Oblique transition in high-speed separated boundary layers

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Wall-bounded turbulence workshop; Newton Institute
Separated boundary layers

- **Ubiquitous in high-speed flows**
  - involve shock/boundary layer interactions (SBLI)
  - characterized by separation/reattachment shocks

![Diagram of separated boundary layer with labels: S (leading edge shock), R (reattachment shock), θ₁ (angle), θ₂ (angle), separation bubble, SBLI on double-wedge.](image-url)
Dominant flow structures

- **STEADY STREAKS NEAR REATTACHMENT**
  - precursor to turbulence

![Diagram showing steady streaks near reattachment](image)

- **Oil flow**
  \( \theta_1/\theta_2 = 2\degree/12\degree \)

- **Temperature sensitive paint**
  \( \theta_1/\theta_2 = 0\degree/15\degree \)

*Dwivedi, Broslawski, Candler, Bowersox*  
*Exp. Fluids ’17*

*Chuvakhov, Borovoy, Egorov, Radchenko, Olivier, Roghelia*  
*AIAA Aviation ’20*
OBJECTIVE

study the origin of reattachment streaks in separated flows
An adiabatic double-wedge

Free stream conditions

\[
\begin{align*}
M_\infty & \quad 5 \\
U_\infty & \quad 792.35 \text{ m/s} \\
p_\infty & \quad 1.22 \text{ kPa} \\
T_\infty & \quad 62.5 \text{ K} \\
U_\infty / \nu_\infty & \quad 13.6 \times 10^6 / \text{m}
\end{align*}
\]

- realistic experimental conditions

Yang, Zare-Behtash, Erdem, Contis
Exp. Therm. Fluid Sci. ’12

- 2D base flow: computed in US3D
Global stability analysis

- **Spectrum of linearization around 2D base flow**

- 2D base flow is **globally stable** (to 3D perturbations)

*Sidharth, Dwivedi, Candler, Nichols, PRF ’18*
Input-output analysis

\[
\frac{\partial \psi}{\partial t} = \mathcal{F}(\Psi_0 + \psi) + d
\]

Fluctuation Dynamics
(around 2D base flow \( \Psi_0 \))
Input-output analysis

\[ \frac{\partial \psi}{\partial t} = F(\Psi_0 + \psi) + d \]

Fluctuation Dynamics
(around 2D base flow \( \Psi_0 \))

\( d \) – external source of momentum, mass, and energy
\( \psi \) – velocity, density, and temperature fluctuations
\( F \) – generator of compressible NS dynamics

2D base flow: \( F(\Psi_0) = 0 \)
SMALL AMPLITUDE FORCING

\[ \frac{\partial \psi(x, t)}{\partial t} = \mathcal{F}(\Psi_0(x) + \psi(x, t)) + \epsilon d_1(x, t) \]

weakly-nonlinear analysis

\[ \psi(x, t) = \epsilon \psi_1(x, t) + \epsilon^2 \psi_2(x, t) + \mathcal{O}(\epsilon^3) \]
\textbf{DYNAMICS AT } \mathcal{O}(\epsilon) \\
\star \text{ linearized flow equations (driven by } d_1) \\
\frac{\partial \psi_1}{\partial t} = A(\Psi_0) \psi_1 + d_1 \\
A(\Psi_0) \quad \text{-- linearized generator}
**DYNAMICS AT $\mathcal{O}(\epsilon^2)$**

- **linear equations**

\[
\frac{\partial \psi_2}{\partial t} = A(\Psi_0) \psi_2 + N(\psi_1)
\]

(input $N(\psi_1)$) \quad (output $\psi_2$)

(driven by $N(\psi_1)$)
Response to deterministic forcing

**Forcing:**
- **Deterministic** in $x$ and $y$
- **Harmonic** in $t$ and $z$

\[
d_1(x, y, z, t) = \hat{d}_1(x, y; k_z, \omega) e^{i\omega t} e^{ik_z z}
\]
**Linearized Dynamics**

- **steady-state response**

\[
\psi_1(x, y, z, t) = \hat{\psi}_1(x, y; k_z, \omega) e^{i\omega t} e^{ik_z z}
\]
**Spatio-Temporal Frequency Response**

- Operator in $x$ and $y$

$$\hat{d}_1(\cdot, \cdot; k_z, \omega)$$

$$\mathcal{H}(k_z, \omega)$$

$$(i\omega I - A(k_z))^{-1}$$

$$\hat{\psi}_1(\cdot, \cdot; k_z, \omega)$$

**Resolvent Analysis**
Worst-case amplification

- Determined by the largest singular value

\[ G(k_z, \omega) = \max \frac{\text{output energy}}{\text{input energy}} = \sigma_{max}^2(\mathcal{H}(k_z, \omega)) \]

- Fluctuations’ energy: Chu’s energy norm

\[ \int_{\Omega} \left( \bar{\rho} |u'|^2 + \frac{\bar{\rho}}{\bar{\rho}^2} \rho'^2 + \frac{\bar{\rho} C_v}{T} T'^2 \right) d\Omega \]

Hanifi, Schmid, Henningson, Phys. Fluids ’96
• **Spatial Localization of Dominant Inputs/Outputs**
  
  * inputs: upstream of corner
  
  * outputs: downstream of corner
Two strongly amplified regions

- steady streaks: triggered by upstream vortical inputs

  Dwivedi, Sidharth, Nichols, Candler, Jovanović, JFM ’19

- unsteady oblique waves
Hypersonic wind-tunnels

- unsteady free-stream disturbances

Schneider, Prog. Aero. Sc. ’15

can unsteady disturbances trigger steady streaks?
**Oblique Wave Forcing**

\[
d(x, y, z, t) = \epsilon \left( d_{+1}(x, y) e^{i\omega t} + d_{-1}(x, y) e^{-i\omega t} \right) e^{ikzz}
\]

*Linear response*

\[
\Psi_0(x, y) + \epsilon \left( \psi_{+1}(x, y) e^{i\omega t} + \psi_{-1}(x, y) e^{-i\omega t} \right) e^{ikzz}
\]
**Oblique Wave Forcing**

\[
d(x, y, z, t) = \epsilon \left( d_{+1}(x, y) e^{i\omega t} + d_{-1}(x, y) e^{-i\omega t} \right) e^{ikzz}
\]

- weakly-nonlinear response

\[
\begin{align*}
\Psi_0(x, y) &+ \epsilon \left( \psi_{+1}(x, y) e^{i\omega t} + \psi_{-1}(x, y) e^{-i\omega t} \right) e^{ikzz} + \\
\psi_{2,0}(x, y) &+ \psi_{+2}(x, y) e^{2i\omega t} + \psi_{-2}(x, y) e^{-2i\omega t} e^{2ikzz}
\end{align*}
\]

- laminar base flow
- oblique waves
- steady streaks


**Weakly-Nonlinear Response**

- Unsteady upstream forcing
- $\mathcal{H}(k_z, \pm \omega)$
  - $\mathcal{O}(\epsilon)$
  - Oblique waves
- $\mathcal{H}(2k_z, 0)$
  - $\mathcal{O}(\epsilon^2)$
  - Steady streaks
Weakly nonlinear analysis

**GRAPHICAL ILLUSTRATION**

\[ \mathcal{O}(\epsilon^0) \]

M = 5

Corner separation bubble

\[ \nabla \bar{\rho} \]

\[ \epsilon (d_{+1}e^{i\omega t} + d_{-1}e^{-i\omega t}) e^{ik_z z} \]

\[ \mathcal{H}(k_z, \pm \omega) \]

\[ \mathcal{H}(2k_z, 0) \]

\[ \mathcal{O}(\epsilon^1) \]

Oblique wave response \( u' \)

\[ \mathcal{O}(\epsilon^2) \]

Steady streak response \( u' \)
Steady forcing from unsteady disturbances

**Steady forcing at** $\mathcal{O}(\epsilon^2)$

- unsteady interactions generate steady vortical forcing
- localized downstream

in contrast to steady primary forcing
DNS with varying forcing amplitude

weakly nonlinear analysis: captures DNS results
**Representation in terms of resolvent modes**

\[ \psi_{2,0}(x, y) = \sum_n \sigma^{(n)} \langle d^{(n)}(\mathcal{N}(\psi_{\pm 1})) \psi^{(n)}(x, y) \rangle \]

- **Gain** \((\omega = 0, 2k_z)\)
- **Projection coefficient**

\((d^{(n)}, \psi^{(n)})\) — input-output modes of \(\mathcal{H}(2k_z, 0)\)
**Spatial Evolution of Reattachment Streaks**

Reattachment streaks: captured by the 2nd output mode
Oblique transition

- Triggered by unsteady interactions

Dwivedi, Sidharth, Jovanović, arXiv:2111.15153
Summary

- **Streaks in high-speed separated flows**
  - robust response to **steady** and **unsteady** disturbances

- **Steady vortical excitation**
  - quadratic interactions of unsteady oblique waves
  - effective route for triggering transition

- **Physical mechanism**
  - oblique waves: curvature of separated shear layer
  - streaks: streamwise deceleration near reattachment

*Dwivedi, Sidharth, Jovanović, arXiv:2111.15153*
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