zeta = r\left((c^2 - 2a)I - 2bL\right).$$

Remark 1. The controller of Theorem 1 represents an example of localized distributed controllers that can be used to achieve an optimal stabilization (in the LQR sense) of formation of vehicles. Many other controllers with localized architectures can meet the same objective. Performance comparison of different localized distributed strategies is beyond the scope of this paper.

3. Peaking in control strategies with uniform convergence rates

In this section, we show that imposing a uniform rate of convergence for all vehicles towards their desired trajectories may generate large control magnitudes. Our results indicate that in very large platoons one needs to select the control gains judiciously to avoid the excessive use of control effort. To demonstrate that these issues are not caused by the specific control strategy, we consider three different designs: (a) design of Section 2, (b) LQR design for a platoon of M vehicles arranged in a circle, and (c) 'controller with information of lead and preceding vehicles', originally proposed by Hedrick et al. [18], and subsequently studied by Swaroop and Hedrick [31,33]. In all three cases we assume that each vehicle has a limited amount of control effort at its disposal, that is $u_n(t) \in [-u_{\max}, u_{\max}]$, for all $t \geq 0$, with $u_{\max} > 0$. Examples of physically relevant initial conditions are provided to establish that fast stabilization of large-scale platoons leads to large transient peaks.

3.1. Design of Section 2

We first consider a platoon of vehicles (2) with controller (3) and (4) and $\{a > 0, b > 0, c \geq \sqrt{2(a + b\lambda_1(L))}\}$. It is readily shown that under these conditions the eigenvalues of the closed-loop A-matrix are uniformly bounded (away from the origin).

In particular, we study the situation in which at $t = 0$ the string of vehicles cruises at the desired velocity v_d. We also assume that the distance between the vehicles indexed by n and $n-1$, for every $n \in \{2, \ldots, M\}$, $M \geq 2$, is equal to $\delta + \mu$. In other words, we consider the following initial condition

$$\left.\begin{aligned} x_n(0) &= -n(\delta + \mu)\\ \dot{x}_n(0) &= v_d \end{aligned}\right\} \quad n \in \{1, \ldots, M\}, \tag{9}$$

which translates into:

$$\left.\begin{aligned} \xi_n(0) &= -n\mu\\ \zeta_n(0) &= 0 \end{aligned}\right\} \quad n \in \{1, \ldots, M\}. \tag{10}$$

For this choice of initial condition the initial amount of control effort is given by

$$u_n(0) = \begin{cases} (a - b)\mu & n = 1,\\ na\mu & n \in \{2, \ldots, M-1\},\\ (aM + b)\mu & n = M, \end{cases}$$

which implies that for any choice of *positive* design parameters a and b there exist $m \leq M$ such that $|u_n(0)| > u_{\max}$, for every $n > m$, provided that M is large enough. For example, if $a = 1, b = 2, c = 5, \mu = 0.5, u_{\max} = 5$, for $M = 50, u_{10}(0) = u_{\max} = 5$, and $|u_n(0)| > u_{\max}, \forall n > 10$. Simulation results for this choice of design parameters and initial conditions given by (10), using controller (3) and (4), are shown in Fig. 4.

3.2. LQR design for a platoon of vehicles arranged in a circle

In this section, we consider a *spatially invariant* LQR design for a platoon of M vehicles arranged in a circle (see Fig. 5), which is an idealization of the case of equally spaced vehicles on a closed track. By exploiting the *spatial invariance*, we *analytically* establish that *any* LQR design leads to large control signals for the appropriately selected, physically relevant set of initial conditions.

The control objective is the same as in Section 2: to drive the entire platoon at the constant cruising velocity v_d, and to keep the distance between the neighboring vehicles at a pre-specified constant level δ. Clearly, this is possible only if the radius of a circle is given by $r_M = M\delta/2\pi$. We rewrite system (1) for

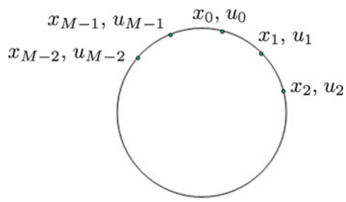

Fig. 5. Circular platoon of M vehicles.

$n \in \{0, \ldots, M-1\}$ in terms of a state-space realization of the form

$$\begin{bmatrix} \dot{\xi}_n \\ \dot{\zeta}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_n \\ \zeta_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n =: A_n \varphi_n + B_n u_n, \tag{11}$$

where $\xi_n(t) := x_n(t) - v_d t - n\delta$ and $\zeta_n(t) := \dot{x}_n(t) - v_d$ denote the absolute position and velocity errors of the nth vehicle, respectively. We propose the following cost functional

$$J := \frac{1}{2}\int_0^\infty \sum_{n=0}^{M-1}\sum_{m=0}^{M-1} \varphi_n^*(t) Q_{n-m} \varphi_m(t)\mathrm{d}t + \frac{1}{2}\int_0^\infty \sum_{n=0}^{M-1}\sum_{m=0}^{M-1} u_n^*(t) R_{n-m} u_m(t)\mathrm{d}t, \tag{12}$$

where

$$\sum_{n=0}^{M-1}\sum_{m=0}^{M-1} \varphi_n^* Q_{n-m}\varphi_m \geq 0, \qquad Q_{-n}^* = Q_n,$$

for all sequences φ_n, and

$$\sum_{n=0}^{M-1}\sum_{m=0}^{M-1} u_n^* R_{n-m} u_m > 0, \qquad R_{-n}^* = R_n,$$

for all non-zero sequences u_n. All arithmetic with indices is done (mod M).

We utilize the fact that system (11) has spatially invariant dynamics over a circle. This implies that the Discrete Fourier Transform (DFT) can be used to convert analysis and quadratic design problems into those for a parameterized family of second order systems [6]. DFT is defined by: $\hat{x}_k := \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1} x_n \mathrm{e}^{-j\frac{2\pi nk}{M}}$, $k \in \{0, \ldots, M-1\}$, and the inverse DFT is defined by: $x_n := \frac{1}{\sqrt{M}}\sum_{k=0}^{M-1} \hat{x}_k \mathrm{e}^{j\frac{2\pi nk}{M}}$, $n \in \{0, \ldots, M-1\}$. Using this, system (11) and quadratic performance index (12) transform to

$$\begin{bmatrix} \dot{\hat{\xi}}_k \\ \dot{\hat{\zeta}}_k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\xi}_k \\ \hat{\zeta}_k \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{u}_k =: \hat{A}_k \hat{\varphi}_k + \hat{B}_k \hat{u}_k, \quad k \in \{0, \ldots, M-1\}, \tag{13}$$

and

$$J = \frac{\sqrt{M}}{2} \sum_{k=0}^{M-1} \int_0^\infty \left(\hat{\varphi}_k^*(t) \hat{Q}_k \hat{\varphi}_k(t) + \hat{u}_k^*(t) \hat{R}_k \hat{u}_k(t) \right) \mathrm{d}t,$$

where, for every $k \in \{0, \ldots, M-1\}$, $\hat{R}_k > 0$ and

$$\hat{Q}_k := \begin{bmatrix} \hat{q}_{11k} & \hat{q}_{21k}^* \\ \hat{q}_{21k} & \hat{q}_{22k} \end{bmatrix} \geq 0.$$

Clearly, the pair $(\hat{A}_k, \hat{B}_k)$ is stabilizable for every $k \in \{0, \ldots, M-1\}$. On the other hand, the pair $(\hat{Q}_k, \hat{A}_k)$ is detectable if and only if $\hat{q}_{11k} > 0$ for every $k \in \{0, \ldots, M-1\}$. These conditions are necessary and sufficient for the existence of a stabilizing optimal solution to the LQR problem (11) and (12). For additional information about proper formulation of quadratically optimal (i.e., LQR, H_2, H_∞) distributed control problems for large-scale and infinite vehicular platoons, we refer the reader to [20].

It is readily shown that for $\dot{x}_n(0) \equiv v_d$, i.e. for $\zeta_n(0) \equiv 0$, we have:

$$\sum_{n=0}^{M-1} u_n^2(0) = \sum_{k=0}^{M-1} \frac{\hat{q}_{11k}}{\hat{R}_k} \hat{\xi}_k^*(0)\hat{\xi}_k(0),$$

which in turn implies

$$\inf_k \frac{\hat{q}_{11k}}{\hat{R}_k} \sum_{n=0}^{M-1} \xi_n^2(0) \leq \sum_{n=0}^{M-1} u_n^2(0) \leq \sup_k \frac{\hat{q}_{11k}}{\hat{R}_k} \sum_{n=0}^{M-1} \xi_n^2(0).$$

Thus, we have established the lower and upper bounds on the initial amount of control effort for a formation that cruises at the desired velocity v_d. These bounds are determined by the deviations of vehicles from their absolute desired trajectories at $t = 0$, and by the LQR design parameters $\hat{q}_{11k}$ and $\hat{R}_k$. Clearly, since $\hat{q}_{11k} > 0$ (for detectability) $\inf_k \hat{q}_{11k}/\hat{R}_k$ is always greater than zero. We note that this quantity can be made smaller by increasing the control penalty. In particular, for $x_n(0) = n(\delta - \mu)$, $0 < \mu < \delta$, we have

$$\sum_{n=0}^{M-1} u_n^2(0) \geq \frac{\mu^2}{6} M(M-1)(2M-1) \inf_k \frac{\hat{q}_{11k}}{\hat{R}_k},$$

which illustrates an unfavorable scaling of the initial amount of control effort with the number of vehicles in formation. Hence, unless $u_{\max} \geq \mu^2(M-1)(2M-1)\inf_k \hat{q}_{11k}/6\hat{R}_k$, there exist at least one vehicle for which $|u_n(0)| > u_{\max}$.

3.3. Hedrick et al. [18] controller

In this section, we show that the controller of Hedrick et al. [18] faces the very same issues as the previously discussed control strategies with uniform convergence rates. We study this controller because of its superior properties compared to other 'look-ahead vehicle following algorithms' (see Table 1 of [33] for comparison of different constant spacing platoon strategies).

The controller of Hedrick et al. [18] for system (1) with an additional lead vehicle, $\ddot{x}_0 = u_0$, is given by (see Section 3.4 of [33]):

$$u_n = \frac{1}{1+q_3}(\ddot{x}_{n-1} + q_3\ddot{x}_0 - (q_1+\lambda)\dot{\eta}_n - q_1\lambda\eta_n - (q_4+\lambda q_3)(\dot{x}_n - \dot{x}_0) - \lambda q_4(x_n - x_0 + n\delta)),$$

where q_1, q_3, q_4, and λ represent positive design parameters. Equivalently, for every $n \in \{1, \dots, M\}$, u_n is determined by the following recursive relation

$$u_n = (1 - \varrho)u_0 + \varrho u_{n-1} - \varrho f_n, \tag{14}$$

where

$$\begin{aligned} f_n &:= (q_1 + \lambda)\dot{\eta}_n + q_1 \lambda \eta_n \\ &\quad + (q_4 + \lambda q_3) \sum_{k=1}^{n} \dot{\eta}_k + \lambda q_4 \sum_{k=1}^{n} \eta_k \\ &= (q_1 + \lambda)(\dot{\xi}_n - \dot{\xi}_{n-1}) + q_1 \lambda (\xi_n - \xi_{n-1}) \\ &\quad + (q_4 + \lambda q_3)(\dot{\xi}_n - \dot{\xi}_0) + \lambda q_4 (\xi_n - \xi_0), \end{aligned}$$

and $\varrho := 1/(1 + q_3) < 1$. Finally, using (14) we express u_n as

$$u_n = u_0 - \sum_{k=1}^{n} \varrho^{n-k+1} f_k, \quad n \in \{1, \dots, M\}.$$

In particular, for initial condition (10), with $\{\xi_0(0) = 0, \zeta_0(0) = 0\}$, u_0 is identically equal to 0. On the other hand, the initial amount of control effort for the remaining vehicles is given by

$$u_n(0) = \lambda\mu \left(n \frac{q_4 \varrho}{1 - \varrho} + \frac{q_1 \varrho (1 - \varrho^n)}{1 - \varrho} - \frac{q_4 \varrho^2 (1 - \varrho^n)}{(1 - \varrho)^2} \right).$$

Thus, for any choice of design parameters q_1, q_3, q_4, and λ there exists $m \le M$ such that $|u_n(0)| > u_{\max}$, for every $n > m$, provided that M is large enough.

4. On avoiding peaking

The results of Section 3 illustrate that certain physically relevant initial conditions in combination with fast stabilization can lead to peaking in control. Faster rates of convergence towards the desired formation cause larger transient peaks. Thus, in very large platoons one needs to take into account the initial distance of vehicles from their desired trajectories and to adjust the control gains accordingly to avoid the excessive control magnitudes.

In this section, we determine the conditions that control gains need to satisfy to provide operation within the imposed saturation limits. The peaking problem is solved by *trajectory generation*:

- *for any given set of initial conditions, explicit constraints on convergence rates are derived to avoid peaking in velocity and control and to guarantee the desired quality of position transient response.*

We demonstrate that fast convergence rates lead to small position transients. On the other hand, slow convergence rates are necessary to provide small velocity and control transients. Therefore, the position overshoots and settling times can be significantly increased in the presence of stringent requirements on velocity and control saturation limits. We utilize this to remark on some of the basic limitations and tradeoffs that need to be addressed in the control of vehicular platoons.

In Section 4.1, we consider the problem of steering the nth vehicle towards its desired absolute position $v_d t - n\delta$ and velocity v_d. We assume the perfect knowledge of initial conditions and pretend that this vehicle is not a part of the platoon. Under these assumptions, we establish explicit constraints on feedback gains – *for any given set of initial conditions* – to assure the desired quality of position transients without magnitude and rate peaking. These requirements are used to generate the trajectory around which the nth vehicle can be driven towards its desired position and velocity without the excessive use of control effort.

The procedure of Section 4.1 leads to fully decentralized strategies that can be utilized for the control of isolated vehicles (i.e., the vehicles that are not a part of the platoon). However, since these control strategies do not account for the inter-vehicular spacings, they are not safe for implementation in automated highway systems. Because of this, in Section 4.2 we design a control law for each vehicle in the platoon to provide asymptotic convergence towards the desired trajectories generated in Section 4.1. The control law for the nth vehicle u_n is obtained as a superposition of:

- *fully decentralized nominal control* $\bar{u}_n$ *of* Section 4.1,
- *distributed control* $\tilde{u}_n$ *of* Section 4.2.

If initial conditions are exactly known then $u_n \equiv \bar{u}_n$, and each vehicle cruises towards its desired absolute position and velocity along the trajectory generated in Section 4.1. On the other hand, the role of distributed control $\tilde{u}_n$ is to account for:

- *the fact that a vehicle is a part of the platoon*,
- *the discrepancies in the initial conditions due to measurement imperfections*.

In Section 4.2, we redesign controller (3) and (4) to provide the convergence to trajectories generated in Section 4.1. However, we note that $\tilde{u}_n$ can be obtained using any other control tool (e.g. H_2 or H_∞).

4.1. Trajectory generation

In this section, we generate the trajectories around which the vehicles can be driven towards their desired positions and velocities without the excessive use of control effort. We rewrite system (1) as

$$\ddot{\bar{x}}_n = \bar{u}_n, \quad n \in \{1, \dots, M\}. \tag{15}$$

We want to drive each vehicle towards its desired absolute position $v_d t - n\delta$, and its desired velocity v_d. For the time being we are not concerned with the relative spacing between the vehicles. If we introduce the error variable, $r_n(t) := \bar{x}_n(t) - v_d t + n\delta$, we can rewrite (15) as

$$\ddot{r}_n = \bar{u}_n, \quad n \in \{1, \dots, M\}, \tag{16}$$

and choose $\bar{u}_n$ to meet the control objective. In particular, we take $\bar{u}_n$ of the form

$$\bar{u}_n = -p_n^2 r_n - 2 p_n \dot{r}_n, \quad n \in \{1, \dots, M\}, \tag{17}$$

where, for every $n \in \{1, \dots, M\}$, p_n represents a positive design parameter. With this choice of control, the solution of system (16) and (17) for any $n \in \{1, \ldots, M\}$ is given by

$$r_n(t) = (c_n + d_n t)\mathrm{e}^{-p_n t}, \tag{18a}$$

$$\dot{r}_n(t) = (d_n - c_n p_n - d_n p_n t)\mathrm{e}^{-p_n t}, \tag{18b}$$

$$\bar{u}_n(t) = (c_n p_n^2 - 2 d_n p_n + d_n p_n^2 t)\mathrm{e}^{-p_n t}, \tag{18c}$$

where

$$\begin{aligned} c_n &:= r_n(0) = \bar{x}_n(0) + n\delta, \\ d_n &:= r_n(0)p_n + \dot{r}_n(0) \\ &= (\bar{x}_n(0) + n\delta)p_n + (\dot{\bar{x}}_n(0) - v_d). \end{aligned} \tag{19}$$

We want to determine conditions that the sequence of positive numbers $\{p_n\}$ has to satisfy to guarantee

$$|r_n(t)| \le r_{n,\max}, \quad \forall t \ge 0, \tag{20a}$$

$$|\dot{r}_n(t)| \le v_{\max}, \quad \forall t \ge 0, \tag{20b}$$

$$|\bar{u}_n(t)| \le u_{\max}, \quad \forall t \ge 0, \tag{20c}$$

with $\{r_{n,\max} > 0, \forall n \in \{1, \ldots, M\}\}$, $v_{\max} > 0$, and $u_{\max} > 0$ being the pre-specified numbers. For notational convenience, we have assumed that all vehicles have the same velocity and control saturation limits, given by $v_{\max}$ and $u_{\max}$, respectively. Typically, the sequence $\{r_{n,\max}\}$ is given in terms of position initial conditions $\{r_n(0)\}$ as $\{r_{n,\max} := \gamma_n |r_n(0)|\}$, where the sequence of numbers $\{\gamma_n > 1, \forall n \in \{1, \ldots, M\}\}$ determines the allowed overshoot with respect to the desired position trajectory of the nth vehicle. Clearly, for this choice of $\{r_{n,\max}\}$, $\{r_n(0)\}$ satisfies (20a). Based on (18a), $r_n(t)$ asymptotically goes to zero, so we only need to determine conditions under which (20a) is violated for finite non-zero times. If (18a) achieves an extremum for some $\bar{t}_n \in (0, \infty)$, the absolute value of r_n at that point is given by:

$$|r_n(\bar{t}_n)| = \frac{|d_n|}{p_n}\mathrm{e}^{-p_n \bar{t}_n} \le \frac{|d_n|}{p_n} \le \frac{|\dot{r}_n(0)|}{p_n} + |r_n(0)|.$$

Therefore, if a sequence of positive numbers $\{p_n\}$ is chosen such that

$$\frac{|\dot{r}_n(0)|}{p_n} + |r_n(0)| \le r_{n,\max}, \quad \forall n \in \{1, \ldots, M\}, \tag{21}$$

condition (20a) will be satisfied for every $t \ge 0$. This implies that, for good position transient response (that is, for small position overshoots), design parameters p_n have to assume large enough values determined by (21).

Clearly, (20b) will be violated unless $|\dot{r}_n(0)| \le v_{\max}$, for every $n \in \{1, \ldots, M\}$. If $\dot{r}_n$ has a maximum or a minimum at some non-zero finite time $\bar{t}_n$, the absolute value of (18b) at that point can be upper bounded by

$$|\dot{r}_n(\bar{t}_n)| = |d_n|\mathrm{e}^{-p_n \bar{t}_n} \le |d_n| \le |r_n(0)|p_n + |\dot{r}_n(0)|.$$

Thus, if a sequence of positive design parameters $\{p_n\}$ are small enough to satisfy

$$|r_n(0)|p_n + |\dot{r}_n(0)| \le v_{\max}, \quad \forall n \in \{1, \ldots, M\}, \tag{22}$$

the velocity saturation will be avoided.

Finally, to rule out saturation in control we need to make sure that condition (20c) is satisfied for both $t = 0$ and $\bar{t}_n > 0$, where the potential extremum of $\bar{u}_n$ takes place. The absolute values of (18c) at these two time instants are, respectively, given by $|\bar{u}_n(0)| = |-r_n(0)p_n^2 - 2\dot{r}_n(0)p_n| \le |r_n(0)|p_n^2 + 2|\dot{r}_n(0)|p_n$, and $|\bar{u}_n(\bar{t}_n)| = |d_n|p_n \mathrm{e}^{-p_n \bar{t}_n} \le |d_n|p_n \le |r_n(0)|p_n^2 + |\dot{r}_n(0)|p_n$. Since $p_n > 0$, if

$$|r_n(0)|p_n^2 + 2|\dot{r}_n(0)|p_n \le u_{\max}, \tag{23}$$

for every $n \in \{1, \ldots, M\}$, then condition (20c) is satisfied.

Inequalities (21)–(23) establish conditions for positive design parameters p_n to prevent saturation in velocity and control, and guarantee a good position transient response. We remark that these conditions can be somewhat conservative, but they are good enough to illustrate the major point. Clearly, for small excursions from the desired position trajectories control gains have to assume large values, determined by (21). On the other hand, for small velocity deviations and small control efforts these gains have to be small enough to satisfy (22) and (23). These facts illustrate some basic tradeoffs that the designer faces in the control of vehicular platoons. In particular, the set of control gains that satisfies (22) and (23) determines the maximal position deviations and the rates of convergence towards the desired trajectories. In other words, the position overshoots and settling times can be significantly increased in the presence of stringent requirements on velocity and control saturation limits. For a platoon that cruises at the desired velocity v_d at $t = 0$, any initial condition can be represented by

$$\left.\begin{aligned} \bar{x}_n(0) &= -\left(n\delta + \sum_{k=1}^{n} \mu_k\right) \\ \dot{\bar{x}}_n(0) &= v_d \end{aligned}\right\} \quad n \in \{1, \ldots, M\}. \tag{24}$$

In this case, condition (21) is always satisfied, which implies that the largest deviation for all vehicles from their desired absolute positions takes place at $t = 0$. Therefore, the chosen initial conditions do not impose any lower bounds on the control gains. On the other hand, conditions (22) and (23), respectively, dictate the following upper bounds on $\{p_n\}$:

$$p_n \le \frac{v_{\max}}{\left|\sum_{k=1}^{n} \mu_k\right|}, \quad p_n \le \sqrt{\frac{u_{\max}}{\left|\sum_{k=1}^{n} \mu_k\right|}}.$$

In particular, the following choice of $\{p_n\}$

$$p_n = \min\left\{\frac{\varrho_n v_{\max}}{\left|\sum_{k=1}^{n} \mu_k\right|}, \sqrt{\frac{\sigma_n u_{\max}}{\left|\sum_{k=1}^{n} \mu_k\right|}}\right\}, \tag{25}$$

with $\{0 < \varrho_n \le 1, 0 < \sigma_n \le 1, \forall n \in \{1, \ldots, M\}\}$, clearly satisfies the above requirements. For the example considered in Section 3, $\{\mu_k = \mu, \forall k \in \{1, \ldots, M\}\}$, and (25) simplifies to

$$p_n = \min\left\{\frac{\varrho_n v_{\max}}{n|\mu|}, \sqrt{\frac{\sigma_n u_{\max}}{n|\mu|}}\right\}. \tag{26}$$

Thus, for the example presented in Section 3, if the control gains scale as $1/\sqrt{n}$ the peaking in control will be precluded; if they scale as $1/n$ the peaking in velocity will be precluded as well.

Fig. 6 illustrates the solution of system (16) and (17) for initial conditions determined by (24) with $M = 50$ and $\mu_n = 0.5$, for every $n \in \{1, \ldots, M\}$. The control gains are chosen using (26) with $v_{\max} = u_{\max} = 5$, $\{\varrho_n = 1, \sigma_n = 0.8, \forall n \in \{1, \ldots, M\}\}$, to prevent reaching imposed velocity and control saturation limits. The dependence of these gains on discrete spatial variable n is also illustrated in Fig. 6.

Thus we have shown that controller (17) with the gains satisfying (21)–(23) precludes saturation in both velocity and control and takes into account the desired quality of position transient response. However, this control strategy does not account for the inter-vehicular spacings. Because of that, in Section 4.2 we redesign controller (3) and (4) by incorporating the constraints imposed by (20) in the synthesis.

4.2. Controller redesign

In this section, we redesign controller (3) and (4) to provide the convergence to trajectories generated in Section 4.1. We again consider system (1), and introduce the following error variables

$$\begin{aligned}\tilde{\xi}_n(t) &:= x_n(t) - v_d t + n\delta - r_n(t)\\ &= \xi_n(t) - r_n(t),\\ \tilde{\zeta}_n(t) &:= \dot{x}_n(t) - v_d - \dot{r}_n(t)\\ &= \zeta_n(t) - \dot{r}_n(t),\end{aligned}$$

with $r_n(t)$ being defined by (19) and (18a), and $\{p_n\}$ satisfying (21)–(23). The initial conditions on these two variables are given by:

$$\tilde{\xi}_n(0) = x_n(0) - \bar{x}_n(0),$$
$$\tilde{\zeta}_n(0) = \dot{x}_n(0) - \dot{\bar{x}}_n(0),$$

where $\{x_n(0), \dot{x}_n(0)\}$ and $\{\bar{x}_n(0), \dot{\bar{x}}_n(0)\}$ represent the actual and the measured initial conditions, respectively. If perfect information about the initial positions and velocities is available, then clearly $\tilde{\xi}_n(0) = \tilde{\zeta}_n(0) \equiv 0$. However, since initial condition uncertainties are always present we want to design a controller to guard against them. We note that for the example presented in Section 3 the above coordinate transformation removes unfavorable scaling of the initial conditions with spatial variable n.

In the new coordinates, system (1) can be represented in terms of its state-space realization of the form

$$\begin{aligned}\begin{bmatrix}\dot{\tilde{\xi}}\\ \dot{\tilde{\zeta}}\end{bmatrix} &= \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\begin{bmatrix}\tilde{\xi}\\ \tilde{\zeta}\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix}\tilde{u}\\ &=: A\tilde{\psi} + B\tilde{u},\end{aligned} \tag{27}$$

where $\tilde{u}_n := u_n - \bar{u}_n$, $\tilde{\xi} := \operatorname{col}\{\tilde{\xi}_n\}$, $\tilde{\zeta} := \operatorname{col}\{\tilde{\zeta}_n\}$, and $\tilde{u} := \operatorname{col}\{\tilde{u}_n\}$. In particular, this system can be stabilized by the following feedback

$$\tilde{u} = \tilde{K}\tilde{\psi} = -\begin{bmatrix}aI + bL & cI\end{bmatrix}\tilde{\psi}. \tag{28}$$

It is noteworthy that, if L is given by (4) and if a, b, and c satisfy conditions of Theorem 1, then controller (4) and (28) has the same properties as controller (3) and (4). For the same choices of design parameters a, b, and c, these two control strategies are only distinguished by the regions from where the states of systems (2) and (27) have to be brought to the origin. Namely, due to different formulations of control objectives, the initial states of system (2) may occupy a portion of the state-space that is significantly larger than a region to which the initial conditions of system (27) belong. In the former case, this region is determined by the maximal deviations from the desired absolute trajectories at $t = 0$, whereas, in the latter case, it is determined by the precision of measurement devices, that is their ability to yield an accurate information about the initial positions and velocities. As illustrated in Section 3, the initial conditions may have an unfavorable scaling with discrete spatial variable n, which may result in the very large initial position deviations (and consequently, a large amount of the initial control effort) for large n's, unless the size of the initial conditions is explicitly accounted for. We have shown in Section 4 how to generate the initial condition dependent trajectories around which the states of vehicular platoon can be driven to zero without extensive use of control effort and large position and velocity overshoots.

Using the definition of $\tilde{u}_n$, we finally give the expressions for u_n:

$$u_n = \bar{u}_n + \tilde{u}_n, \quad n \in \{1, \ldots, M\}, \tag{29}$$

where $\bar{u}_n$ and $\tilde{u}_n$ are, respectively, given by (17) and (28). We remark that $u_n \equiv \bar{u}_n$ if perfect information about the initial conditions is available. The role of $\tilde{u}_n$ is to account for the discrepancies in the initial conditions due to measurement imperfections. If some information about accuracy of sensors is available, the conditions on parameters a, b, and c can be easily derived to provide operation within the imposed saturation bounds and asymptotic convergence of the platoon of vehicles to its desired cruising formation using controller (29).

Simulation results of the platoon system with 50 vehicles ($M = 50$) using controller (29) with $a = 1, b = 2$, and $c = 5$ are shown in Fig. 7. The measured initial condition is given by (24) with $\mu_n \equiv 0.5$, whereas $\tilde{\xi}_n(0)$ and $\tilde{\zeta}_n(0)$ that determine the actual positions and velocities at $t = 0$ are randomly selected. The rates of convergence towards the origin are chosen using (25) with $v_{\max} = u_{\max} = 5$, $\{\varrho_n = 1, \ \sigma_n = 0.8, \forall n \in \{1, \ldots, M\}\}$, to prevent reaching imposed velocity and control saturation limits. These convergence rates are shown in the far right plot in Fig. 6. Clearly, the desired control objective is successfully accomplished with the quality of the transient response determined by the prescribed saturation bounds.

5. Concluding remarks

We illustrate some fundamental design limitations and tradeoffs in automated highway systems. We discuss the peaking phenomenon, and demonstrate that fast stabilization of large-scale platoons can suffer from large control magnitudes. This implies that in very large platoons the designer needs to

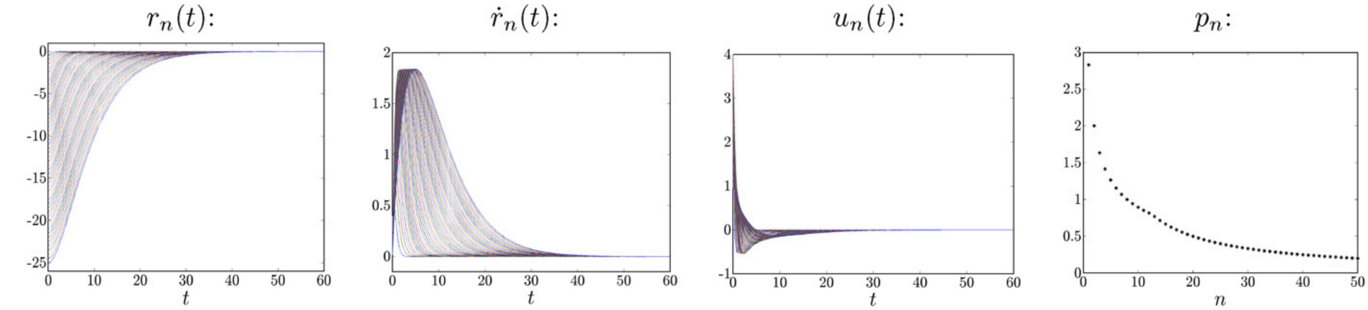

Fig. 6. Solution of system (16) and (17) for initial conditions determined by (24) with $M = 50$ and $\mu_n \equiv 0.5$. The control gains (far right plot) are determined using (26) with $\upsilon_{\max} = u_{\max} = 5$, and $\{\varrho_n = 1,\ \sigma_n = 0.8, \forall n \in \{1, \ldots, M\}\}$.

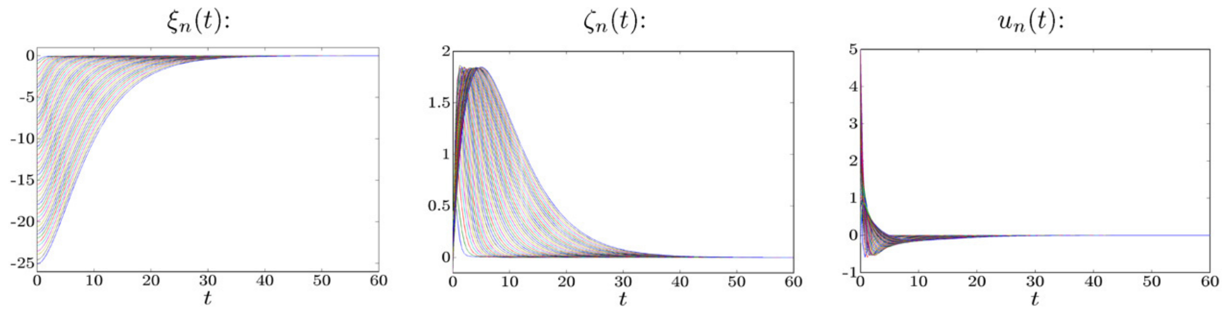

Fig. 7. Simulation results of platoon with 50 vehicles ($M = 50$) using controller (29) with $a = 1$, $b = 2$, and $c = 5$. The measured initial condition is given by (24) with $\mu_n \equiv 0.5$, whereas $\tilde{\xi}_n(0)$ and $\tilde{\zeta}_n(0)$ are randomly selected.

pay attention to the initial deviations of vehicles from their desired trajectories when selecting control gains. We establish explicit constraints on these gains – for any given set of initial conditions – to assure the desired quality of position transients without magnitude and rate saturation. These requirements are used to generate the trajectories around which the states of the platoon system are driven towards their desired values without the excessive use of control effort.

Ongoing research effort is directed towards the design of distributed localized controllers for vehicular platoons using the *path-following* framework [11,1,2]. Path-following represents an alternative to reference tracking [1] and it appears to be well suited for avoiding peaking using the strategy of saturating actuators [35,28]. The main drawback of the control strategy employed in this paper is that it represents an off-line scheme that uses trajectory generation to guard against unfavorable initial conditions. The path-following design will also provide satisfactory performance in the presence of external disturbances and the desired trajectories will be generated in an on-line fashion.

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