

Oblique transition in high-speed separated boundary layers

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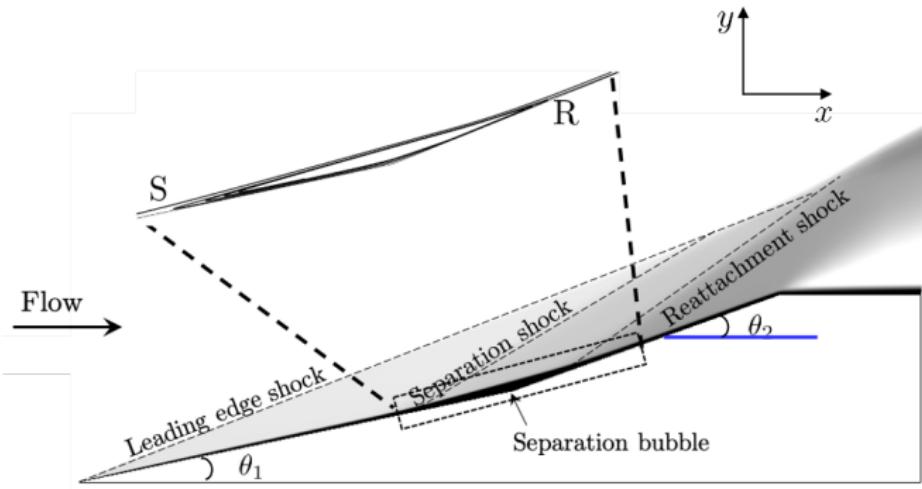


G. S. Sidharth

Wall-bounded turbulence workshop; Newton Institute

Separated boundary layers

- UBIQUITOUS IN HIGH-SPEED FLOWS
 - ★ involve shock/boundary layer interactions (SBLI)
 - ★ characterized by separation/reattachment shocks

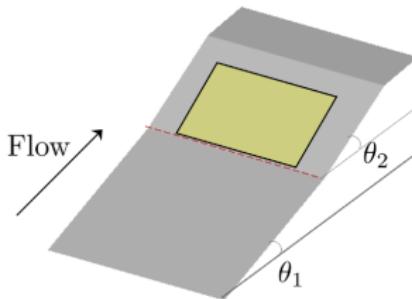


SBLI on
double-wedge

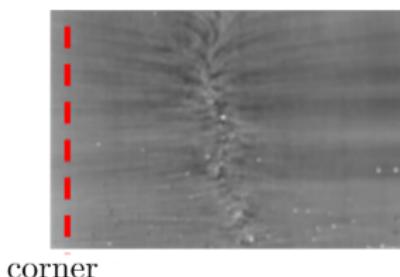
Dominant flow structures

- STEADY STREAKS NEAR REATTACHMENT

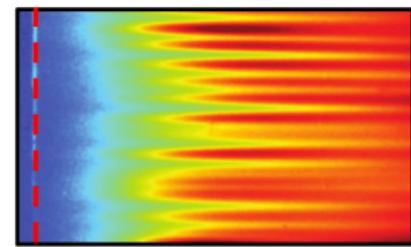
* precursor to turbulence



Oil flow
($\theta_1/\theta_2 = 2^\circ/12^\circ$)



Temperature sensitive paint
($\theta_1/\theta_2 = 0^\circ/15^\circ$)



Dwivedi, Broslawski, Candler, Bowersox

AIAA Aviation '20

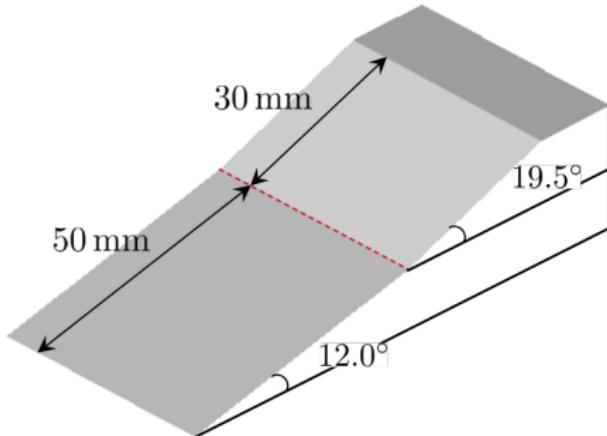
Chuvakhov, Borovoy, Egorov, Radchenko, Olivier, Roghelia

Exp. Fluids '17

OBJECTIVE

study the origin of **reattachment streaks** in **separated flows**

An adiabatic double-wedge



Free stream conditions

M_∞	5
U_∞	792.35 m/s
p_∞	1.22 kPa
T_∞	62.5 K
U_∞/ν_∞	13.6×10^6 /m

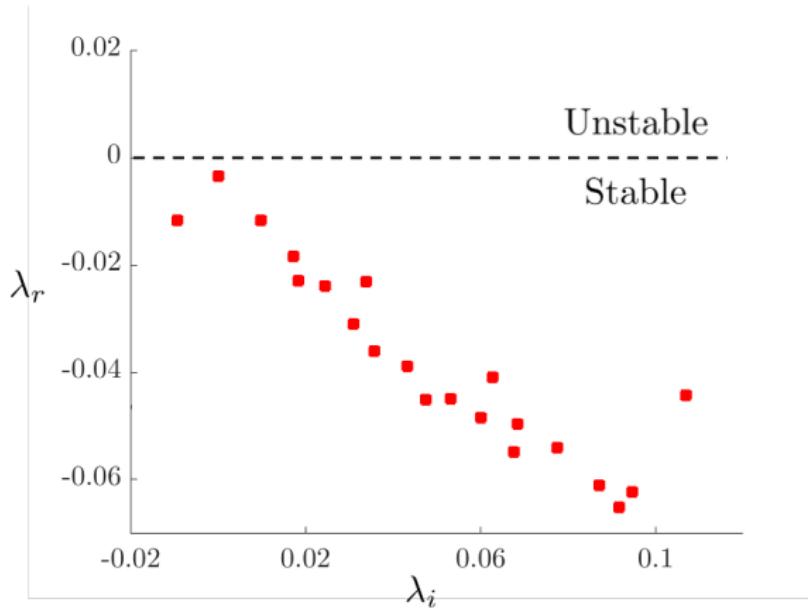
- * realistic experimental conditions

*Yang, Zare-Behtash, Erdem, Contis
Exp. Therm. Fluid Sci. '12*

- * 2D base flow: computed in US3D

Global stability analysis

- SPECTRUM OF LINEARIZATION AROUND 2D BASE FLOW

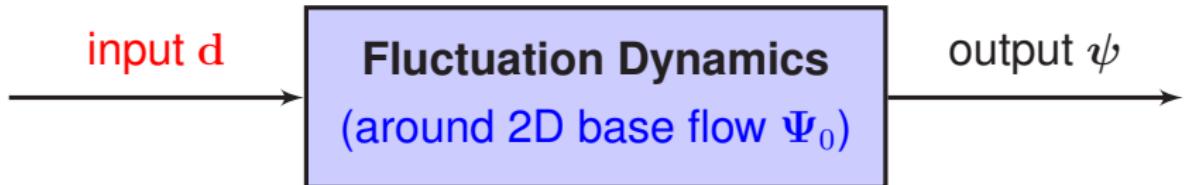


- * 2D base flow is **globally stable** (to 3D perturbations)

Sidharth, Dwivedi, Candler, Nichols, PRF '18

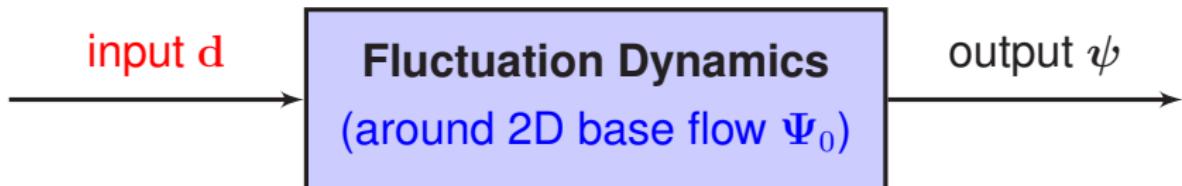
Input-output analysis

$$\frac{\partial \psi}{\partial t} = \mathcal{F}(\Psi_0 + \psi) + \mathbf{d}$$



Input-output analysis

$$\frac{\partial \psi}{\partial t} = \mathcal{F}(\Psi_0 + \psi) + \mathbf{d}$$



\mathbf{d} – external source of momentum, mass, and energy

ψ – velocity, density, and temperature fluctuations

\mathcal{F} – generator of compressible NS dynamics

2D base flow: $\mathcal{F}(\Psi_0) = 0$

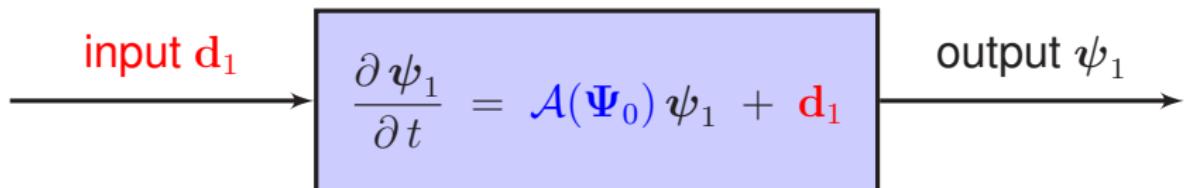
- SMALL AMPLITUDE FORCING

$$\frac{\partial \psi(\mathbf{x}, t)}{\partial t} = \mathcal{F}(\Psi_0(\mathbf{x}) + \psi(\mathbf{x}, t)) + \epsilon \mathbf{d}_1(\mathbf{x}, t)$$

weakly-nonlinear analysis

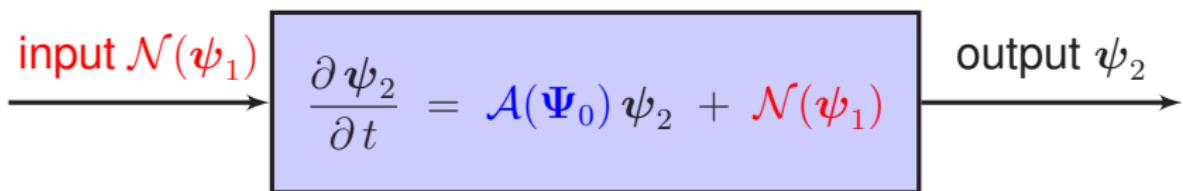
$$\psi(\mathbf{x}, t) = \epsilon \psi_1(\mathbf{x}, t) + \epsilon^2 \psi_2(\mathbf{x}, t) + \mathcal{O}(\epsilon^3)$$

- DYNAMICS AT $\mathcal{O}(\epsilon)$
 - * linearized flow equations (driven by \mathbf{d}_1)

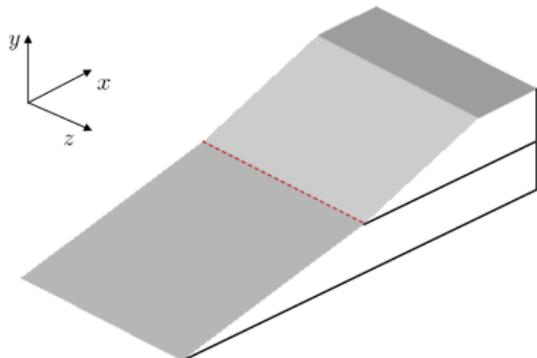


$\mathcal{A}(\Psi_0)$ – linearized generator

- DYNAMICS AT $\mathcal{O}(\epsilon^2)$
 - * linear equations (driven by $\mathcal{N}(\psi_1)$)



Response to deterministic forcing



forcing:

deterministic in x and y

harmonic in t and z

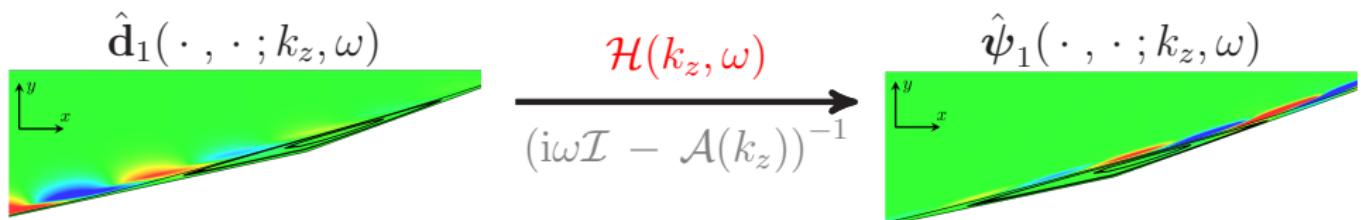
$$\mathbf{d}_1(x, y, z, t) = \hat{\mathbf{d}}_1(x, y; k_z, \omega) e^{i\omega t} e^{ik_z z}$$

- LINEARIZED DYNAMICS
 - * steady-state response

$$\psi_1(x, y, z, t) = \hat{\psi}_1(x, y; k_z, \omega) e^{i\omega t} e^{ik_z z}$$

- SPATIO-TEMPORAL FREQUENCY RESPONSE

- * operator in x and y



Worst-case amplification

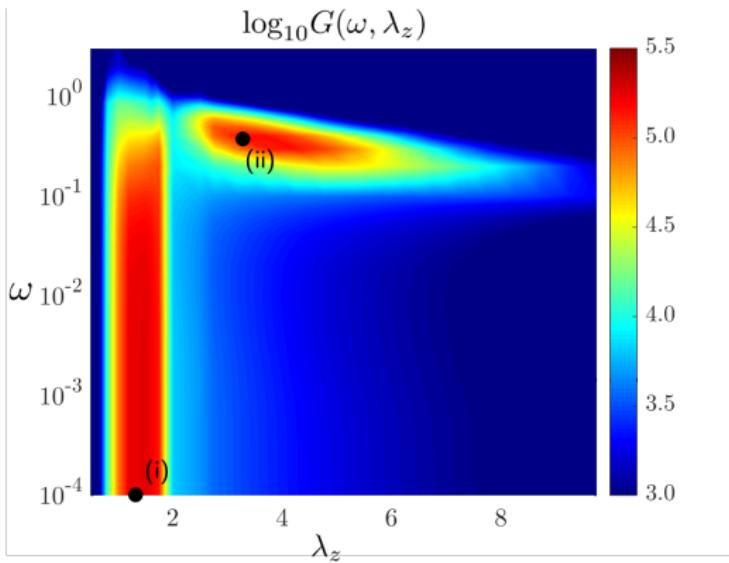
- DETERMINED BY THE LARGEST SINGULAR VALUE

$$G(k_z, \omega) = \max \frac{\text{output energy}}{\text{input energy}} = \sigma_{\max}^2(\mathcal{H}(k_z, \omega))$$

- * fluctuations' energy: Chu's energy norm

$$\int_{\Omega} \left(\bar{\rho} |\mathbf{u}'|^2 + \frac{\bar{p}}{\bar{\rho}^2} \rho'^2 + \frac{\bar{\rho} C_v}{\bar{T}} T'^2 \right) d\Omega$$

Hanifi, Schmid, Henningson, Phys. Fluids '96



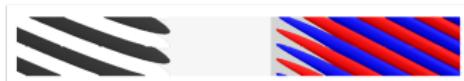
(i) $(\omega, \lambda_z) = (0.0, 1.5)$

Streamwise
vorticity input
 d_1



Temperature
output ϕ_1

(ii) $(\omega, \lambda_z) = (0.4, 3.0)$



- SPATIAL LOCALIZATION OF DOMINANT INPUTS/OUTPUTS

- inputs: upstream of corner
- outputs: downstream of corner

- TWO STRONGLY AMPLIFIED REGIONS
 - * steady streaks: triggered by upstream vortical inputs
Dwivedi, Sidharth, Nichols, Candler, Jovanović, JFM '19
 - * unsteady oblique waves

- HYPERSONIC WIND-TUNNELS

- * unsteady free-stream disturbances

Schneider, Prog. Aero. Sc. '15

can unsteady disturbances trigger steady streaks?

- OBLIQUE WAVE FORCING

$$\mathbf{d}(x, y, z, t) = \epsilon (\mathbf{d}_{+1}(x, y) e^{i\omega t} + \mathbf{d}_{-1}(x, y) e^{-i\omega t}) e^{ik_z z}$$

* linear response

laminar
base flow

$$\overbrace{\Psi_0(x, y)}$$

oblique
waves

$$+ \overbrace{\epsilon (\psi_{+1}(x, y) e^{i\omega t} + \psi_{-1}(x, y) e^{-i\omega t}) e^{ik_z z}}$$

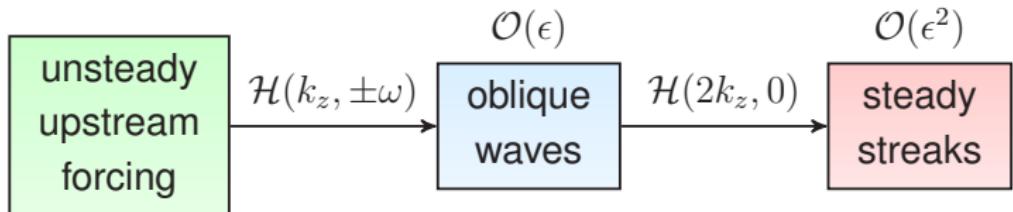
- OBLIQUE WAVE FORCING

$$\mathbf{d}(x, y, z, t) = \epsilon (\mathbf{d}_{+1}(x, y) e^{i\omega t} + \mathbf{d}_{-1}(x, y) e^{-i\omega t}) e^{ik_z z}$$

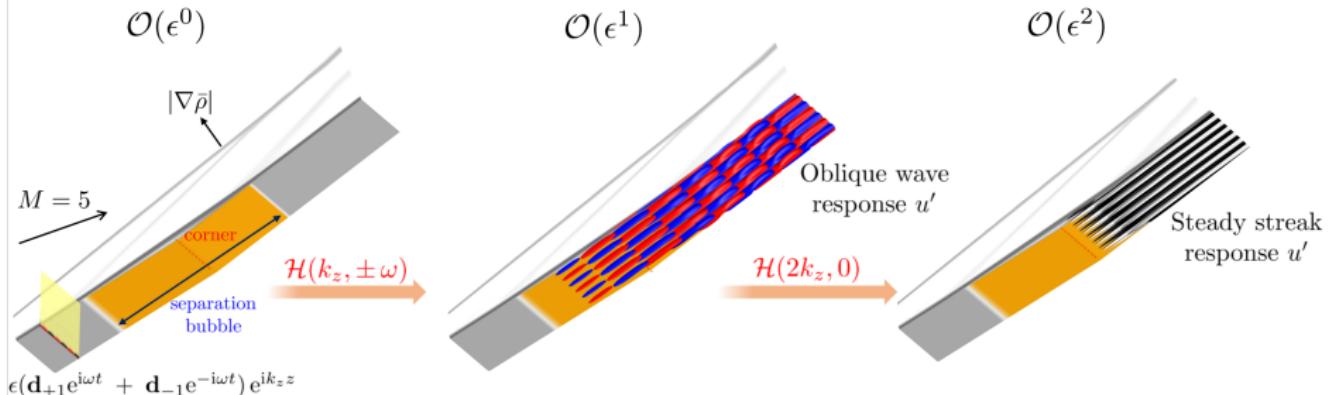
- * weakly-nonlinear response

<p>laminar base flow</p>	<p>oblique waves</p>	<p>+</p>
$\overbrace{\Psi_0(x, y)}$	$\overbrace{\epsilon (\psi_{+1}(x, y) e^{i\omega t} + \psi_{-1}(x, y) e^{-i\omega t}) e^{ik_z z}}$	$+$
$\epsilon^2 \left(\underbrace{\psi_{2,0}(x, y)}_{\text{steady streaks}} + \psi_{+2}(x, y) e^{2i\omega t} + \psi_{-2}(x, y) e^{-2i\omega t} \right) e^{2ik_z z}$		

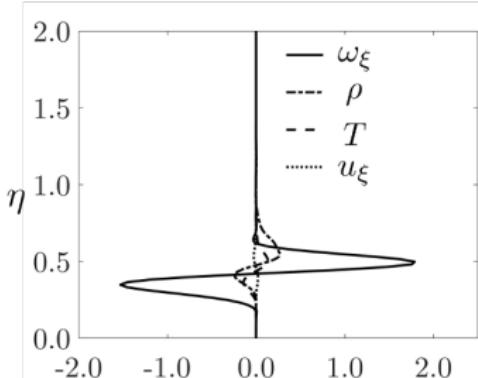
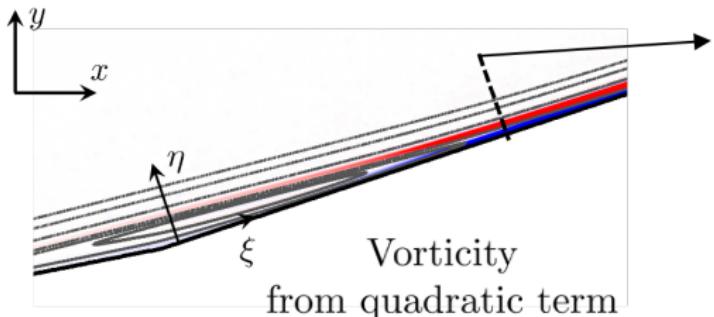
- WEAKLY-NONLINEAR RESPONSE



• GRAPHICAL ILLUSTRATION

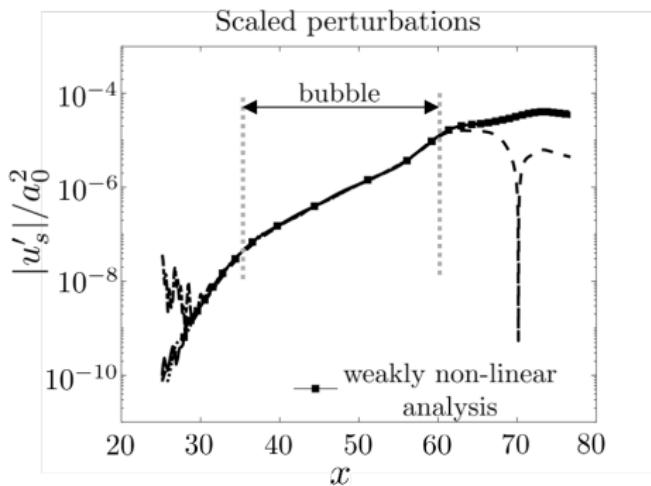
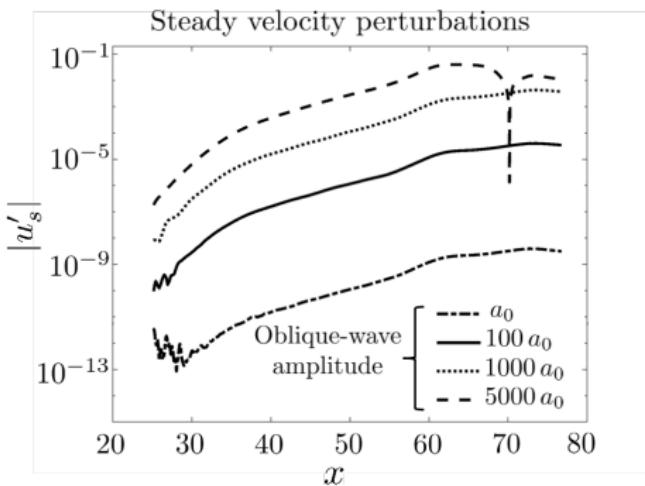


- STEADY FORCING AT $\mathcal{O}(\epsilon^2)$



- * unsteady interactions generate steady vortical forcing
- * localized downstream
in contrast to steady primary forcing

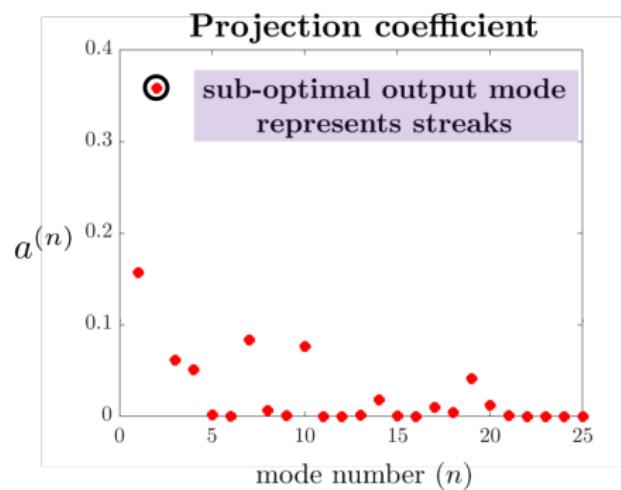
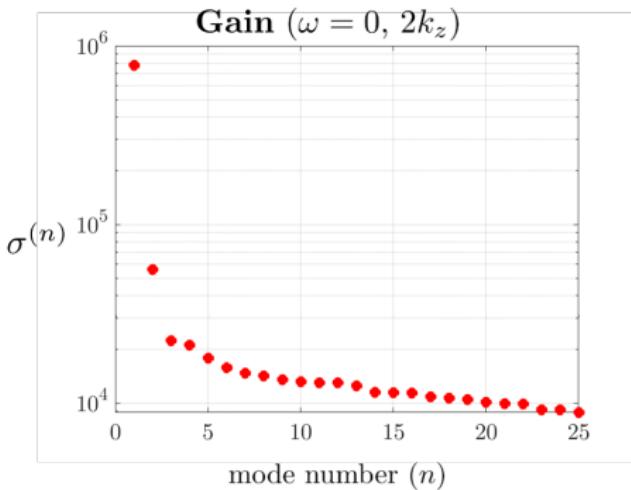
- DNS WITH VARYING FORCING AMPLITUDE



weakly nonlinear analysis: captures DNS results

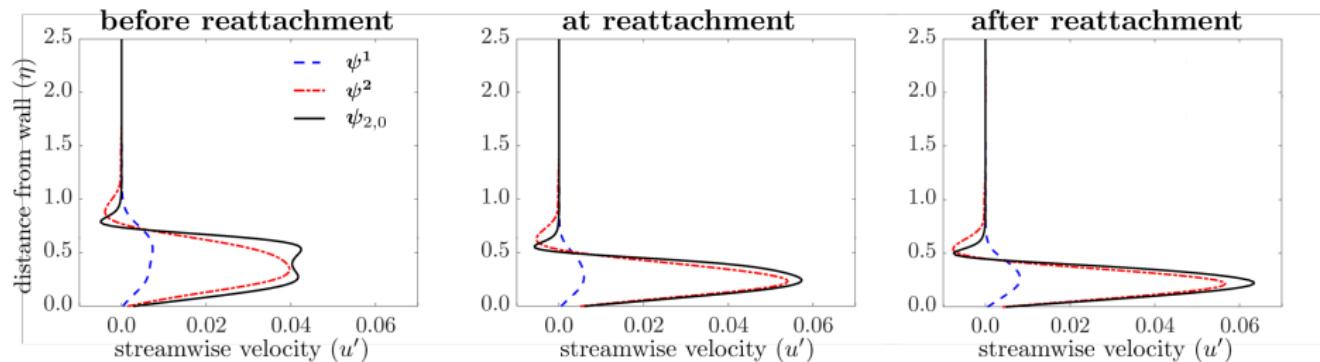
- REPRESENTATION IN TERMS OF RESOLVENT MODES

$$\psi_{2,0}(x, y) = \sum_n \underbrace{\sigma^{(n)} \langle \mathbf{d}^{(n)}, \mathcal{N}(\psi_{\pm 1}) \rangle}_{a^{(n)}} \psi^{(n)}(x, y)$$



$(\mathbf{d}^{(n)}, \psi^{(n)})$ – input-output modes of $\mathcal{H}(2k_z, 0)$

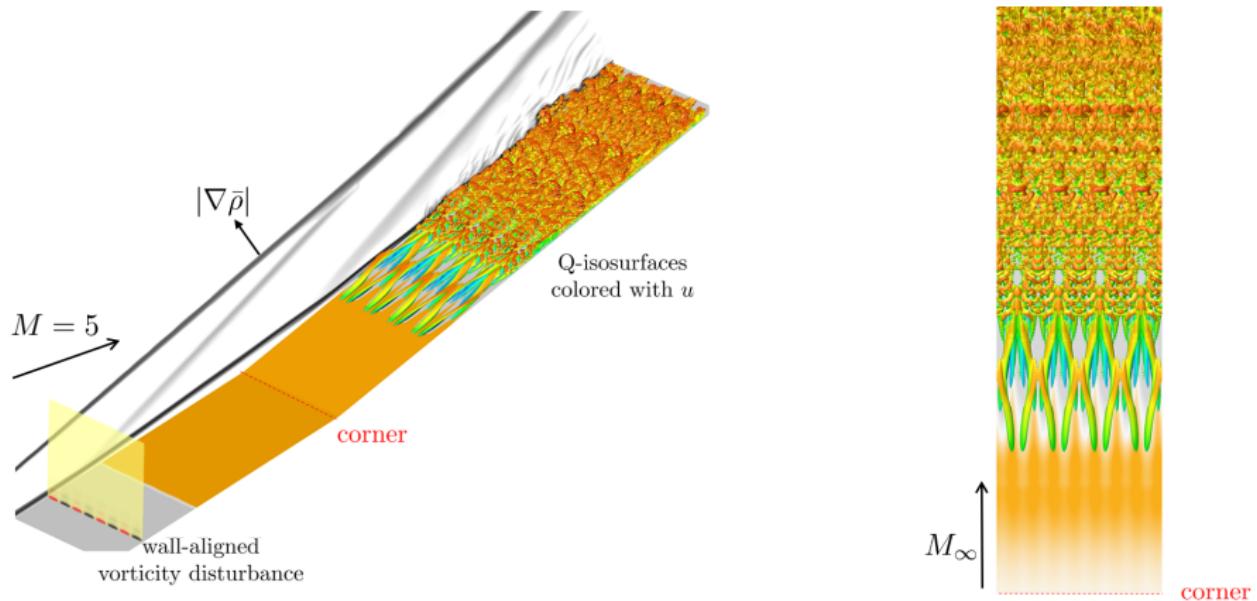
- SPATIAL EVOLUTION OF REATTACHMENT STREAKS



reattachment streaks: captured by the 2nd output mode

Oblique transition

- TRIGGERED BY UNSTEADY INTERACTIONS



Dwivedi, Sidharth, Jovanović, arXiv:2111.15153

Summary

- STREAKS IN HIGH-SPEED SEPARATED FLOWS
 - * robust response to **steady** and **unsteady** disturbances
- STEADY VORTICAL EXCITATION
 - * quadratic interactions of unsteady oblique waves
 - * effective route for triggering transition
- PHYSICAL MECHANISM
 - * **oblique waves:** curvature of separated shear layer
 - * **streaks:** streamwise deceleration near reattachment

Dwivedi, Sidharth, Jovanović, arXiv:2111.15153

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G. S. Sidharth

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