Topology identification and design of distributed integral action in power networks

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Abstract— **Recently distributed integral controllers relying on averaging and communication have been proposed as effective means for optimal frequency regulation in power systems, load balancing of network flows, and as natural extensions to static consensus controllers. Typically, only the questions of stability, disturbance rejection, and steady-state resource allocation are addressed in the literature, and the problems of transient performance and optimal communication network design remain open. In this paper we consider the optimal frequency regulation problem and propose a principled heuristic to identify the topology and gains of the distributed integral control layer. We employ an ℓ_1-regularized $\mathcal{H}_2$-optimal control framework as a means for striking a balance between network performance and communication requirements. The resulting optimal control problem is solved using the alternating direction method of multipliers algorithm. For the IEEE 39 New England benchmark problem, we demonstrate that the identified sparse and distributed integral controller can achieve reasonable performance relative to the optimal centralized controller. Interestingly, the identified control architecture is directed and correlates with the generator rotational inertia and cost coefficients.**

Index Terms— **Alternating direction method of multipliers, distributed PI-control, power systems, sparsity-promoting optimal control, topology identification.**

I. Introduction

The basic task of power system operation is to match load and generation. In an AC power grid, the synchronous frequency is a direct measure of the load-generation imbalance, which makes frequency control the fundamental power balancing mechanism. This task is traditionally accomplished by adjusting generation in a hierarchical three-layer structure: primary (droop control), secondary (automatic generation control) and tertiary (economic dispatch) layer, from fast to slow timescales, and from decentralized to centralized architectures [1], [2].

From a control-theoretic perspective, the three frequency control layers essentially correspond to proportional-integral (PI) control and set-point scheduling to solve a resource allocation problem. A broad range of research efforts have recently been put forward to decentralize these control tasks. While the primary layer is typically being implemented by means of proportional droop control, the secondary and

tertiary integral and set-point controllers can be realized in a plug-and-play fashion through discrete-time averaging algorithms [3], continuous-time optimization approaches [4], or distributed averaging-based proportional-integral (DAPI) controllers [5]; see [6] for a recent literature review. Here, we focus on the simple yet effective DAPI controllers advocated, among others, in [5]–[9] to coordinate the action of multiple integral controllers through continuous averaging of the marginal injection costs to arrive at an optimal solution for a tertiary resource allocation problem.

More generally, PI control is a simple and effective method, it is well known for its ability to eliminate the influence of static control errors and constant disturbances, and it is commonly used in many industrial applications [10], [11]. For large-scale distributed systems DAPI-type control strategies have been used successfully for stabilization, disturbance rejection, and resource allocation, as summarized above for power systems [5]–[9] as well as for general network flow problems and other applications [12], [13]. DAPI-type control strategies have also been studied from a pure theoretic perspective as natural extension to proportional consensus control; see [14].

A common theme of the above studies on various DAPI-type controllers is that the communication network among the integral controllers needs to be connected to achieve stable disturbance rejection and resource allocation. However, to the best of our knowledge, there are no studies addressing the question of how to optimally design the *cyber* integral control network relative to the *physical* dynamics and interactions. Here, we pursue this question for the special case of frequency regulation in a power system and using the DAPI controllers advocated in [5]–[9].

In this paper, we identify topology of the integral control communication graph and design the corresponding edge weights for the DAPI controller. Our proposed approach allows us to identify stabilizing and optimal integral controllers with a sparse communication architecture. As a preliminary pre-processing step, we introduce a coordinate transformation to enforce the structural constraints on the rotor angles and auxiliary integral states. In the new set of coordinates, the system dynamics are amenable to both standard linear quadratic regulator tools as well as a ℓ_1 regularized version of the standard $\mathcal{H}_2$ optimal control problem. We invoke the paradigm of sparsity-promoting optimal control developed in [15]–[17] and seek a balance between system performance and sparsity of the integral controller. An alternating direction method of multipliers (ADMM) algorithm is used to iteratively solve the static output-feedback control problem.

Similar techniques have recently been used to solve wide-area control problems in bulk power grids [18]–[22]. For the New England example, we show that distributed integral control can achieve reasonable performance compared to the optimal centralized controller. The optimal communication topology for the distributed integral controller is directed and related to the rotational inertia and cost coefficients of the synchronous generators.

The remainder of the paper is organized as follows. Section II reviews the problem setup in power system frequency regulation. Section III defines the proposed distributed PI-controller and formulates the optimal static output-feedback control problem. Augmented Lagrangian method is used to design the optimal centralized controller in Section III-C. In Section IV, we introduce the sparsity-promoting optimal control algorithm and describe the iterative ADMM steps. We apply the proposed control design strategy on the IEEE 39 New England model in Section V to illustrate our development. Finally, Section VI concludes the paper.

II. SYNCHRONOUS FREQUENCY AND POWER SHARING

In this section, we briefly summarize background material on synchronous frequency and economic load sharing. In the linearized *swing equations* [1]

$$M\dot{\omega} = -L_p\theta - D\omega + \eta + u, \qquad (1)$$

$(\theta, \omega) \in \mathbb{R}^{2n}$ are the generator rotor angles and frequencies, $u \in \mathbb{R}^n$ is the governor control action, and $\eta \in \mathbb{R}^n$ is a disturbance input accounting for stochastic fluctuations in generation and load, which we model as white noise signals. The diagonal matrices M and D are positive definite with diagonal elements being the generator inertia and damping coefficients, and $L_p = L_p^T \in \mathbb{R}^{n \times n}$ is the network suscep-tance matrix, We assume that the network is connected so that $L_p\mathbb{1} = \mathbb{O}$, where $\mathbb{1}$ and $\mathbb{O}$ are vectors of unit entries and zeros of appropriate sizes.

If one assumes the existence of a synchronous steady-state with $\dot{\theta}_i = \omega_{\text{sync}} \in \mathbb{R}$ for all $i \in \{1, \ldots, n\}$, then by summing all equations in (1) in steady state, we obtain the synchronous frequency explicitly as

$$\omega_{\text{sync}} = \frac{\sum_{i=1}^n u_i}{\sum_{i=1}^n D_i} + \frac{\sum_{i=1}^n \eta_i}{\sum_{i=1}^n D_i}. \qquad (2)$$

The control objective is to design a secondary control strategy so that the frequency deviations converge to zero.

Aside from driving the frequency deviations to zero it is also desirable to schedule the injections $u_i(t)$ to balance load and generation while minimizing the operational cost [2]:

$$\begin{aligned} \underset{u}{\text{minimize}} \quad & \sum_{i=1}^n E_i u_i^2 \\ \text{subject to} \quad & \sum_{i=1}^n (u_i + \eta_i) = 0. \end{aligned} \qquad (3)$$

Here $E_i > 0$ is the cost coefficient for source $i \in \{1, \ldots, n\}$. The optimization problem (3) is convex and the essential

insight from the optimality conditions is that all units should produce at *identical marginal costs* of generation:

$$E_i u_i^* = E_j u_j^* \quad \text{for all} \quad i, j \in \{1, \ldots, n\}. \qquad (4)$$

Observe that the budget constraint equation in (3) also guarantees a zero frequency deviation in (2). A special case of the identical marginal cost requirement is the classical *proportional power sharing* [23] criterion

$$\frac{u_i^*}{\overline{P_i}} = \frac{u_j^*}{\overline{P_j}}, \qquad (5)$$

where $\overline{P_i}$ is the rating of source i. Clearly, the power sharing objective is a special case of the resource allocation problem (3) if one sets each cost coefficient E_i to $1/\overline{P_i}$.

III. DISTRIBUTED INTEGRAL CONTROL

In this section, we first introduce the problem setup and describe a model for frequency control of power systems. We then formulate the design of distributed integral action as a static output-feedback control problem. In the absence of sparsity constraints, we use an augmented Lagrangian method to determine optimal centralized integral controller.

A. Problem setup

The frequency error can in principle be driven to zero via decentralized integral action of the form

$$\begin{aligned} u &= -K_1 s \\ \dot{s} &= \omega, \end{aligned} \qquad (6)$$

where s denotes the auxiliary integral states, and K_1 is a di-agonal feedback matrix. It is well known, however, that such decentralized integral controllers do not achieve steady-state optimality [6]. Furthermore, they are prone to instabilities that may arise from biased measurement errors [8].

To remedy these shortcomings, we consider the distributed averaging-based integral controller also used in [5]–[9]

$$\begin{aligned} u &= -E^{-1}z \\ \dot{z} &= \tilde{K}_1 \omega - L_I z. \end{aligned} \qquad (7)$$

Here, z is the vector of auxiliary distributed integral states, E and $\tilde{K}_1$ are diagonal matrices of cost coefficients and positive gains, respectively, L_I is the Laplacian matrix of a connected communication graph in the integral controller. Since $\sum_{i=1}^n \dot{z}_i = \sum_{i=1}^n \tilde{K}_{1,i}\omega_i$, any steady-state solution of (7) satisfies $\omega_i = 0$, i.e., the frequency deviations are driven to zero. Because of $L_I z = -L_I E u = 0$, any steady-state solution of (7) also satisfies the identical marginal cost criterion (4). Hence, the controller (7) achieves optimal frequency regulation.

By substituting (7) to (1) yields the closed-loop system

$$\begin{aligned} \dot{\theta} &= \omega \\ M\dot{\omega} &= -L_p\theta - D\omega - E^{-1}z + \eta \\ \dot{z} &= \tilde{K}_1\omega - L_I z. \end{aligned} \qquad (8)$$

In this paper, without loss of generality, we assume that integral controllers are installed on all the generators. We

also assume that $\tilde{K}_1$ is a known diagonal matrix and confine our attention to the design of the Laplacian matrix L_I. Our objective is to identify topology of L_I and to design the corresponding edge weights in order to optimally enhance performance of the closed-loop network (8) in the presence of stochastic disturbances η.

B. Static output-feedback control problem

The design of L_I can be formulated as a static output-feedback problem for a system with a state-space model

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}_1\eta + \hat{B}_2 v, \qquad (9)$$

where $\hat{x} = \begin{bmatrix} \theta^T & \omega^T & z^T \end{bmatrix}^T$ is the state vector, and the auxiliary control is defined as $v = -G\hat{C}_2\hat{x}$. Here, $G := L_I$ is the control gain to be designed, and the matrices in (9) are partitioned conformably with the state $\hat{x}$

$$\hat{A} = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}L_p & -M^{-1}D & -(EM)^{-1} \\ 0 & \tilde{K}_1 & 0 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} 0 \\ M^{-1} \\ 0 \end{bmatrix}, \ \hat{B}_2 = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \ \hat{C}_2 = \begin{bmatrix} 0 & 0 & I \end{bmatrix}.$$
$$(10)$$

The closed-loop system resulting from (9) is given by,

$$\dot{\hat{x}} = (\hat{A} - \hat{B}_2 G \hat{C}_2)\hat{x} + \hat{B}_1\eta$$
$$y = \begin{bmatrix} Q^{1/2} \\ -R^{1/2} G \hat{C}_2 \end{bmatrix} \hat{x}. \qquad (11)$$

Here, y is the performance output, $R = R^T \succ 0$ is the control weight, and the state weight $Q = Q^T \succeq 0$ is selected as

$$Q = \begin{bmatrix} Q_\theta & 0 & 0 \\ 0 & Q_\omega & 0 \\ 0 & 0 & Q_z \end{bmatrix}$$

with $Q_\theta = Q_z = I - (1/n)\mathbb{1}\mathbb{1}^T$ and $Q_\omega = M$. The performance output y in (11) accounts for deviations from the averages of θ and z, as well as the kinetic energy and the control effort of the system. The choice of performance indices is inspired by [21] for designing wide-area controller. Hence, $\|y\|_2^2 = x^T Q x$ penalizes frequency deviations and non-identical integral states similar to the distributed averaging-based integral controller (7) thereby accelerating the convergence of the integral error state. Together with the frequency penalty Q_ω, the penalty Q_θ on non-identical angle variables aids in the convergence of the dynamics (1) as in [19], [21]. Finally, inspired by the quadratic criterion (3) a suitable choice for the control weight is $R = E$.

In a power system without a slack bus, the generator rotor angles are only defined in a relative frame of reference, as can be observed in the linearized swing equations (1). Thus, all rotor angles θ can be rotated by a uniform amount without changing the dynamics (1). Since only differences between the components of $\theta(t) \in \mathbb{R}^n$ enter into (8), this rotational symmetry is preserved in the closed-loop system (10) as well.

By introducing a coordinate transformation [21], [24]

$$\theta = U\psi + \mathbb{1}\bar{\theta}, \qquad (12a)$$

we can eliminate the marginally stable average mode $\bar{\theta} = \mathbb{1}^T\theta/n$ from (8) and the preserve rotational symmetry. Here, $\psi \in \mathbb{R}^{n-1}$ and the columns of the matrix $U \in \mathbb{R}^{n\times(n-1)}$ form an orthonormal basis of the subspace orthogonal to span$(\mathbb{1})$. For example, the columns of U can be obtained from the $(n-1)$ eigenvectors of the projector matrix $\left(I - (1/n)\mathbb{1}\mathbb{1}^T\right)$. The matrix U has the following properties

$$U^T U = I, \ U U^T = I - (1/n)\mathbb{1}\mathbb{1}^T, \ U^T\mathbb{1} = \mathbb{0}.$$

Furthermore, since the Laplacian matrix of the integral controller satisfies $L_I\mathbb{1} = \mathbb{0}$, we can use similar coordinate transformation on the auxiliary integral states z to ensure the Laplacian property of L_I in our control design,

$$z = U\phi + \mathbb{1}\bar{z}, \qquad (12b)$$

where $\bar{z} = \mathbb{1}^T z/n$ is the average integral state. Note that, in contrast to $\bar{\theta}$, the average of the integral state $\bar{z}$ actually enters into the closed-loop dynamics (8).

The structural constraints on θ and z are enforced by the following conditions

$$Q_\theta\mathbb{1} = \mathbb{0}, \qquad L_p\mathbb{1} = \mathbb{0}$$
$$Q_z\mathbb{1} = \mathbb{0}, \qquad L_I\mathbb{1} = \mathbb{0}.$$

As an additional benefit, the above choice of Q_z penalizes the z variable relative to the vector $\mathbb{1}$, and thus facilitates the achievement of the identical marginal cost criterion (4).

To eliminate the marginally stable average-angle-mode $\bar{\theta}$ and preserve the relative information exchange requirement for the dynamics of z, we combine (12a) and (12b) to obtain the following coordinate transformation

$$\underbrace{\begin{bmatrix} \theta \\ \omega \\ z \end{bmatrix}}_{\hat{x}} = \underbrace{\begin{bmatrix} U & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & \mathbb{1} & U \end{bmatrix}}_{T_1} \underbrace{\begin{bmatrix} \psi \\ \omega \\ \bar{z} \\ \phi \end{bmatrix}}_{x} + \begin{bmatrix} \mathbb{1} \\ 0 \\ 0 \\ 0 \end{bmatrix}\bar{\theta}.$$
$$(13)$$

Equivalently, x can be expressed in terms of $\hat{x}$ as

$$\underbrace{\begin{bmatrix} \psi \\ \omega \\ \bar{z} \\ \phi \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} U^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & (1/n)\mathbb{1}^T \\ 0 & 0 & U^T \end{bmatrix}}_{T_2} \underbrace{\begin{bmatrix} \theta \\ \omega \\ z \end{bmatrix}}_{\hat{x}}. \qquad (14)$$

The properties of the matrix U imply that the matrices T_1 and T_2 satisfy $T_2 T_1 = I$ and

$$T_1 T_2 = \begin{bmatrix} I - (1/n)\mathbb{1}\mathbb{1}^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

In the new set of coordinates, the closed-loop system (7)

takes the form

$$\dot{x} = (A - B_2 F C_2)\, x \,+\, B_1\, \eta$$
$$y = \left[\begin{array}{c} Q^{1/2} \\ -R^{1/2} F C_2 \end{array} \right] x \qquad (15)$$

where

$$A = T_2\, \hat{A}\, T_1, \qquad B_1 = T_2\, \hat{B}_1, \qquad Q = T_1^T\, \hat{Q}\, T_1,$$

and $B_2^T = \left[\begin{array}{cccc} 0 & 0 & 0 & U \end{array}\right]^T$, $C_2 = \left[\begin{array}{cccc} 0 & 0 & 0 & I \end{array}\right]$. The matrices B_2 and C_2 are partitioned conformably with the partition of the state vector x. The feedback matrices G and F (in the $\hat{x}$ and x coordinates, respectively) are related by

$$F = G U \quad \Leftrightarrow \quad G = F U^T.$$

For this static-output feedback problem (15), the control objective is to achieve a desirable tradeoff between the $\mathcal{H}_2$ performance of (15) and the sparsity of the feedback gain G. The $\mathcal{H}_2$ norm from the disturbance η to the output y, which quantifies the steady-state variance (energy) of y of the stochastically forced system (15), is defined as

$$J(F) := \left\{ \begin{array}{ll} \text{trace}\left(B_1^T P(F) B_1\right) & F \text{ stabilizing} \\ \infty & \text{otherwise} \end{array} \right.$$

where the closed-loop observability Gramian P satisfies the Lyapunov equation

$$(A - B_2 F C_2)^T P \,+\, P(A - B_2 F C_2) = $$
$$-\, (Q + C_2^T F^T R F C_2).$$

While the performance is expressed in terms of the feedback gain matrix F, we will enhance sparsity of the Laplacian matrix $G = L_I$ in the original coordinates; see Section IV.

C. Optimal design of the centralized integral action

We first focus on the design the centralized integral controller $G = L_I$ that minimizes the $\mathcal{H}_2$ norm of the closed-loop system, we follow the augmented Lagrangian approach for structured feedback synthesis [25]. Since the matrix $\hat{C}_2$ in (15) only contains zero and identity submatrices, we can formulate the static output-feedback problem (15) as a structured state-feedback optimal control problem

$$\dot{x} = (A - B_2 K)\, x \,+\, B_1\, \eta$$
$$y = \left[\begin{array}{c} Q^{1/2} \\ -R^{1/2} K \end{array} \right] x \qquad (16)$$

where K satisfies the following structural constraint

$$K := \left[\begin{array}{cccc} K_\psi & K_\omega & K_{\bar{z}} & K_\phi \end{array}\right] = \left[\begin{array}{cccc} 0 & 0 & 0 & F \end{array}\right]. \quad (17)$$

Finding a solution of the structured optimal control problem (16) amounts to solving

$$\begin{array}{cl} \underset{K}{\text{minimize}} & J(K) \\ \text{subject to} & K \in \mathcal{S}, \end{array} \qquad (18)$$

where $J(K)$ is the $\mathcal{H}_2$ norm of system (16) parameterized as a function of K, and $\mathcal{S}$ is a set of stabilizing feedback gains K satisfying the structural constraint (17). The algebraic

characterization of the structural constraint is given by

$$K \in \mathcal{S} \quad \Leftrightarrow \quad K \circ I_{\mathcal{S}} = K,$$

where $\circ$ is the elementwise matrix multiplication and

$$I_{\mathcal{S}} = \left[\begin{array}{cccc} 0 & 0 & 0 & \mathbb{1}\mathbb{1}^T \end{array}\right]$$

is partitioned conformably with the partition of the state x.

The augmented Lagrangian method developed in [25] solves a sequence of unstructured problems iteratively, and the minimizers of the unstructured problems converge to a minimizer of the optimal control problem (18). The resulting centralized $L_I = G = F U^T$ can be used as a warm-start for the sparsity-promoting optimal control problem that is discussed next.

IV. SPARSITY-PROMOTING OPTIMAL CONTROL

A sparsity-promoting optimal control framework for finding a state feedback that simultaneously optimizes the closed-loop variance and induces a sparse control architecture was developed in [15]–[17]. In this section, we extend this approach to a static output-feedback optimal control problem.

While we want to minimize the $\mathcal{H}_2$ norm in terms of the feedback matrix F in the new set of coordinates, we would like to promote sparsity of the Laplacian matrix $G = L_I$ in the physical domain. This procedure is used to identify sparse structure of the integral control layer. This is accomplished by considering the regularized optimal control problem

$$\begin{array}{cl} \underset{F,\, G}{\text{minimize}} & J(F) \,+\, \gamma\, g(G) \\ \text{subject to} & F U^T \,-\, G = 0. \end{array} \qquad \text{(SP)}$$

The regularization term in (SP) is determined by

$$g(G) := \sum_{i,\, j} W_{ij}\, |G_{ij}|$$

which is an effective proxy for inducing elementwise sparsity in the feedback gain G [26]. The weights W_{ij}'s are updated iteratively using the solution to (SP) from the previous iteration; see [26] for details. In (SP), γ is positive regularization parameter that characterizes the emphasis on the sparsity level of the feedback matrix G.

Next we describe the ADMM algorithm for solving (SP), see [17], [24] for additional details.

1) Initialization: We follow the augmented Lagrangian approach introduced in Section III-C to design an optimal $F_0 = G U$ to initialize the iterative procedure.

2) Form augmented Lagrangian:

$$\mathcal{L}_\rho(F, G, \Lambda) = J(F) \,+\, \gamma\, g(G) \,+$$
$$\text{trace}\left(\Lambda^T(F U^T \,-\, G)\right) + \frac{\rho}{2}\, \|\, F U^T - G \,\|_F^2$$

where Λ denotes the matrix of Lagrange multipliers and $\|\cdot\|_F$ is the Frobenius norm of a matrix.

3) Iterative ADMM algorithm:

$$F^{m+1} = \underset{F}{\text{argmin}}\; \mathcal{L}_\rho\left(F, G^m, \Lambda^m\right)$$
$$G^{m+1} = \underset{G}{\text{argmin}}\; \mathcal{L}_\rho\left(F^{m+1}, G, \Lambda^m\right)$$
$$\Lambda^{m+1} = \Lambda^m + \rho\left(F^{m+1} U^T \,-\, G^{m+1}\right).$$

Here, m represents the iteration index. Using the fact that $U^T U = I$, it is readily shown that the F-minimization step amounts to solving the following optimization problem

$$F^{m+1} = \underset{F}{\operatorname{argmin}} \left(J(F) + \frac{\rho}{2} \|F - H^m\|_F^2 \right)$$

where $H^m := (G^m - (1/\rho)\Lambda^m)U$. We apply the KKT necessary conditions [27] for optimality of $\mathcal{L}_\rho(F, G^m, \Lambda^m)$, and the following equations need to be satisfied

$$(A - B_2 F C_2)L + L(A - B_2 F C_2)^T = -B_1 B_1^T$$
$$(A - B_2 F C_2)^T P + P(A - B_2 F C_2) =$$
$$- (Q + C_2^T F^T R F C_2)$$
$$2(R F C_2 - B_2^T P)L C_2^T + \rho(F - H^m) = 0.$$

The resulting set of the matrix-valued equations is solved using the iterative procedure developed in [17].

Similarly, properties of the matrix U can be used to bring the G-minimization problem into the following form

$$G^{m+1} = \underset{G}{\operatorname{argmin}} \left(\gamma g(G) + \frac{\rho}{2} \|G - V^m\|_F^2 \right)$$

where $V^m := F^{m+1}U^T + (1/\rho)\Lambda^m$ and the unique solution is obtained via the soft thresholding operator,

$$G_{ij}^{;+1} = \begin{cases} (1 - a/|V_{ij}^m|) V_{ij}^m & |V_{ij}^m| > a \\ 0 & |V_{ij}^m| \le a. \end{cases}$$

Here, $a := (\gamma/\rho)W_{ij}$ and, for a given V_{ij}^m, G_{ij}^{m+1} is either set to zero or it is obtained by moving V_{ij}^m towards zero with the amount $(\gamma/\rho)W_{ij}$.

4) Stopping criterion:

$$\|F^{m+1}U^T - G^{m+1}\| \le \epsilon, \quad \|G^{m+1} - G^m\| \le \epsilon$$

The ADMM algorithm stops when both primal and dual residuals are smaller than specified thresholds.

5) Polishing step: Finally, we fix the sparsity pattern of G identified using ADMM and solve the optimal control problem with the identified structural constraints. This polishing step improves the $\mathcal{H}_2$ performance relative to the feedback gain identified by ADMM; see [17] for additional details.

V. CASE STUDY: IEEE 39 NEW ENGLAND MODEL

The IEEE 39 New England Power Grid model consists of 39 buses and 10 detailed two-axis generator models; see Fig. 1. All loads are modeled as constant power loads. As previously mentioned, we assume that all the generators are equipped with integral controllers. We extract network susceptance matrix L_p and inertia matrix M of the IEEE 39 New England model from Power System Toolbox [28]. We set the the damping coefficients D_i of each generator to be $0.1M_i$, and the diagonal positive control gain matrix $\tilde{K}_1$ to be identity matrix. The values of the cost coefficients E_i are chosen to be $E_i = 0.9$ for $i \in \{1, 2, 3, 4, 6, 7, 8, 9, 10\}$, $E_5 = 0.1$, i.e., we assume that generator 5 cost the least to operate while all other generators have the same cost coefficients. The state matrices and performance indices are defined as outlined in Section III-B.

Next, we illustrate that our proposed static output feedback sparsity-promoting optimal control framework is an efficient way to achieve a balance between the system performance and sparsity level of L_I. In Fig 2, we show the sparsity pattern of the feedback matrix $G = L_I \in \mathbb{R}^{10 \times 10}$ for different value of γ. The blue dots denote local feedback control gains, and the red dots identify information that needs to be communicated between different generators. For $\gamma = 0.001$, L_I is dense and recovers the communication pattern of the conventional integral controller as shown in Fig. 2a. When γ increases from 0.001 to 0.101, the 5th column of L_I becomes sparse while the 5th row becomes the only row with all nonzero elements. This indicates that most generators do not care about generator 5 that has the smallest cost coefficient. At the same time, integral controller on generator 5 has to gather information from all other generators to achieve desired performance of the network.
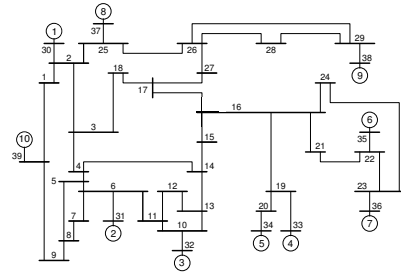

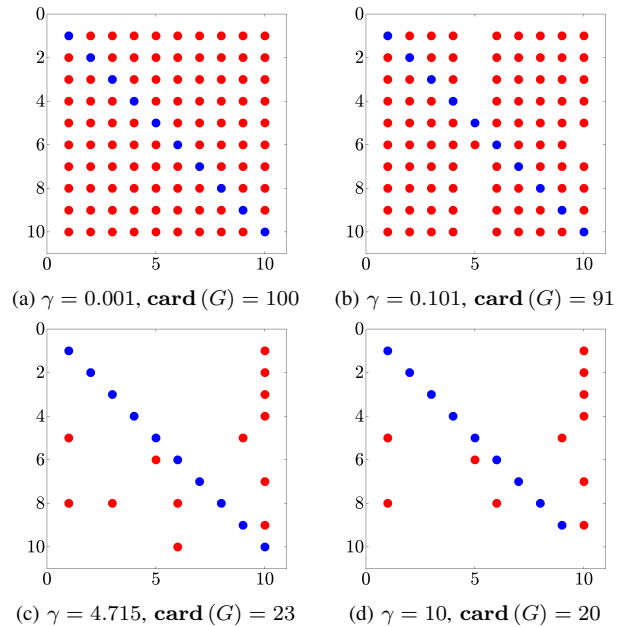Fig. 1: The IEEE 39 New England Power Grid.

(a) $\gamma = 0.001$, **card** $(G) = 100$ (b) $\gamma = 0.101$, **card** $(G) = 91$

(c) $\gamma = 4.715$, **card** $(G) = 23$ (d) $\gamma = 10$, **card** $(G) = 20$

Fig. 2: Sparsity pattern of G resulting from (SP).

By further increasing γ to 4.715, the structure of L_I shows that integral state information of generator $1, 3, 6, 9, 10$, which have the five largest inertia, is gathered by other

integral controllers. Apparently, six other generators need to access information from generator 10, since it has the largest inertia and thus the most reliable frequency measurement in the integral control. Finally, when $\gamma = 10$, only 11 long-range links are required, and integral controller on generator 10 is no longer needed. Since generator 10 is an equivalent aggregated model representing the transmission network of a neighboring area, it has an oversized inertia coefficient and thus also little control agility. Hence, it is not surprising to drop this virtual controller. Our observation shows that the optimal communication architecture correlates with both inertia and cost coefficients.
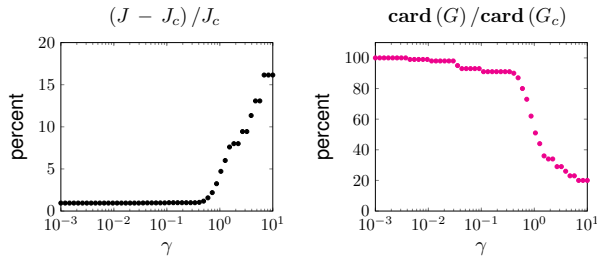

Fig. 3: Performance vs sparsity comparison of sparse G and the optimal centralized controller G_c for 50 logarithmically-spaced points $\gamma \in [\,10^{-3}\,,\,10\,]$.

In Fig. 3, we compare performance degradation and sparsity level for different values of γ. Compared to the optimal centralized integral controller G_c, our sparse G in Fig 2d degrades system performance by only 16.15%. Therefore, by constructing only 11 long-range links for the integral controller architecture, reasonable performance is achieved compared to the optimal centralized feedback gain G_c.

VI. CONCLUDING REMARKS

In this paper, we propose a distributed PI-control strategy for frequency control in power systems. We formulate the topology identification and design of integral controller as a static output-feedback control problem. A coordinate transformation is introduced to enforce the structural constraints on the rotor angles and auxiliary integral states. We find the solution by solving the sparsity-promoting optimal control problem, which balances the tradeoff between system performance and sparsity of the controller. Our development is validated by a benchmark power system example.

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