Controller Architectures: Tradeoffs between Performance and Structure

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Dynamics and Control in Spatio-Temporal Systems; MTNS 2024

Motivating application

- INTER-AREA OSCILLATIONS IN POWER SYSTEMS

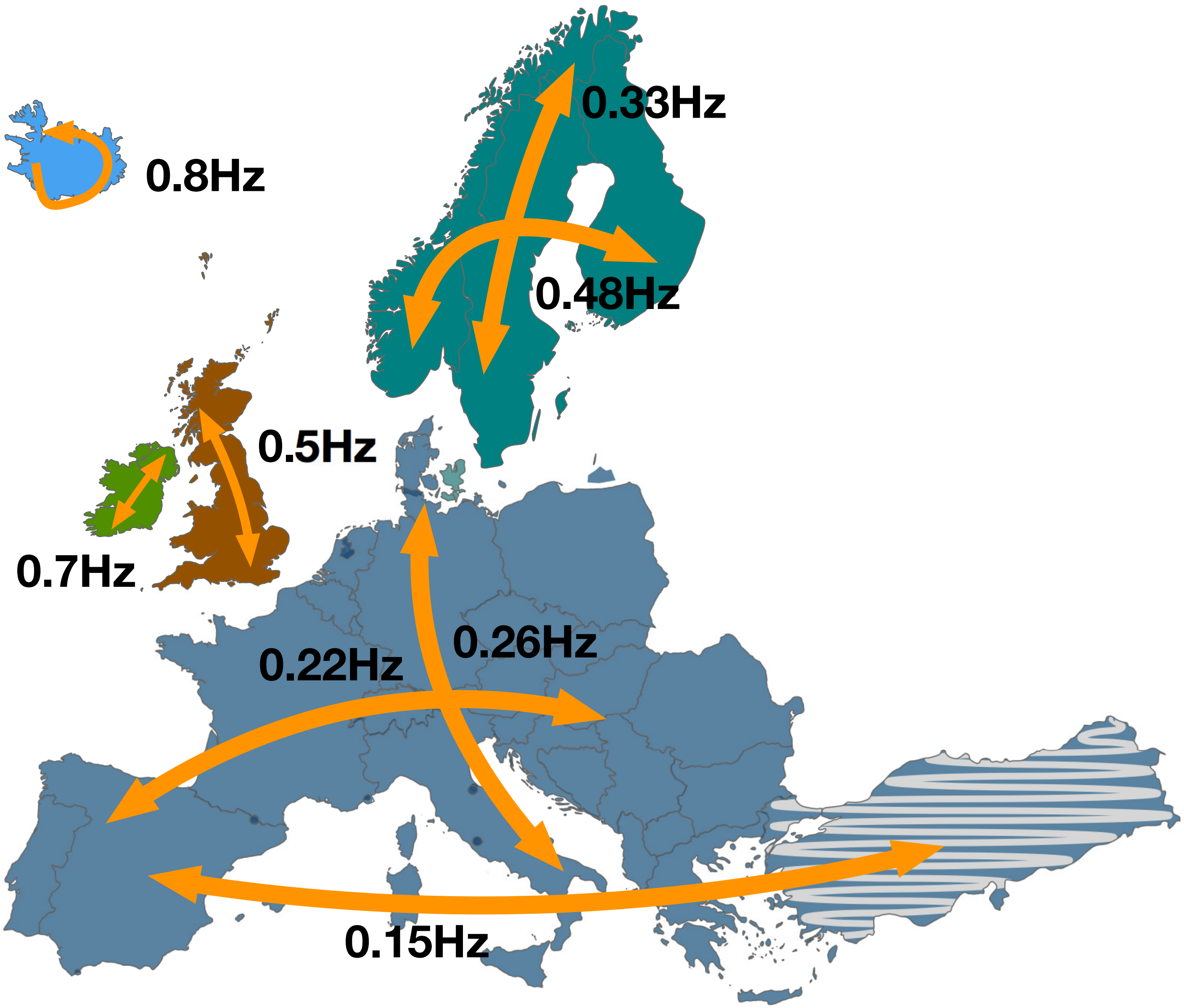
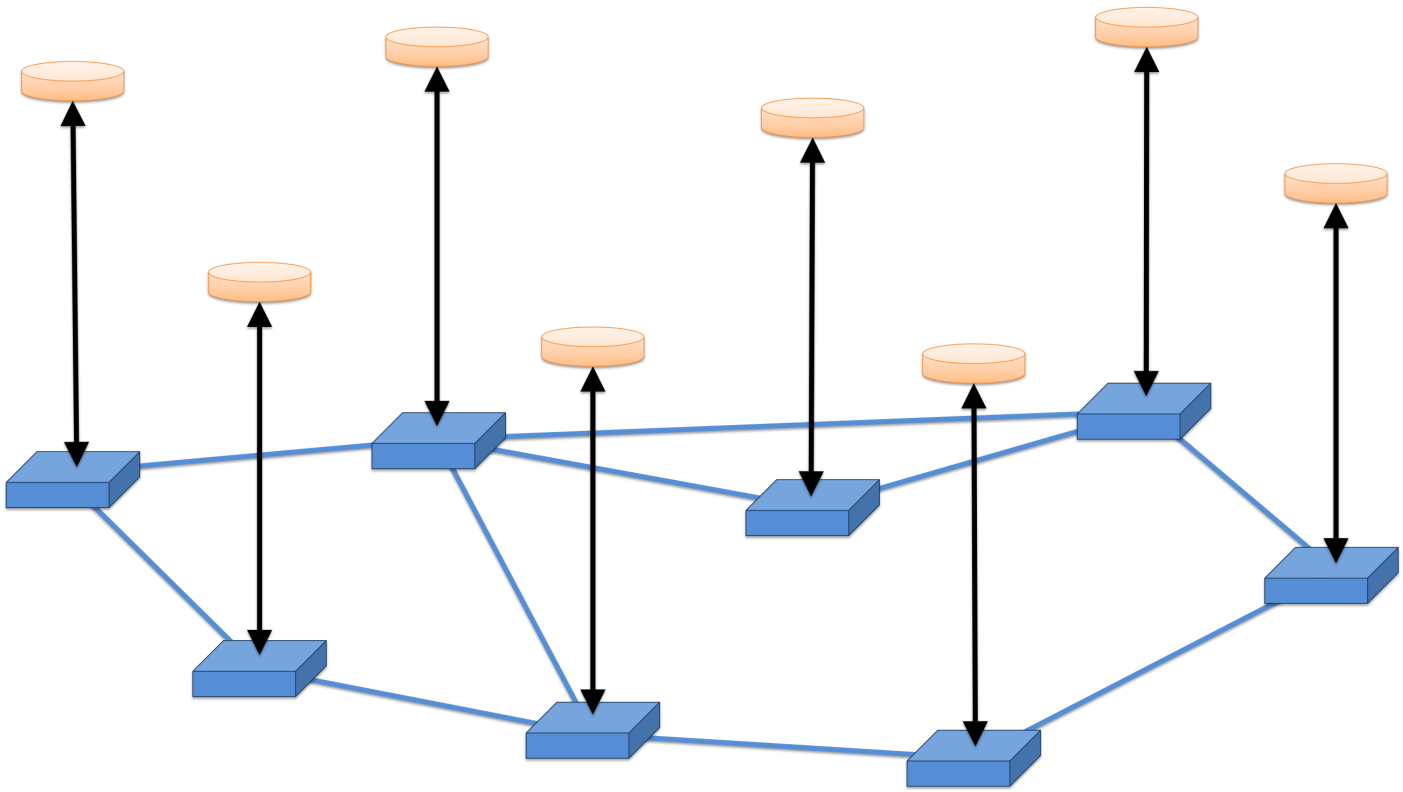


Image credit: Florian Dörfler

Conventional control of generators

fully decentralized controller

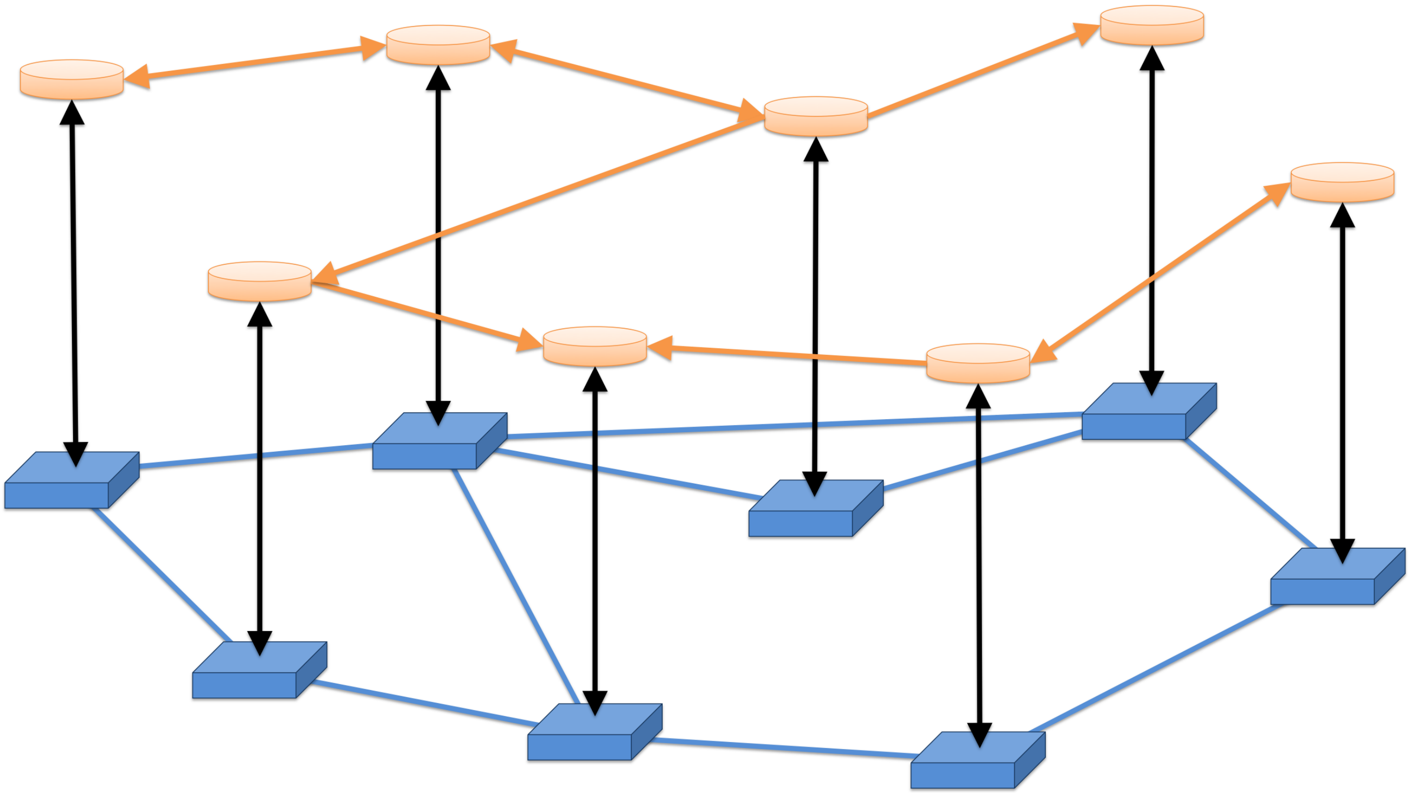


network of generators

- ⋆ local oscillations ✔
- ⋆ inter-area oscillations ✘

Possible alternative

structured dynamical controller



distributed plant and its interaction links

CHALLENGE

design of controller architectures

performance vs **complexity**

Complexity via Regularization

$$\text{minimize} \quad \underbrace{J(K)}_{\text{closed-loop performance}} \quad + \quad \gamma \, \underbrace{g(K)}_{\text{controller complexity}}$$

closed-loop performance — **controller complexity**

$\gamma > 0$ – performance vs complexity tradeoff

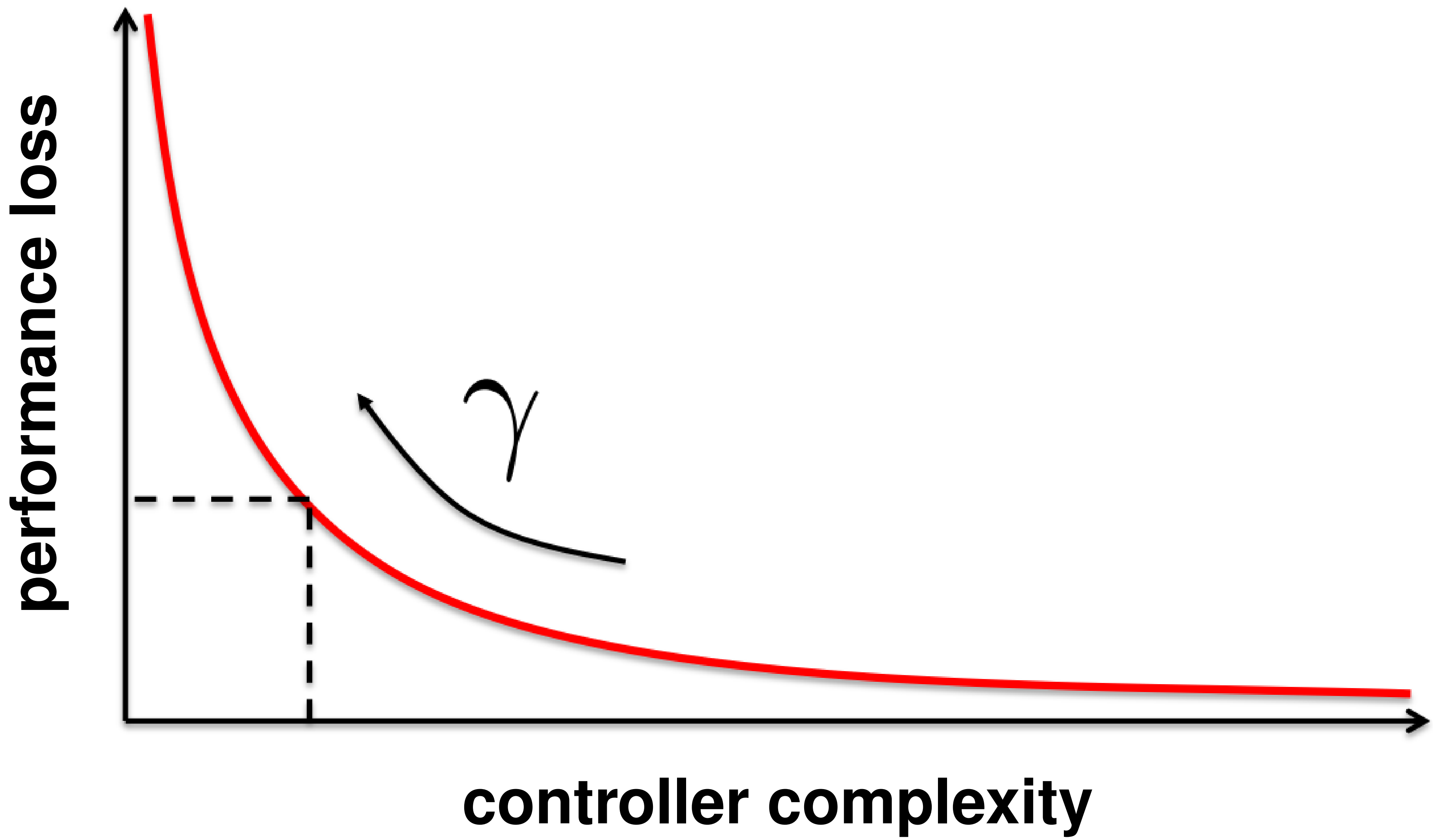
Fardad, Lin, Jovanović, ACC '11

Lin, Fardad, Jovanović, IEEE TAC '13

Jovanović & Dhingra, EJC '16

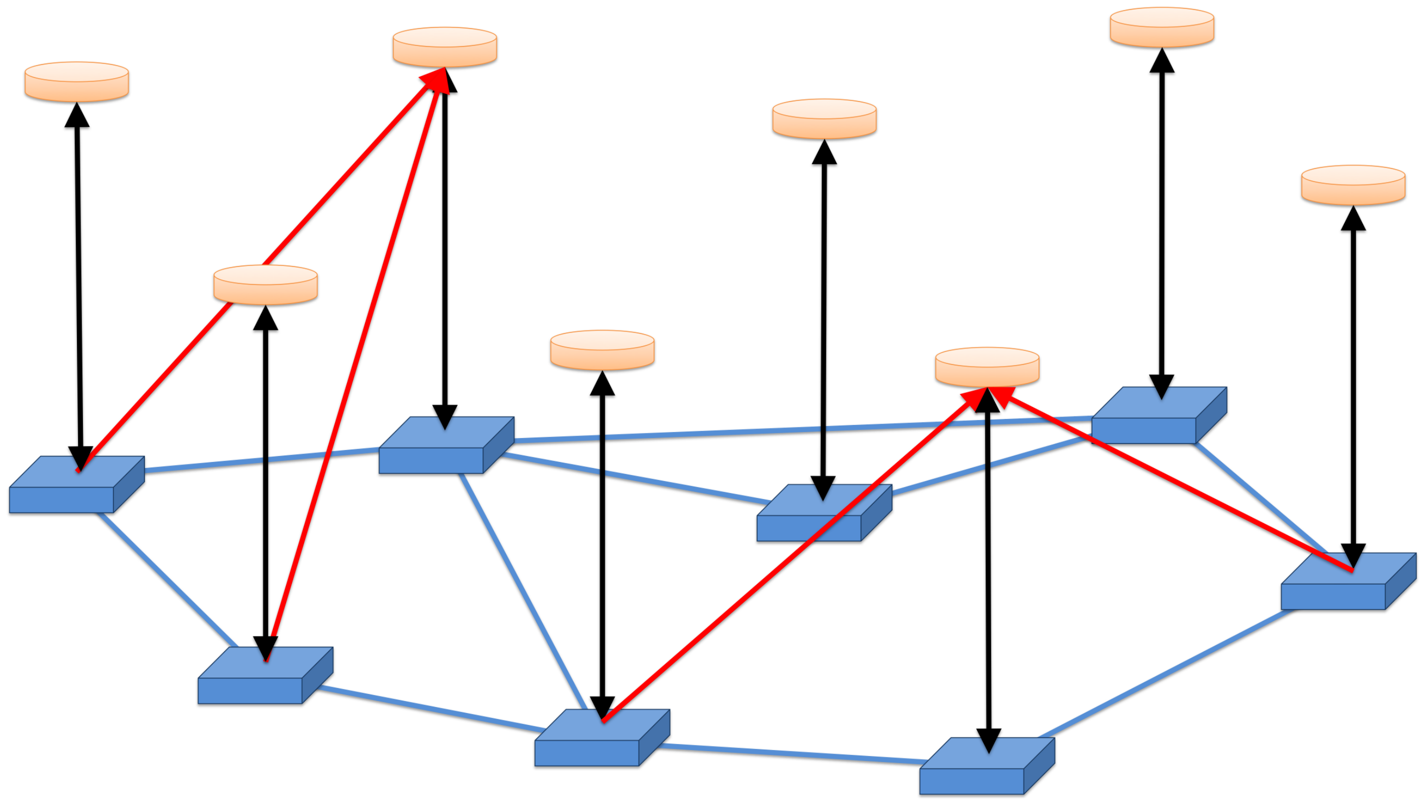
Matni & Chandrasekaran, IEEE TAC '16

- TRADE-OFF CURVE
  - ⋆ **performance vs complexity**



This talk

structured memoryless controller



distributed plant and its interaction links

OBJECTIVE

identification of a signal exchange network

performance vs **sparsity**

CONTROL PROBLEM

Minimum variance state-feedback problem

dynamics: $\dot{x} \;=\; A\,x \;+\; B_1\,d \;+\; B_2\,u$

objective function: $J \;=\; \lim_{t \,\to\, \infty} \mathbf{E}\left(x^T(t)\,Q\,x(t) \;+\; u^T(t)\,R\,u(t)\right)$

memoryless controller: $u \;=\; -F\,x$

- CLOSED-LOOP VARIANCE

J – **non-convex function of** F

No structural constraints

- SDP CHARACTERIZATION

$$\begin{array}{ll} \underset{X,\,F}{\text{minimize}} & \text{trace}\left((Q + F^T R\, F)\, X\right) \\ \text{subject to} & (A - B_2 F)\, X + X\, (A - B_2 F)^T + B_1 B_1^T = 0 \\ & X \succ 0 \end{array}$$

⋆ **change of variables:** $F X = Y$

$$\begin{array}{ll} \underset{X,\,Y}{\text{minimize}} & \text{trace}\,(Q\, X) + \text{trace}\left(R\, Y\, X^{-1}\, Y^T\right) \\ \text{subject to} & (A\, X - B_2\, Y) + (A\, X - B_2\, Y)^T + B_1 B_1^T = 0 \\ & X \succ 0 \end{array}$$

Schur complement $\Rightarrow$ SDP characterization

- RICCATI-BASED-CHARACTERIZATION

globally optimal controller

$$A^T P + P A - P B_2 R^{-1} B_2^T P + Q = 0$$

$$F_c = R^{-1} B_2^T P$$

- STRUCTURAL CONSTRAINTS $F \in \mathcal{S}$

centralized

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

fully-decentralized

$$\begin{bmatrix} * & & & \\ & * & & \\ & & * & \\ & & & * \end{bmatrix}$$

localized

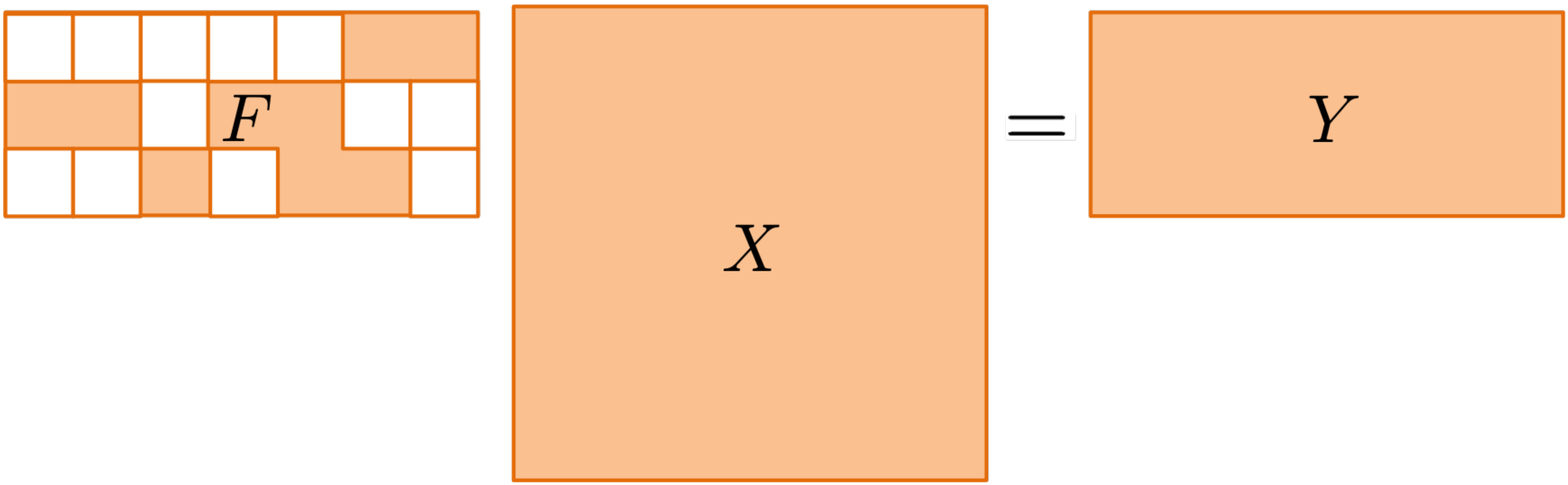
$$\begin{bmatrix} * & * & & \\ * & * & * & \\ & * & * & * \\ & & * & * \end{bmatrix}$$

GRAND CHALLENGE

convex characterization in the face of structural constraints

difficult to establish relation between

$$\left\{ \begin{array}{c} \textbf{structural constraints} \\ \text{on } F \end{array} \right\} \text{ and } \left\{ \begin{array}{c} \textbf{structural constraints} \\ \text{on } X \text{ and } Y \end{array} \right\}$$



Classes of convex problems

- PARTIALLY-NESTED SYSTEMS

  Ho & Chu, IEEE TAC '72

  Voulgaris, ACC '00; ACC '01

- CONE- AND FUNNEL-CAUSAL SYSTEMS

  Voulgaris, Bianchini, Bamieh, SCL '03

  Bamieh & Voulgaris, SCL '05

  Fardad & Jovanović, Automatica '11

- QUADRATICALLY-INVARIANT SYSTEMS

  Rotkowitz & Lall, IEEE TAC '06

- POSET-CAUSAL SYSTEMS

  Shah & Parrilo, IEEE TAC '13

- POSITIVE SYSTEMS

  Tanaka & Langbort, IEEE TAC '11

  Colaneri, Middleton, Chen, Caporale, Blanchini, Automatica '14

  Rantzer, EJC '15; IEEE TAC '16

- **System level synthesis**

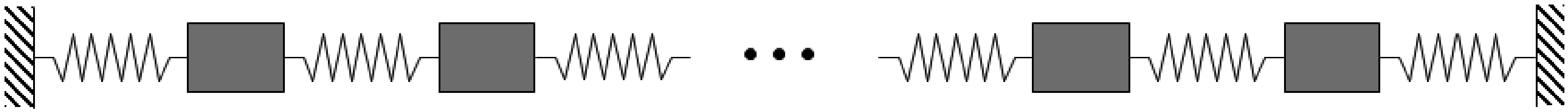
  Wang, Matni, Doyle, IEEE TAC '19

  Anderson, Doyle, Low, Matni, Annu. Rev. Control '19

- **Sparsity invariance**

  Furieri, Zheng, Papachristodoulou, Kamgarpour, IEEE TCNS '20

An example



$$u(t) = -\begin{bmatrix} F_p & F_v \end{bmatrix} \begin{bmatrix} p(t) \\ v(t) \end{bmatrix}$$

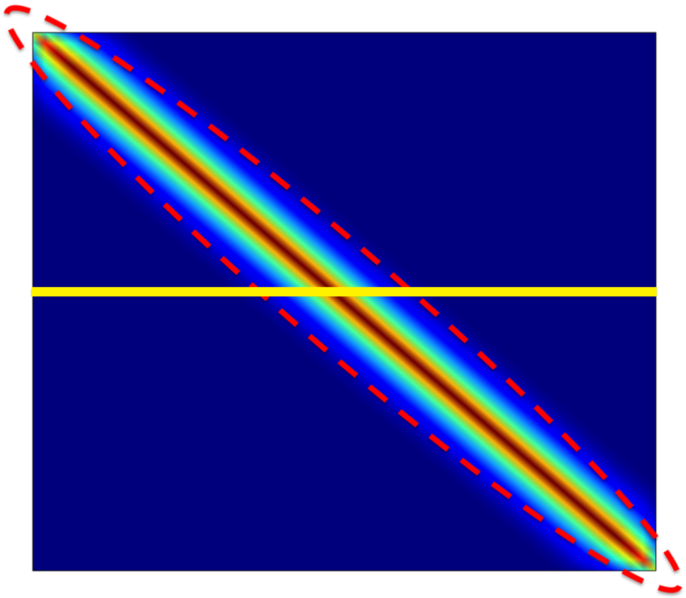
- OBJECTIVE
  - ⋆ minimize steady-state variance of p, v, u

optimal controller – Linear Quadratic Regulator

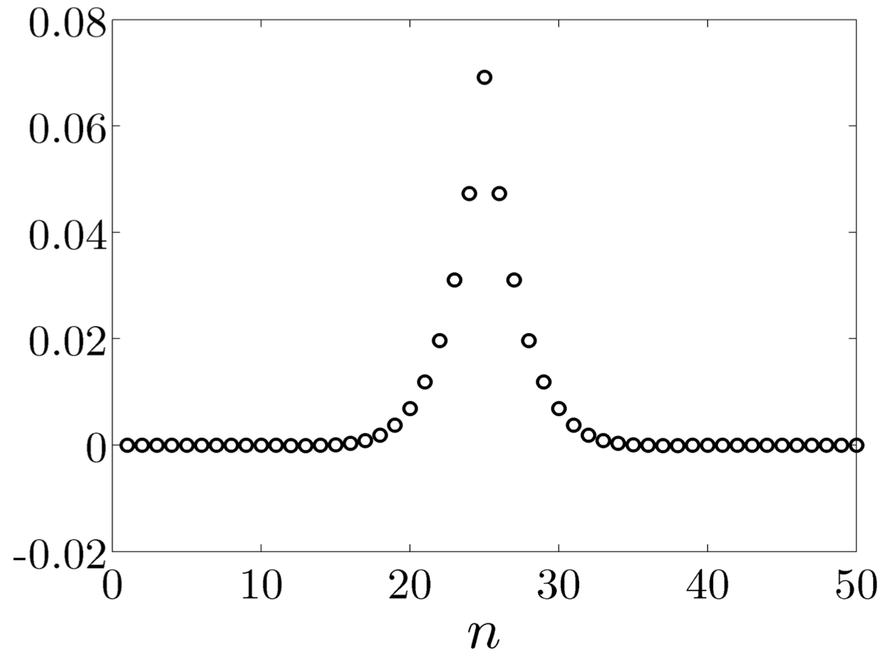
$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} = -\underbrace{\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}}_{F_p} \begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{bmatrix} - \underbrace{\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}}_{F_v} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix}$$

Structure of optimal controller

position feedback matrix



gains for middle mass



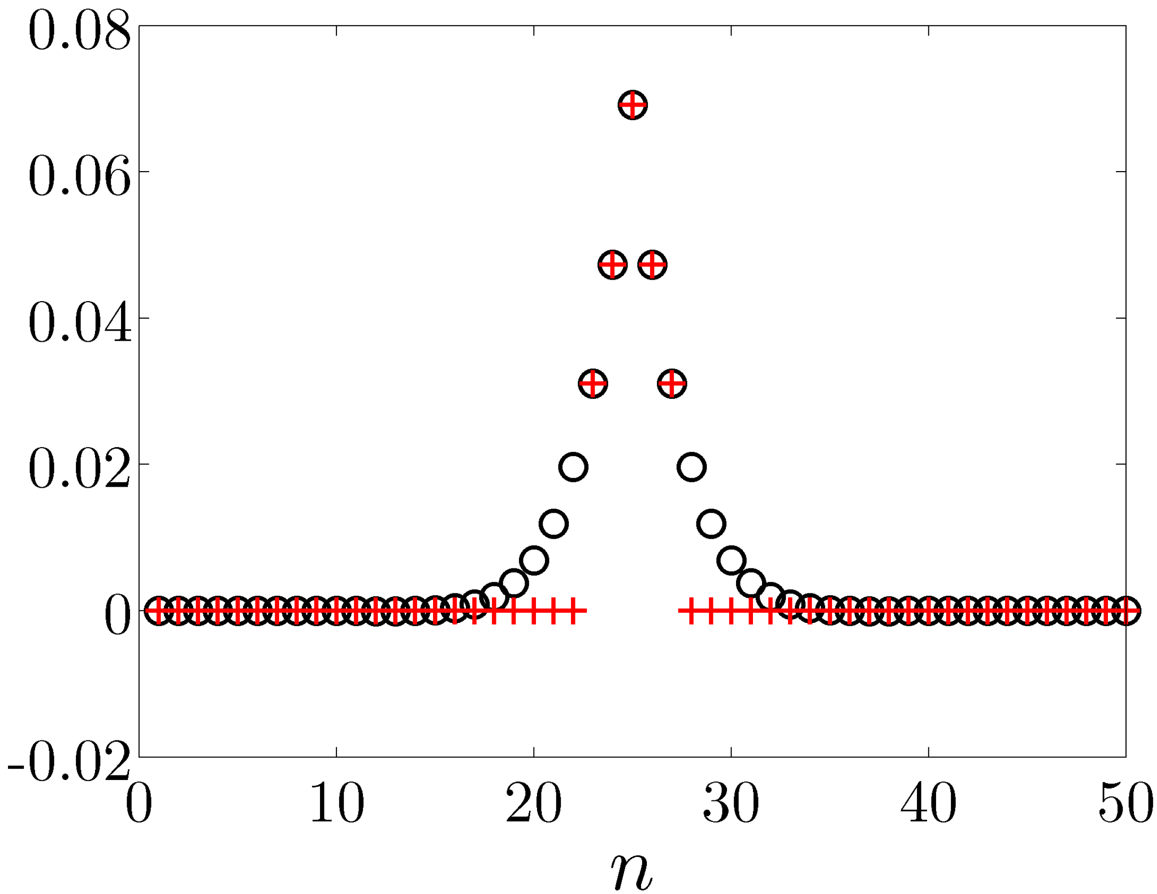
- Observations
  - ⋆ diagonals almost constant (modulo edges)
  - ⋆ off-diagonal decay of a centralized gain

Bamieh, Paganini, Dahleh, IEEE TAC '02

Motee & Jadbabaie, IEEE TAC '08

Enforcing sparsity?

- **One approach: truncate centralized controller**



- DANGERS
  - ⋆ significant performance degradation
  - ⋆ instability

Rest of the talk

- SPARSITY-PROMOTING OPTIMAL CONTROL
  - ⋆ identification and design of sparse feedback gains
- ALGORITHM
  - ⋆ Proximal Augmented Lagrangian Method
- CLASSES OF CONVEX PROBLEMS
  - ⋆ optimal actuator/sensor selection
  - ⋆ optimal design of consensus networks
  - ⋆ diagonal modifications of positive systems
- EXAMPLES
- SUMMARY AND OUTLOOK

SPARSITY-PROMOTING OPTIMAL CONTROL

- **Objective**

  ⋆ **promote sparsity of F**

$$
\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix}
= - \underbrace{\begin{bmatrix} * & * & & & \\ * & * & & & * \\ & * & * & * & \\ & & * & * & \\ & * & & * & * \end{bmatrix}}_{F}
\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}
$$

Sparsity-promoting optimal control

$$\text{minimize} \quad J(F) \quad + \quad \gamma \, \mathbf{card}\,(F)$$

$J(F)$ $\downarrow$ **variance amplification**

$\gamma \, \mathbf{card}\,(F)$ $\downarrow$ **sparsity-promoting penalty function**

$\mathbf{card}\,(F)$ – number of non-zero elements of F

$\gamma > 0$ – performance vs sparsity tradeoff

Fardad, Lin, Jovanović, ACC ’11

Lin, Fardad, Jovanović, IEEE TAC ’13

Convex relaxations of $\mathbf{card}\,(F)$

ℓ_1 norm: $\displaystyle\sum_{i,\,j} |F_{ij}|$

weighted ℓ_1 norm: $\displaystyle\sum_{i,\,j} w_{ij}\,|F_{ij}|, \quad w_{ij} \;\geq\; 0$

- CARDINALITY VS WEIGHTED ℓ_1 NORM

$$\{w_{ij} \;=\; 1/|F_{ij}|, \;\; F_{ij} \;\neq\; 0\} \quad \Rightarrow \quad \mathbf{card}\,(F) \;=\; \sum_{i,\,j} w_{ij}\,|F_{ij}|$$

RE-WEIGHTED SCHEME

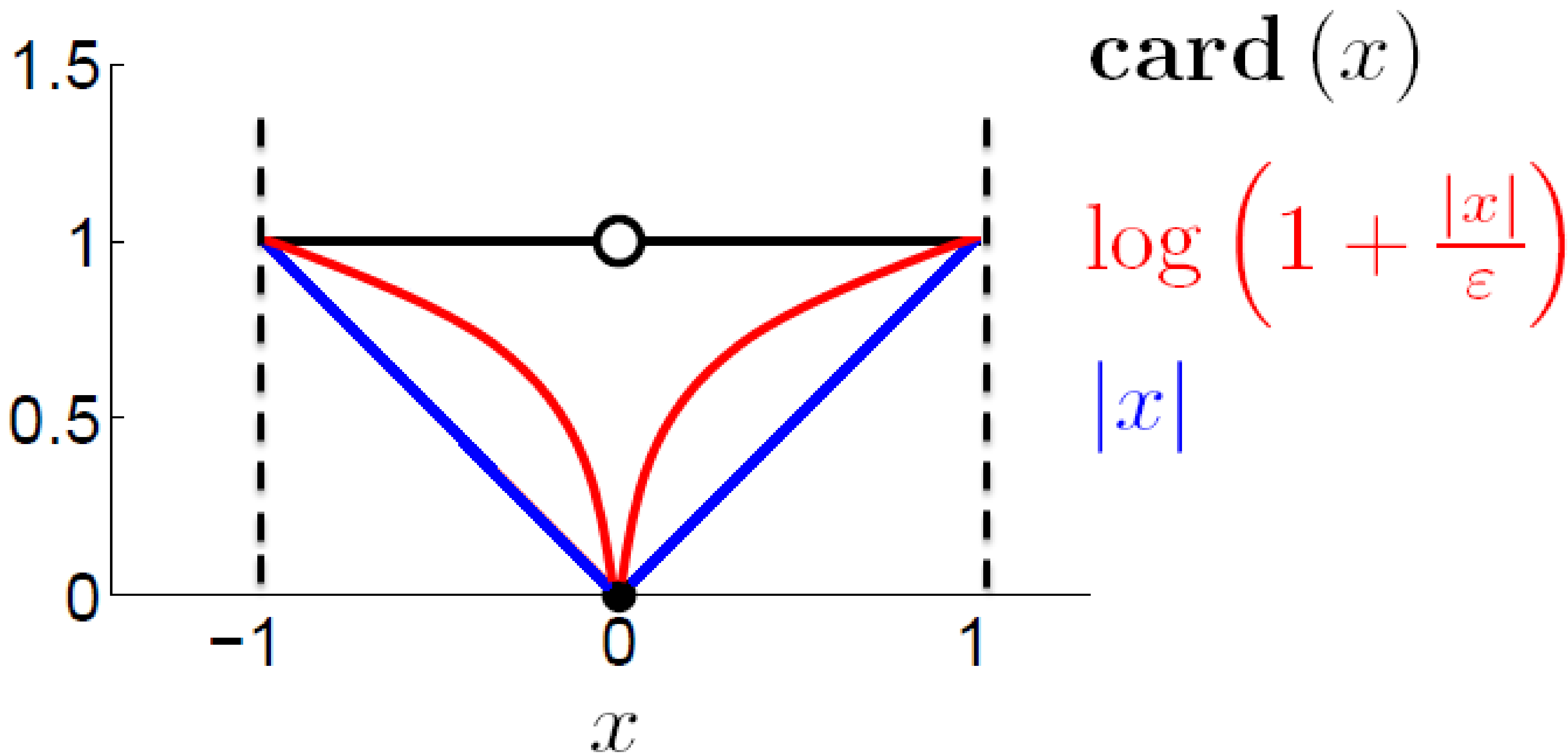
⋆ **use gains from previous iteration to form weights**

$$w_{ij}^{+} \;=\; \frac{1}{|F_{ij}| \;+\; \varepsilon}$$

Candès, Wakin, Boyd, J. Fourier Anal. Appl. '08

A non-convex relaxation of $\mathbf{card}\,(F)$

sum-of-logs: $$\sum_{i,j} \log\left(1 + \frac{|F_{ij}|}{\varepsilon}\right), \qquad 0 < \varepsilon \ll 1$$

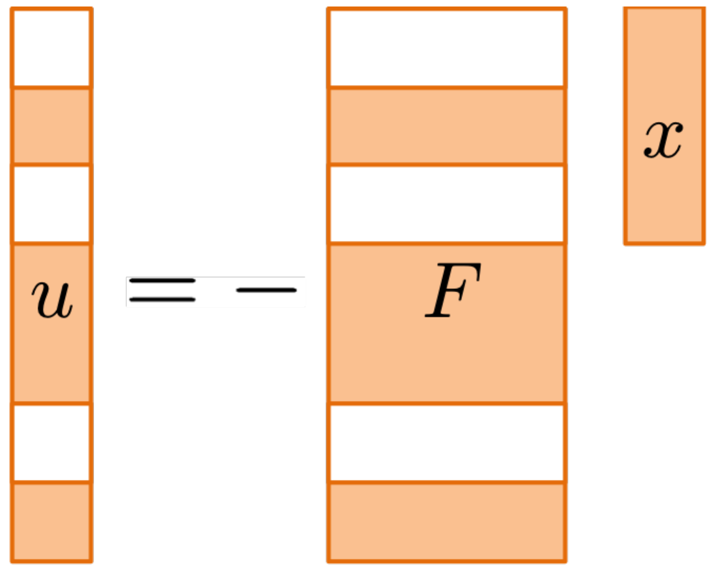


Candès, Wakin, Boyd, J. Fourier Anal. Appl. '08

Classes of convex problems

Optimal actuator/sensor selection

- OBJECTIVE: **identify row-sparse feedback gain**

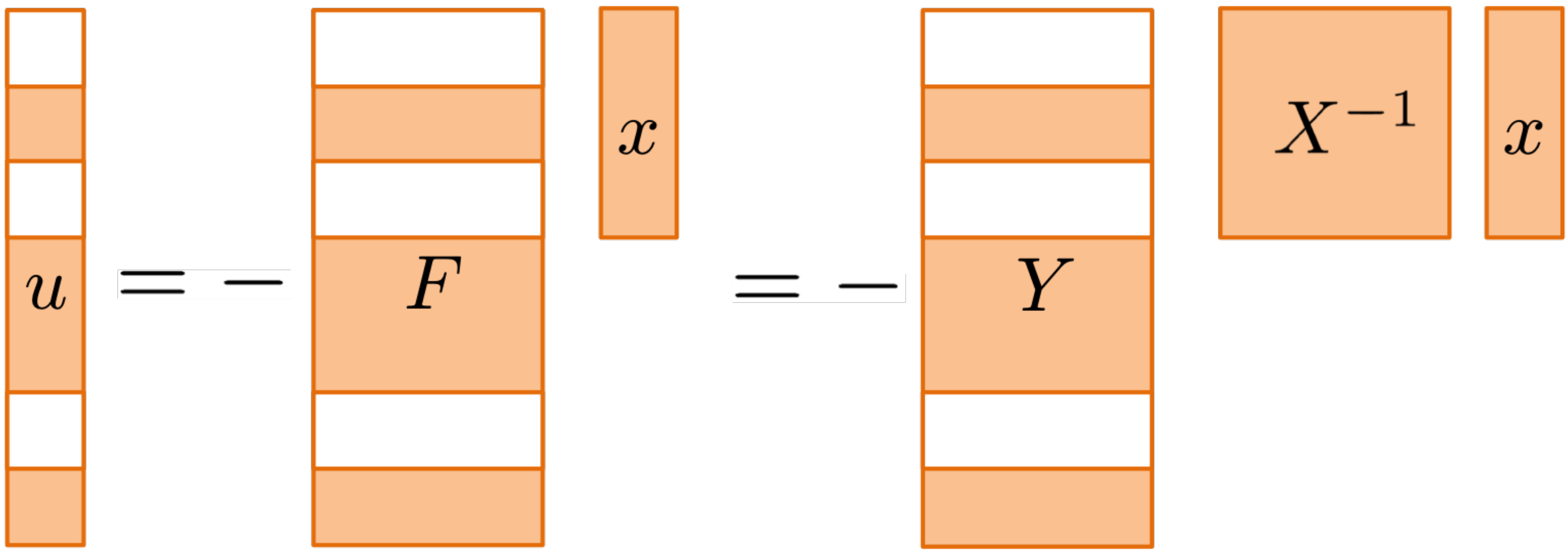


$$\text{minimize} \quad J(F) \quad + \quad \gamma \sum_i \|e_i^T F\|_2$$

$\downarrow$ **variance amplification**

$\downarrow$ **row-sparsity-promoting penalty function**

- CHANGE OF VARIABLES: $Y := F\,X$
  - ⋆ **convex dependence of J on X and Y**
  - ⋆ **row-sparse structure preserved**



- **Optimal actuator selection**

  ⋆ **admits SDP characterization**

$$\text{minimize} \quad \underbrace{J(X,Y)}_{\text{variance amplification}} \quad + \quad \underbrace{\gamma \sum_i \|\mathrm{e}_i^T Y\|_2}_{\text{row-sparsity-promoting penalty function}}$$

Polyak, Khlebnikov, Shcherbakov, ECC '13

Dhingra, Jovanović, Luo, CDC '14

Zare, Mohammadi, Dhingra, Georgiou, Jovanović, IEEE TAC '20

Design of undirected consensus networks

dynamics: $\dot{x} \; = \; -\, L\, x \; + \; d \; + \; u$

control: $u \; = \; -\, F\, x$

objective: $J \; = \; \lim_{t \to \infty} \mathbf{E} \left(x^T(t)\, Q\, x(t) \; + \; u^T(t)\, R\, u(t) \right)$

convex characterization

$$
\begin{array}{ll}
\text{minimize} & \operatorname{trace}(X) \; + \; \gamma\, \mathbb{1}^T\, Y\, \mathbb{1} \\
\text{subject to} & \begin{bmatrix} X & \begin{bmatrix} Q^{1/2} \\ -R^{1/2}\, F \end{bmatrix} \\ \begin{bmatrix} Q^{1/2} & -F\, R^{1/2} \end{bmatrix} & F \; + \; L \; + \; \mathbb{1}\mathbb{1}^T/n \end{bmatrix} \; \succeq \; 0 \\
& F\, \mathbb{1} \; = \; 0, \quad -\, Y_{ij} \; \leq \; W_{ij}\, F_{ij} \; \leq \; Y_{ij}
\end{array}
$$

Lin, Fardad, Jovanović, Allerton '12

Zelazo, Schuler, Allgöwer, SCL '13

Hassan-Moghaddam & Jovanović, IEEE TCNS '18

Diagonal modifications of positive systems

$$\dot{x} \;=\; \left(A \;+\; \sum_k u_k\, D_k\right) x \;+\; d$$

A – Metzler matrix ($A_{ij} \geq 0$, $i \neq j$)

D_k – diagonal matrices

- EXAMPLES

  ⋆ combination drug therapy $\begin{cases} x_i \text{ mutates to } x_j \text{ at rate } A_{ji} \\ u_k \text{ kills } x_i \text{ at rate } (D_k)_{ii} \end{cases}$

  ⋆ leader selection in directed networks

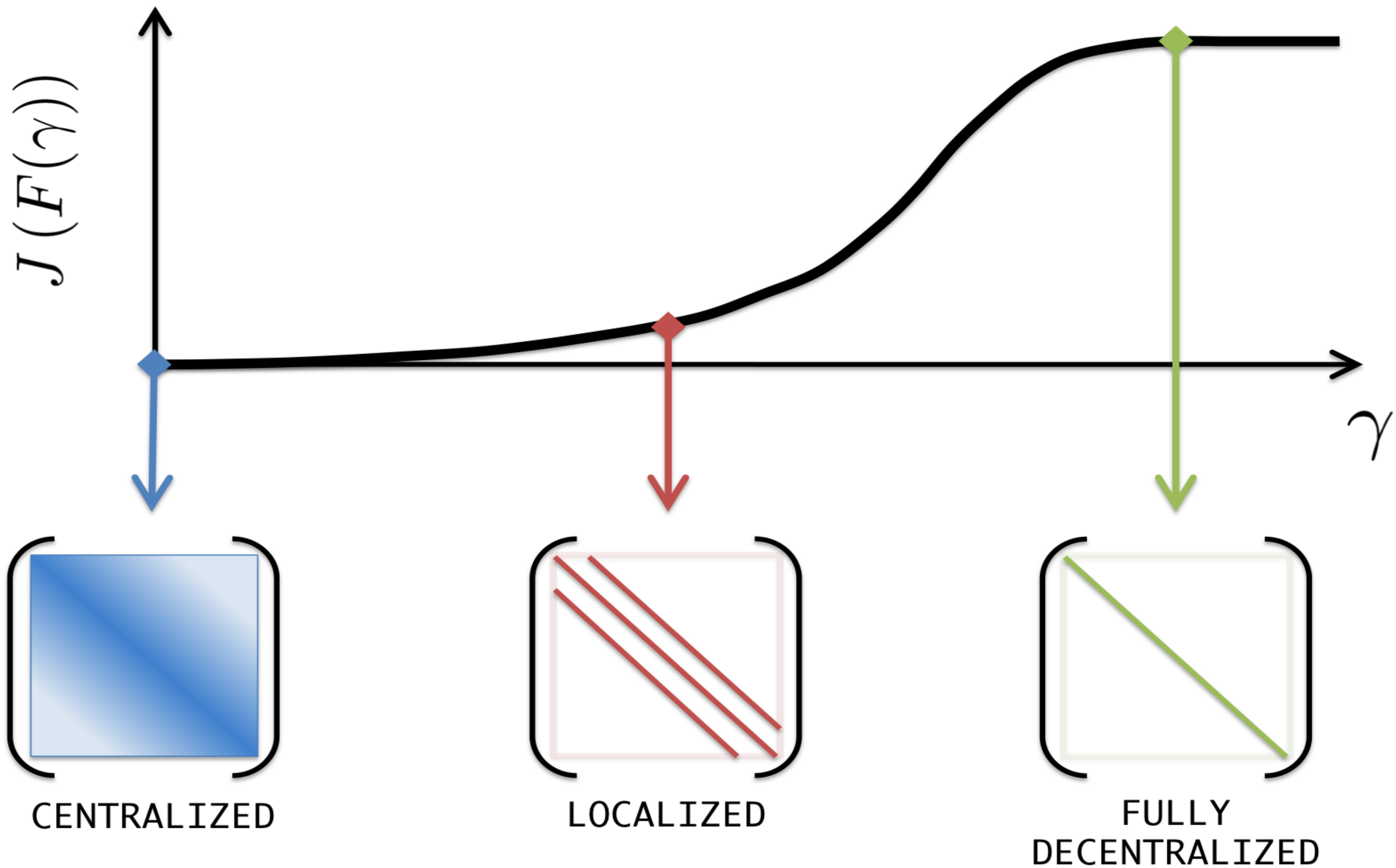
Rantzer & Bernhardsson, CDC '14

Jonsson, Matni, Murray, CDC '14

Dhingra, Colombino, Jovanović, IEEE TCNS '19

Parameterized family of feedback gains

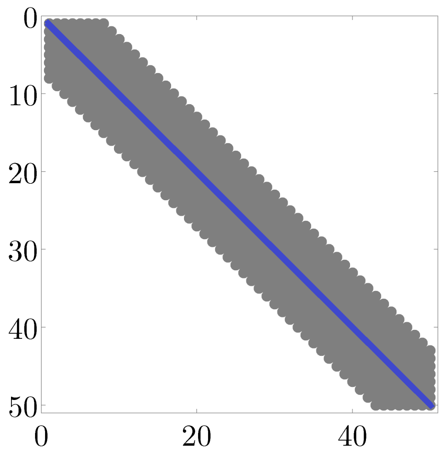
$$F(\gamma) \;:=\; \underset{F}{\operatorname{argmin}}\,(J(F) \;+\; \gamma\, g(F))$$

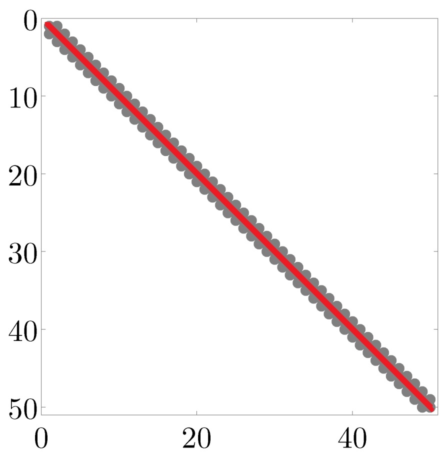


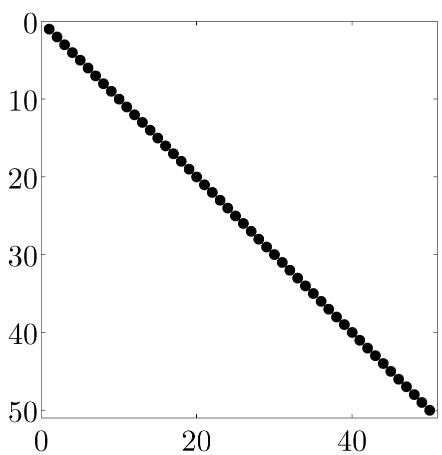
EXAMPLES

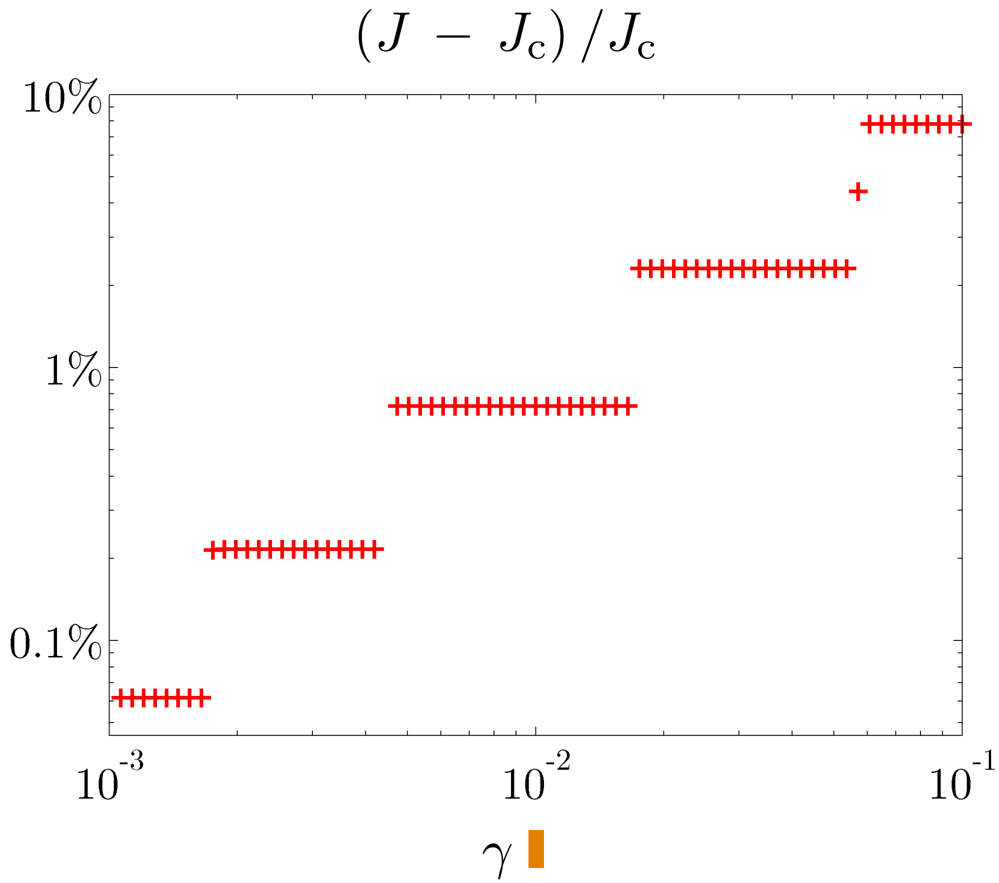
Mass-spring system

Performance comparison: sparse vs centralized





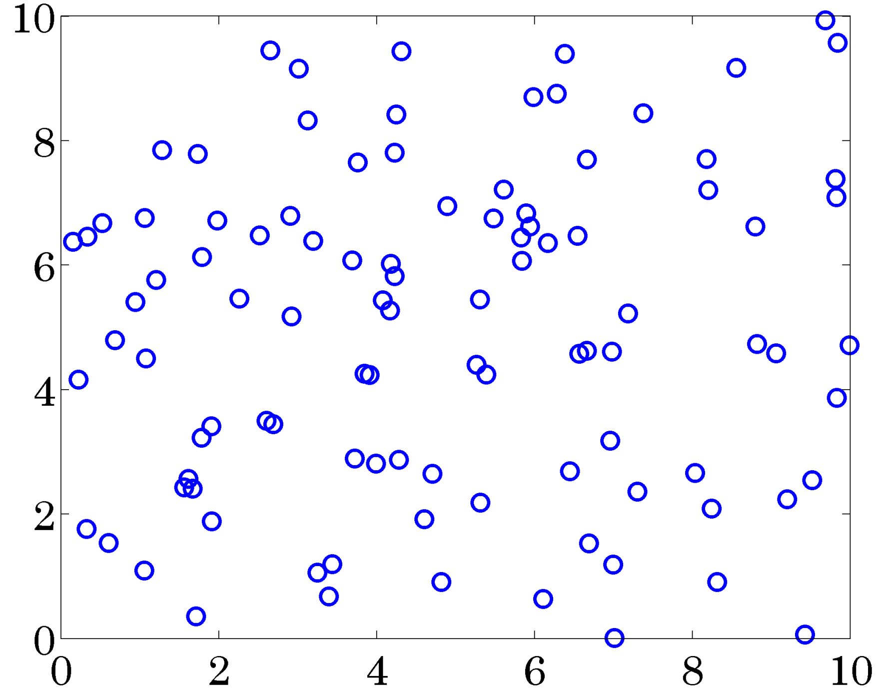




$\mathbf{card}(F)/\mathbf{card}(F_c)$	$(J - J_c)/J_c$
10%	0.75%
6%	2.4%
2%	7.8%

fully-decentralized

Network with 100 nodes

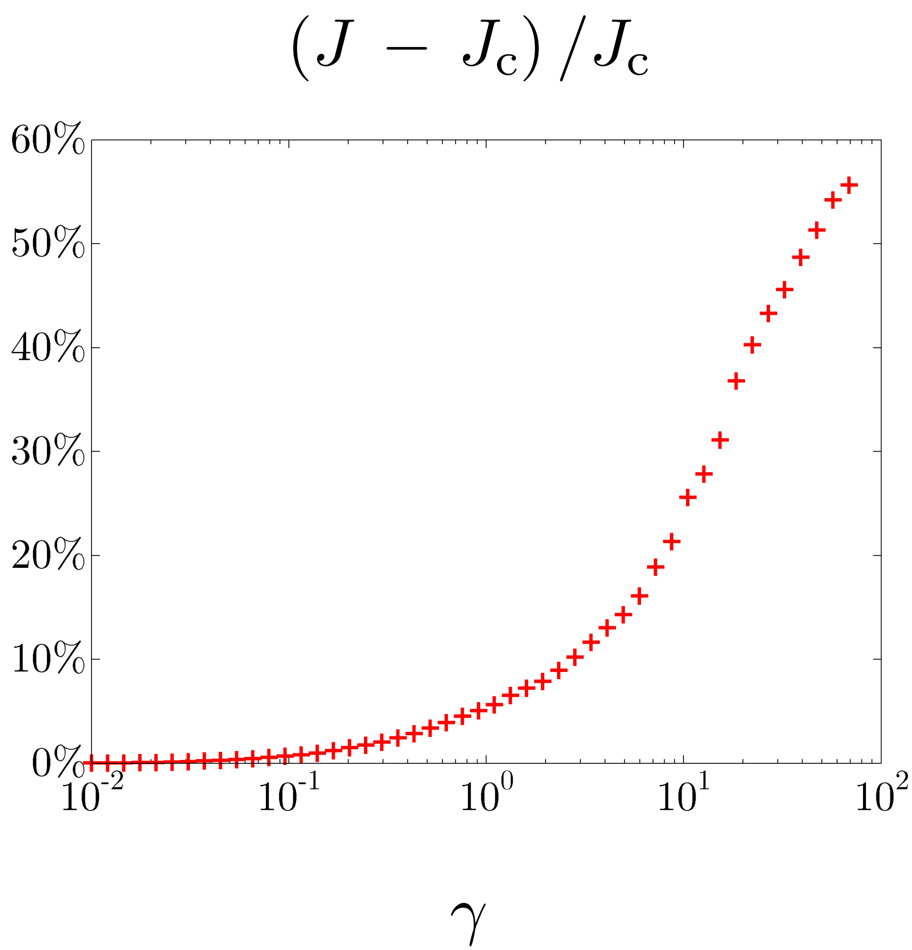


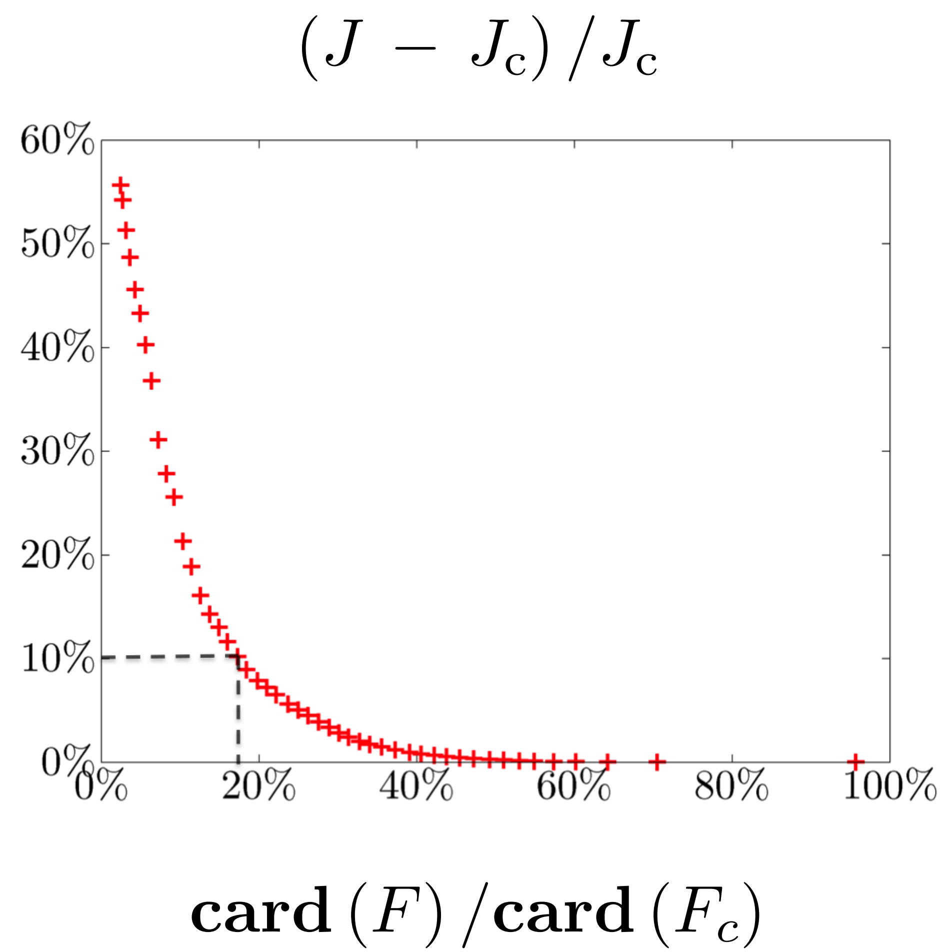
$$\begin{bmatrix} \dot{p}_i \\ \dot{v}_i \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} p_i \\ v_i \end{bmatrix}}_{\text{unstable dynamics}} + \underbrace{\sum_{j \neq i} \mathrm{e}^{-\alpha(i,j)} \begin{bmatrix} p_j \\ v_j \end{bmatrix}}_{\text{coupling}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (d_i + u_i)$$

$\alpha(i, j)$: Euclidean distance between nodes i and j

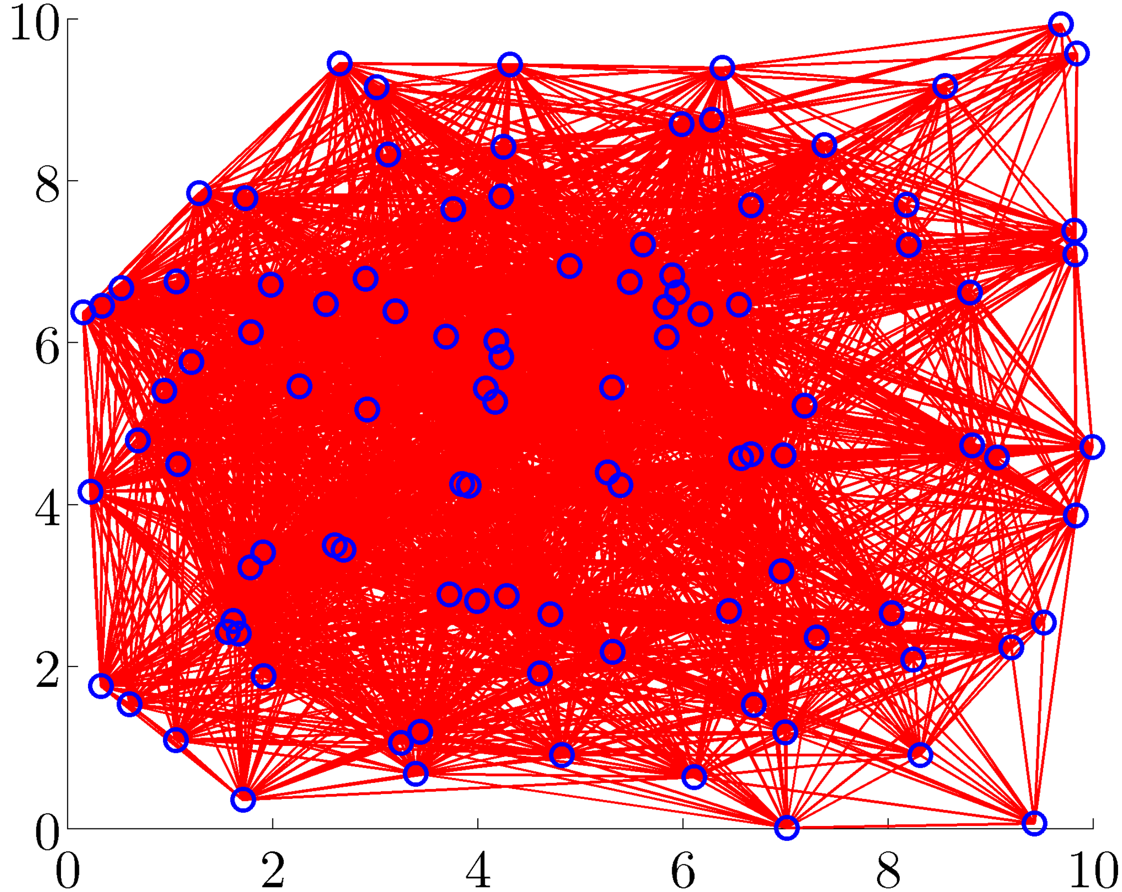
Motee & Jadbabaie, IEEE TAC '08

- **Performance comparison: sparse vs centralized**





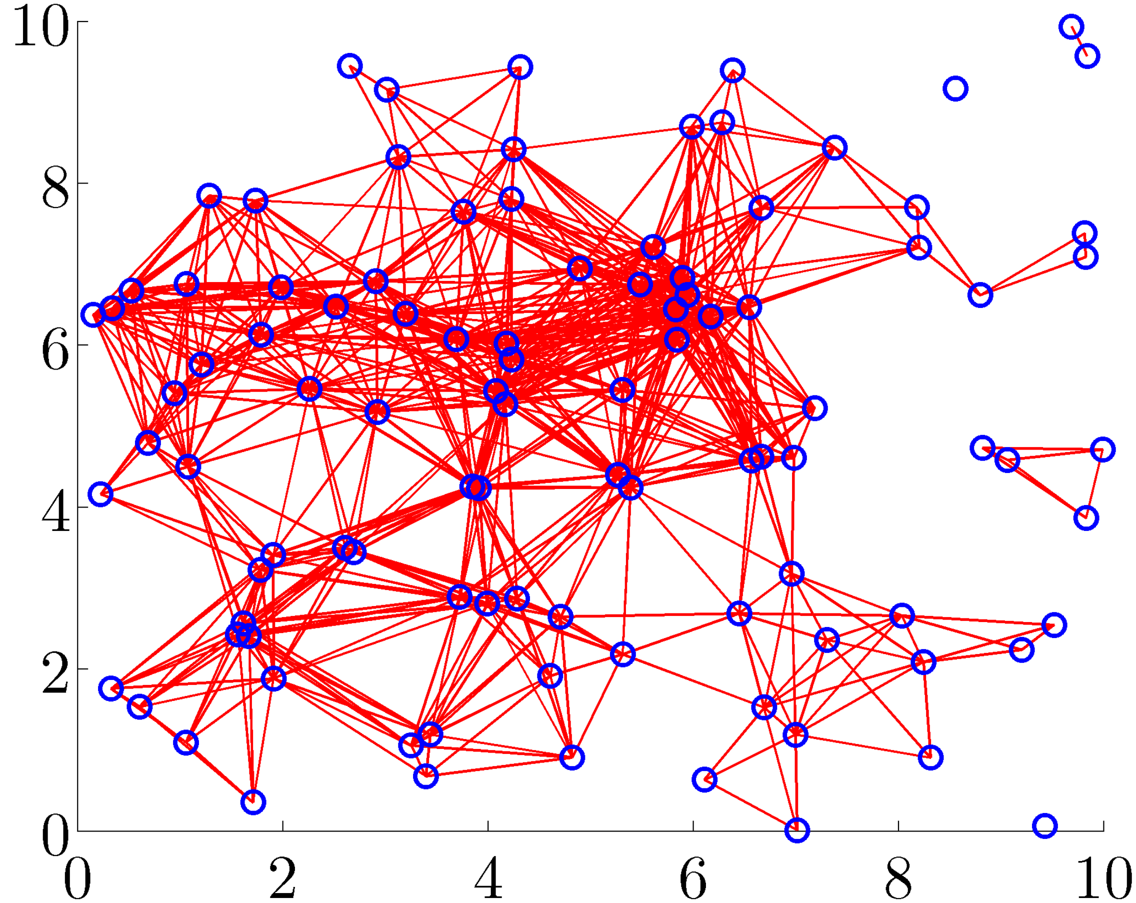
communication graph of a truncated centralized gain



$$\textbf{card}\,(F) \;=\; 7380 \;\; (36.9\%)$$

non-stabilizing

identified communication graph



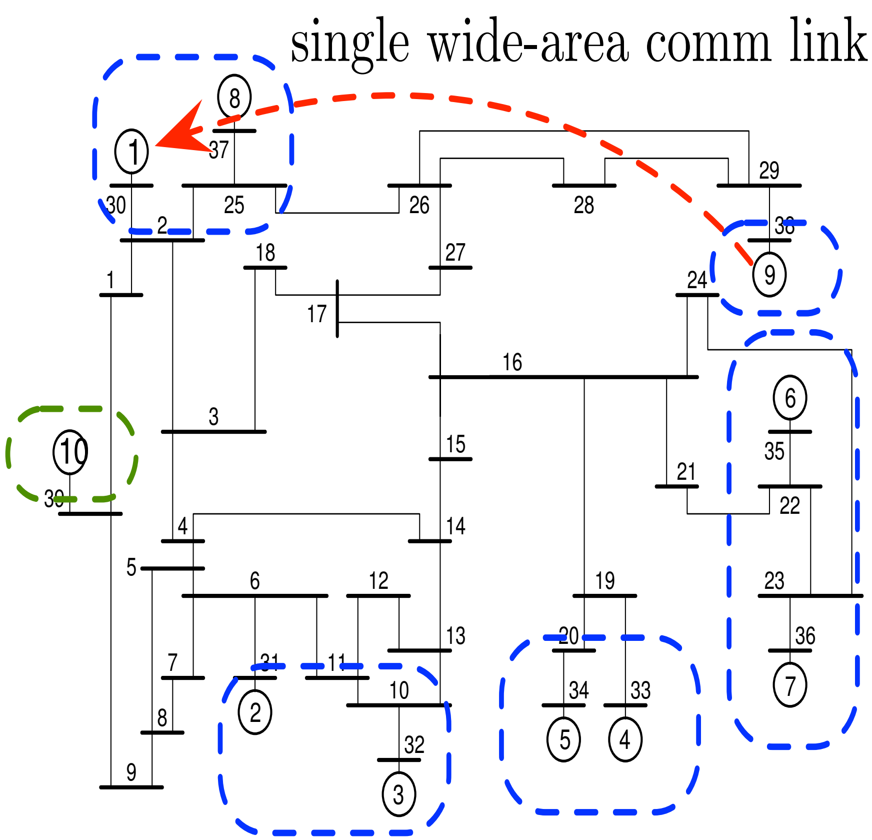
$$\gamma = 5$$

$$\mathbf{card}(F)/\mathbf{card}(F_{\mathrm{c}}) = 8.8\%$$

$$(J - J_{\mathrm{c}})/J_{\mathrm{c}} = 24.6\%$$

Power networks

- SPARSITY-PROMOTING WIDE-AREA CONTROL
  - ⋆ **remedy against inter-area oscillations**



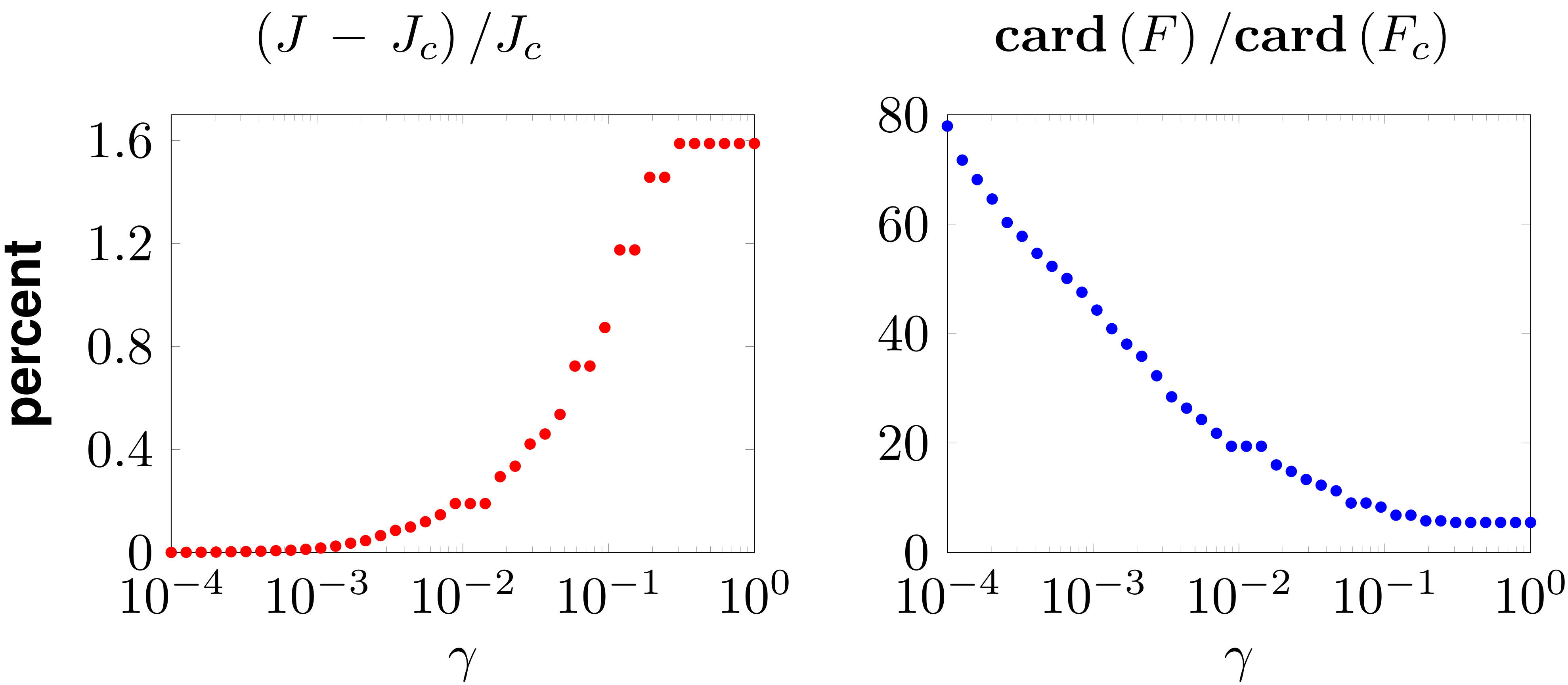
single long-range interaction:

nearly centralized performance

Dörfler, Jovanović, Chertkov, Bullo, ACC '13

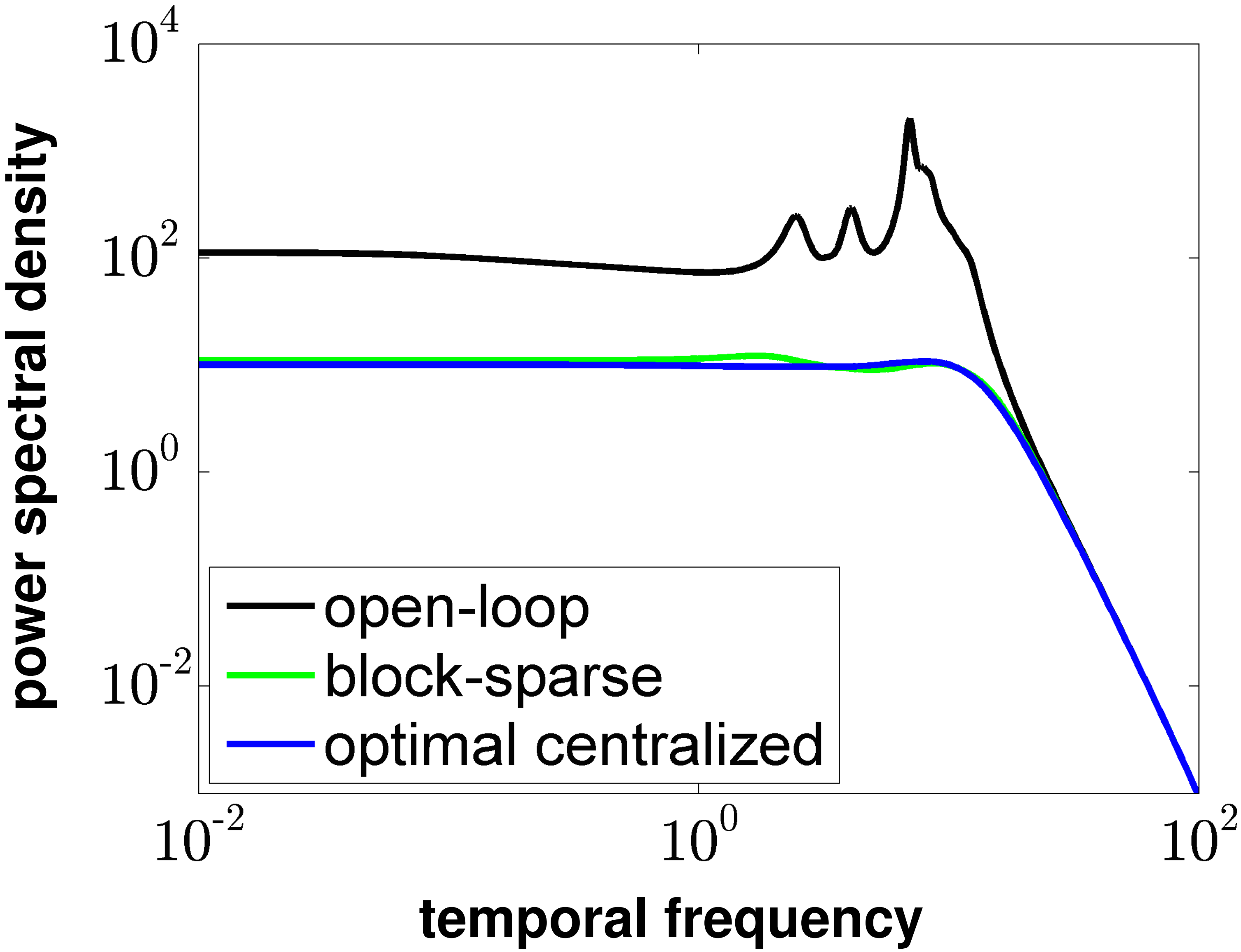
Dörfler, Jovanović, Chertkov, Bullo, IEEE TPWRS '14

- **Performance comparison: sparse vs centralized**



$$\gamma = 1 \xrightarrow{\text{relative to } F_c} \begin{cases} 1.6\,\% & \text{performance loss} \\ 5.5\,\% & \text{non-zero elements in } F \end{cases}$$

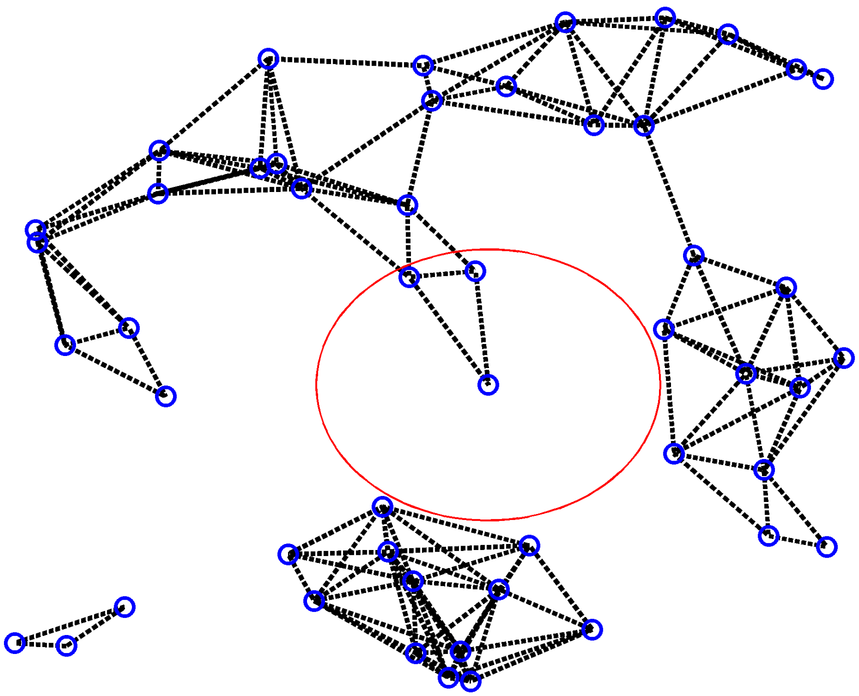
- RE-DESIGN OF FULLY-DECENTRALIZED CONTROLLERS
  - **preserves rotational symmetry**



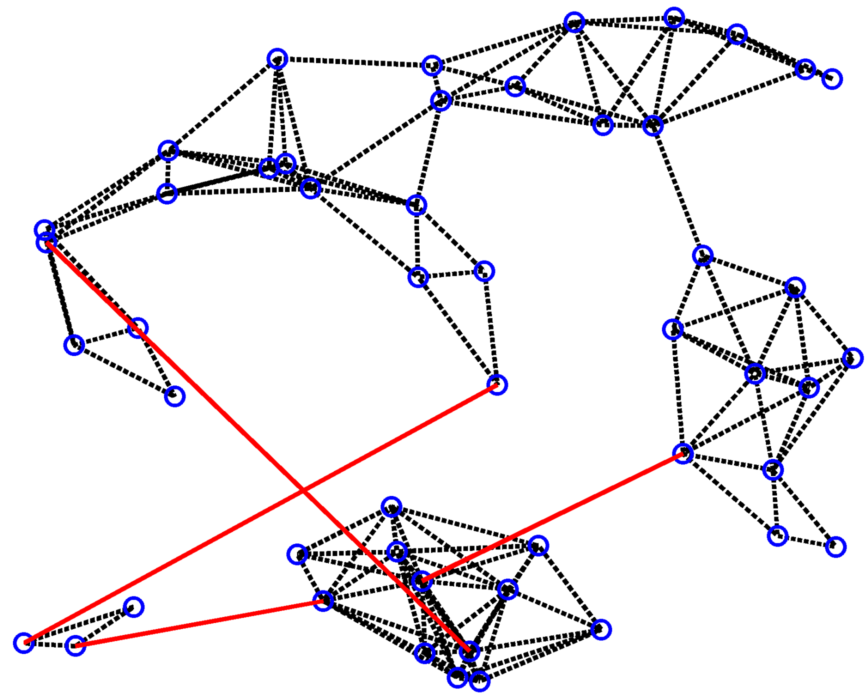
Wu, Dörfler, Jovanović, IEEE TPWRS '16

Sparsity-promoting consensus algorithm

plant graph



identified communication graph



Q := deviation from average

$$\frac{J - J_{\text{all-to-all}}}{J_{\text{all-to-all}}} \approx 82\%$$

Hassan-Moghaddam & Jovanović, IEEE TCNS '18

ALGORITHM

Method of multipliers

$$\text{minimize} \quad J(F) \; + \; \gamma\, g(F)$$

- **Step 1: introduce an additional variable/constraint**

$$\begin{array}{ll} \text{minimize} & J(F) \; + \; \gamma\, g(G) \\ \text{subject to} & F \; - \; G \; = \; 0 \end{array}$$

benefit: decouples J and g

- **Step 2: introduce augmented Lagrangian**

$$\mathcal{L}_\rho(F, G; \Lambda) \; = \; J(F) \; + \; \gamma\, g(G) \; + \; \langle \Lambda, F \; - \; G \rangle \; + \; \frac{\rho}{2} \| F \; - \; G \|_F^2$$

- **Step 3: use MM for augmented Lagrangian minimization**

$$\mathcal{L}_\rho(F, G; \Lambda) \;=\; J(F) \;+\; \gamma\, g(G) \;+\; \langle \Lambda, F \,-\, G \rangle \;+\; \frac{\rho}{2}\, \| F \,-\, G \|_F^2$$

METHOD OF MULTIPLIERS

$$\begin{aligned} \left(F^{k+1}, G^{k+1}\right) \;&:=\; \underset{F,\,G}{\operatorname{argmin}}\; \mathcal{L}_{\rho^k}(F, G; \Lambda^k) \\ \Lambda^{k+1} \;&:=\; \Lambda^k \;+\; \rho^k \left(F^{k+1} \;-\; G^{k+1}\right) \end{aligned}$$

- **Step 4: Polishing – back to structured optimal design**

  ⋆ MM $\begin{cases} \text{identifies sparsity patterns} \\ \text{provides good initial condition for structured design} \end{cases}$

  ⋆ **optimality conditions for the structured problem**

$$
\begin{aligned}
(A - B_2 F)^T P + P (A - B_2 F) &= -\left(Q + F^T R F\right) \\
(A - B_2 F) X + X (A - B_2 F)^T &= -B_1 B_1^T \\
\left[\left(R F - B_2^T P\right) X\right] \circ I_{\mathcal{S}} &= 0
\end{aligned}
$$

$I_{\mathcal{S}}$ - structural identity

$$
F = \begin{bmatrix} * & * & & \\ * & * & * & \\ & * & * & * \\ & & * & * \end{bmatrix} \quad \Rightarrow \quad I_{\mathcal{S}} = \begin{bmatrix} 1 & 1 & & \\ 1 & 1 & 1 & \\ & 1 & 1 & 1 \\ & & 1 & 1 \end{bmatrix}
$$

Lin, Fardad, Jovanović, IEEE TAC '11

The two pillars

- PROXIMAL OPERATOR

$$\mathbf{prox}_{\mu g}(V) := \operatorname*{argmin}_{G} \; g(G) + \frac{1}{2\mu}\|G - V\|_F^2$$

MOREAU ENVELOPE

$$M_{\mu g}(V) := g(\mathbf{prox}_{\mu g}(V)) + \frac{1}{2\mu}\|\mathbf{prox}_{\mu g}(V) - V\|_F^2$$

⋆ **continuously differentiable**
even when g is not

$$\nabla M_{\mu g}(V) = \frac{1}{\mu}\big(V - \mathbf{prox}_{\mu g}(V)\big)$$

Parikh & Boyd, FnT in Optimization '14

Proximal augmented Lagrangian

$$\mathcal{L}_\rho(F, G; \Lambda) \;=\; J(F) + \underbrace{\gamma\, g(G) \;+\; \frac{\rho}{2}\,\|G \,-\, (F \,+\, (1/\rho)\Lambda)\|_F^2}\;-\;\frac{1}{2\rho}\,\|\Lambda\|_F^2$$

⋆ minimize over G

$$G^\star \;=\; \mathbf{prox}_{(\gamma/\rho)g}(F \,+\, (1/\rho)\Lambda)$$

⋆ evaluate $\mathcal{L}_\rho$ at $G^\star$

$$\begin{aligned}\mathcal{L}_\rho(F; \Lambda) \;&:=\; \mathcal{L}_\rho(F, G^\star(F, \Lambda); \Lambda)\\ &=\; J(F) \;+\; \gamma\, M_{(\gamma/\rho)g}(F \,+\, (1/\rho)\Lambda) \;-\; \frac{1}{2\rho}\,\|\Lambda\|_F^2\end{aligned}$$

continuously differentiable

Method of multipliers

$$
\begin{aligned}
F^{k+1} &= \underset{F}{\text{argmin}} \; \mathcal{L}_{\rho^k}(F; \Lambda^k) \\
\Lambda^{k+1} &= \gamma \, \nabla M_{(\gamma/\rho^k)g}(F^{k+1} + (1/\rho^k)\Lambda^k)
\end{aligned}
$$

- FEATURES
  - ⋆ outstanding practical performance
  - ⋆ nonconvex J: convergence to a local minimum
  - ⋆ F-minimization: differentiable problem
  - ⋆ adaptive ρ-update

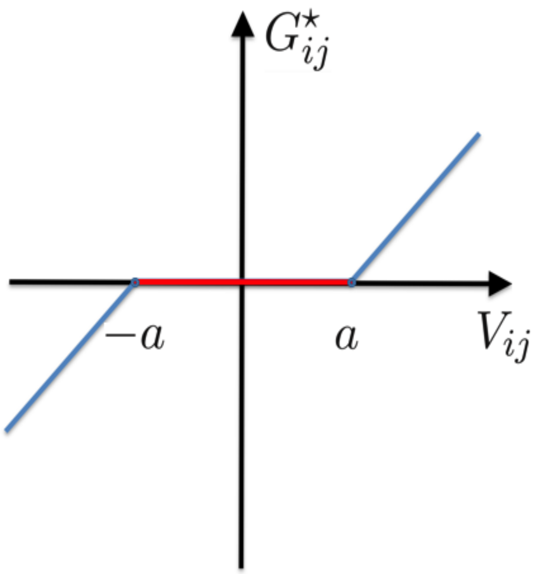
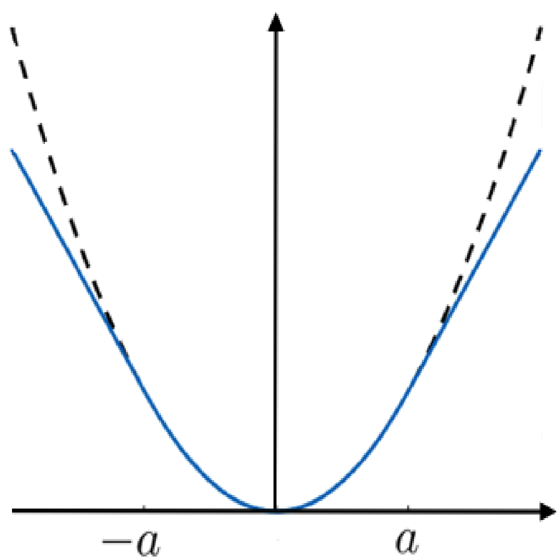
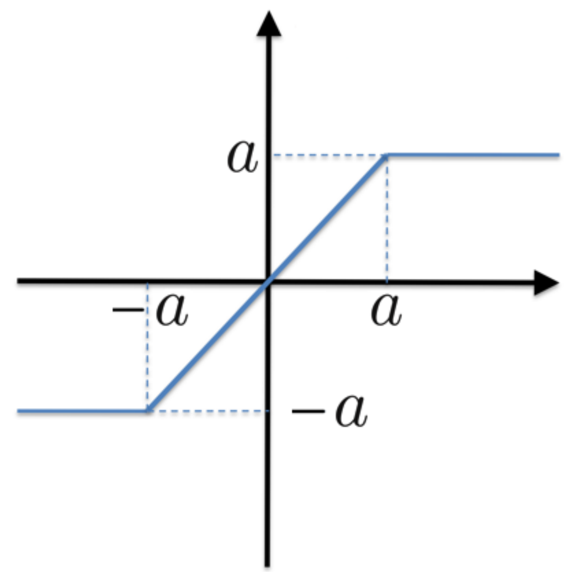
Dhingra & Jovanović, ACC '16

Dhingra, Khong, Jovanović, IEEE TAC '19

• G-UPDATE IN SPARSITY-PROMOTING PROBLEM

$$\underset{G_{ij}}{\text{minimize}} \;\; \sum_{i,\,j} \left(\gamma\, w_{ij}\, |G_{ij}| \;+\; \frac{\rho}{2} (G_{ij} \,-\, V_{ij})^2 \right)$$

separability $\Rightarrow$ **element-wise analytical solution**

prox operator soft-thresholding	**Moreau envelope** Huber function	∇M saturation
		
	$a \;=\; (\gamma/\rho)\, w_{ij}$	

An extension

- Coordinate-specific sparsity requirements

$$\begin{array}{ll} \text{minimize} & J(F) \; + \; \gamma \, g(G) \\ \text{subject to} & \mathcal{T}(F) \; - \; G \; = \; 0 \end{array}$$

Examples

$\star$ $\mathcal{T}(F) \; = \; F\,T$

$\star$ $\mathcal{T}(F)$ – inverse spatial Fourier transform

Zoltowski, Dhingra, Lin, Jovanović, ACC '14

Wu & Jovanović, SCL '17

Related effort

- SPARSITY-PROMOTING H_∞ CONTROL

  Schuler, Li, Lam, Allgöwer, IJC '11

  Schuler, Münz, Allgöwer, IFAC '12

- SYSTEMS WITH SYMMETRIES

  Dhingra & Jovanović, ACC '15

  Dhingra, Wu, Jovanović, NOLCOS '16

- CONVEX RELAXATIONS

  Lavaei, Allerton '13

  Fazelnia, Madani, Lavaei, CDC '14

  Fardad & Jovanović, ACC '14

- ATOMIC NORM REGULARIZATION

  Matni, CDC '13; *IEEE TCNS '17*

  Matni & Chandrasekaran, IEEE TAC '16

Summary

- SPARSITY-PROMOTING OPTIMAL CONTROL
  - ⋆ Performance vs sparsity tradeoff

    Lin, Fardad, Jovanović, IEEE TAC '13

    Jovanović & Dhingra, EJC '16
  - ⋆ Software

    www.umn.edu/∼mihailo/software/lqrsp/

- ADDITIONAL EFFORT
  - ⋆ Leader selection in large dynamic networks

    Lin, Fardad, Jovanović, IEEE TAC '14
  - ⋆ Topology design for consensus networks

    Hassan-Moghaddam & Jovanović, IEEE TCNS '18
  - ⋆ Decentralized control of positive systems

    Dhingra, Colombino, Jovanović, IEEE TCNS '19

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