Low-Overhead Distributed MAC for Serving Dynamic Users over Multiple Channels

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***Abstract*—With the adoption of 5G wireless technology and the Internet-of-Things (IoT) networking, there is a growing interest in serving a dense population of low-complexity devices over shared wireless uplink channels. Different from the traditional scenario of persistent users, in these new networks each user is expected to generate only small bundles of information intermittently. The highly dynamic nature of such demand and the typically low-complexity nature of the user devices calls for a new MAC paradigm that is geared for low-overhead and distributed operation of dynamic users.**

In this work, we address this need by developing a generic MAC mechanism for estimating the number and coordinating the activation of dynamic users for efficient utilization of the time-frequency resources with minimal public feedback from the common receiver. We fully characterize the throughput and delay performance of our design under a basic threshold-based multi-channel capacity condition, which allows for the use of different channel utilization schemes. Moreover, we consider the Successive-Interference-Cancellation (SIC) Multi-Channel MAC scheme as a specific choice in order to demonstrate the performance of our design for a spectrally-efficient (albeit idealized) scheme. Under the SIC encoding/decoding scheme, we prove that our low-overhead distributed MAC can support maximum throughput, which establishes the efficiency of our design. Under SIC, we also demonstrate how the basic threshold-based success model can be relaxed to be adapted to the performance of a non-ideal success model.

***Index Terms*—network stability, dynamic arrivals, distributed random access, efficient spectrum access**

I. Introduction

One of the driving forces behind the emerging 5G and future wireless technologies is the upcoming explosion in the scale and requirements of the diverse mobile devices that will need to be supported over an ultra-wide frequency spectrum (including the 57-71 GHz in the U.S. [1], [2]). For example, new services in diverse domains, including health, transportation, energy, and entertainment, within the broad framework of the so-called Internet-of-Things (IoT) present scenarios where many mobile devices intermittently generate small bundles of information to transmit to a backbone wireless server (such as a base station or router). The distributed, large-scale, dynamic, and intermittent nature of these new service requirements call for the design of new spectrum access strategies. In this work, we aim to respond to this need by developing low-overhead and provably-efficient distributed medium-access-control (MAC) strategies aimed at serving a large (possibly unbounded) population of dynamic users with intermittent service demands.

Uncoordinated distributed MAC has a long history, dating back to basic Slotted-Aloha [3] and a large number of interesting variants (e.g., [4], [5], [6], [7], [8] to name a few). The ideas employed in these works range from aiming to exploit multi-channel availability (e.g., [4], [8]) by developing random access strategies to avoid collisions over a subset of channels, to considering advanced signalling capabilities, such as multi-packet reception (e.g., [5], [6]) or orthogonal-frequency-division-multi-access (OFDMA) capabilities (e.g., [9]). Another interesting development has been in the use of carrier-sense-multi-access (CSMA) strategies (e.g., [7]) that employ asynchronous sensing and adaptive transmission strategies with asymptotic efficiency characteristics.

However, these and a large portion of other works in this domain (which must be omitted for lack of space) either build on sophisticated capabilities of the devices (such as the aforementioned advanced signalling and sensitive timing capabilities), and/or consider static user populations under which efficiency characteristics are investigated. Moreover, as revealed in [10], random access solutions designed for static users are likely to be unstable under the presence of user dynamics. In a related thread of important works (e.g., [11], [12], [13]), adaptive random access solutions for slotted-Aloha are proposed to address this instability problem. These works guarantee stability when arrival rates are less than e^{-1} per slot. While resolving the stability problem, these designs have been facilitated by the relatively tractable nature of the slotted-Aloha protocol, but also have been constrained by its relatively low efficiency of around $\approx 37\%$.

In recent years, there have been exciting advances in the design and analysis of the so-called 'Modern Random Access' strategies that yield provably efficient random access schemes to efficiently share a given number of (virtual) channels (in frequency and/or in time) by a given number of users. Among them, one spectrally-efficient strategy is called *Successive Interference Cancellation (SIC)* (e.g., [14], [15], also see [16]

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and references therein). However, as noted above, such new efficient strategies need to operate in future networks that are envisioned to be composed of many dynamic users/devices that intermittently generate small bundles of information, and then become inactive after its transmission is completed. As such, there is a need for developing efficient Multi-Channel MAC solutions that account for these user dynamics and are also well-suited to distributed operation. Our work addresses this need with the following contributions:

- We consider a generic model (in Section II) of orthogonal time-frequency channels to define a flexible frame structure that is well-suited to implementing various Multi-Channel MAC encoding/decoding strategies. By incorporating the arrival and departure dynamics of future mobile networks, our model exposes the instability characteristics of static Multi-Channel MAC strategies.
- We consider a threshold-based success model which can capture basic characteristics of many spectrally-efficient multi-channel uplink MAC schemes. Under that, we develop provably-stabilizing distributed Multi-Channel MAC strategies for perfect (in Section III) and estimated (in Section IV) state information for dynamic users with a novel low-overhead estimation and feedback mechanism which solves the instability problem of static MAC solutions for the setting of dynamic users.
- We first propose a feedback and activation mechanism under perfect state knowledge for any given MAC strategy that complies with a threshold-based success model, and characterize its maximum stabilizable rate of traffic (Theorem 1). This maximum rate is asymptotically-optimal in fully utilizing the threshold-based success model as the frame length grows. Then, we consider the case of unknown network state to propose an estimation and feedback mechanism. Despite its need to estimate the unknown and dynamic network state of the distributed system, we establish (in Theorem 2) that our design achieves the same stability region as that of the idealized design with perfect state knowledge.
- By investigating particular success/failure characteristics of the SIC operation, we also extend (in Section V-C) our design for the threshold-based success model to the SIC case. Using a simple feedback strategy, this design closes the loop by utilizing the principles and operations developed under the approximate model to be applied successfully to the SIC setting. We validate our designs by performing extensive numerical studies (in Section VI) to investigate the their stability and delay characteristics.

We note that our focus in this work is on achieving network stability under dynamic users under the preferred MAC strategy. While we take SIC as an instance of a spectrally-efficient strategy, our approach can be extended to other MAC solutions with possibly different spectral-efficiency characteristics.

A related interesting work, [17], developed a policy which uses the time elapsed and the number of repeated transmissions to control the transmission probability of users, and is agnostic to the number of users. Also, a recent work, [18], provided a probability adjustment strategy based on estimating the network state by assuming no new arrivals during the period when the estimation is performed. However, these works do not provide theoretical guarantees and exhibit degrading performance under highly loaded conditions. This differs from our approach in that we track the number of dynamic users and use simple feedback to offer provable good performance even in highly loaded conditions.

In the related domain of stabilizing policies for stochastic networks, there have been numerous advances over the last few decades (e.g., [19], [20], [21] to name a few). These and numerous other works (see [22], [23], [24] for more information) employ Lyapunov-drift methods for the design and analysis of provably good adaptive strategies. We employ some of these techniques in the analysis of our novel estimation and feedback mechanism in this paper.

Finally, our work also relates to the thread of works on network stability under dynamic flows, also called *flow-level dynamics* (e.g. [25], [26], [27] just to list a few). However, our work is orthogonal to this thread in that we focus on distributed design for efficient random access, where the feedback and the state estimation components become critical.

II. System Model

A. Network Dynamics

Dynamic User Arrival Model: We consider the operation of a distributed spectrum access system that evolves over discrete-time, whereby dynamic users arrive with packets and exit after successful transmission (see Fig.1). In each time slot $\tau = 1, 2, 3, \ldots$, a random number $R(\tau)$ of new users arrives at the system with content to transmit to a base station over a common (possibly multi-channel) communication environment. For simplicity, we assume that each user has a single packet that it wants to transmit. Hence, in the sequel, we will refer to user and packet interchangeably. We define the indices of users arriving in slot τ as $\mathcal{R}(\tau)$, and assume that the number of users $R(\tau) \triangleq |\mathcal{R}(\tau)|$ that arrive in each time slot are non-negative-valued independent and identically distributed random variables with mean λ and variance σ^2.

Distributed Multi-Channel Spectrum Access Model: All users in the system share $M \geq 1$ physical frequency bands. Also, we organize $K \geq 1$ successive time slots into one *transmission frame*. We distinguish these two time-scales by

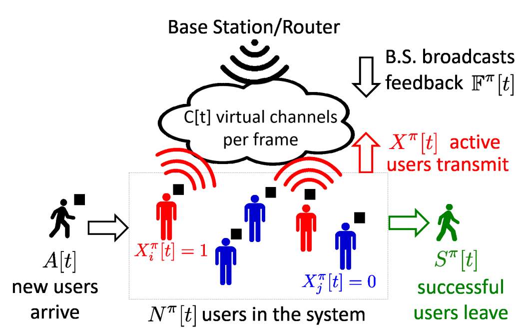

Figure 1. Network dynamics, and Base-Station-User interaction in frame t.

using: parentheses with variable $\tau \in \{1, 2, 3, \dots\}$ when referring to slots; and brackets with variable $t \in \{1, 2, 3, \dots\}$ when referring to frames. Then, we can write the cumulative arrivals in frame t as $A[t] = \sum_{\tau=(t-1)K+1}^{tK} R(\tau)$.

With M frequency bands per time slot, and K time slots per frame, we have a total of $C = MK$ (virtual) *channels* in each frame (see Fig. 2). We assume that each channel can successfully transfer one packet if it does not have any interference, or if the interference can be successfully cancelled (to be further discussed in Section II-B).

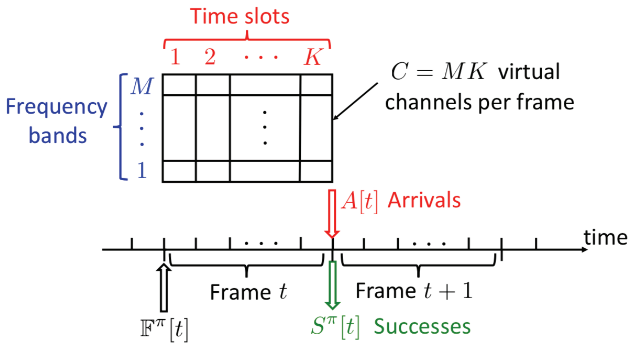

Figure 2. Frame structure, virtual frequency-time channels, timing of signals.

Components of Spectrum Access Policy: At the beginning of each frame each user in the system has to distributedly decide: whether to be active (transmit); and how to encode its content over the C channels using the common feedback broadcast from the base station. At the end of each frame, once active users complete the transmission of their packets, the base station has to decode all decodable packets and broadcast a common feedback to the present users. Accordingly, a *spectrum access policy*, denoted π, is defined by: (i) the common feedback to be transmitted by the base station at the end of each frame; (ii) the activation strategy for the users at the beginning of each frame in response to the last feedback; and (iii) the distributed encoding (performed by the active users) and decoding (performed at the base station) strategies to be employed in the transmission frame. In the sequel, we will use superscript $\cdot^\pi$ to denote the dependence of a variable on a specific policy, π.[1]

Leaving further details of the policy space description to Section II-B, we use $S_i^\pi[t] \in \{0, 1\}$ to indicate the success of user i's potential transmission in frame t under Policy π: $S_i^\pi[t] = 1$ when the base station can successfully decode user i's packet, and $S_i^\pi[t] = 0$ when either the base station fails to decode user i's packet or user i does not transmit.

Network State Dynamics: For a given Policy π, let $\mathcal{N}^\pi[t]$ be the set of all users that are in the system at the beginning of frame t. We define the *network state* $N^\pi[t] \triangleq |\mathcal{N}^\pi[t]|$ as the number of users in the system at the beginning of the frame t. Noting (as depicted in Fig. 1) that new users arrive according to the process $\{A[t]\}_t$ and an existing user $i \in \mathcal{N}^\pi[t]$ leaves only if its packet is successfully decoded, i.e., $S_i^\pi[t] = 1$, the network state evolution (over frames) under Policy π is:

$$N^\pi[t+1] = N^\pi[t] + A[t] - S^\pi[t], \qquad t \geq 0, \qquad (1)$$

where $S^\pi[t] \triangleq \sum_{i \in \mathcal{N}^\pi[t]} S_i^\pi[t]$ gives the total number of users that leave the system at the end of frame t. We say that a policy, say π, stabilizes the system if

$$\limsup_{T \to \infty} \tfrac{1}{T} \textstyle\sum_{t=1}^{T} \mathbb{E}[N^\pi[t]] < \infty.$$

In this paper, our goal is to design stabilizing policies with low-overhead feedback structures that provide high throughput guarantees for the distributed and dynamic spectrum access setting described above. Next, we describe the space of policies over which we will pursue this goal.

B. Distributed MAC Strategy Space Π

As briefly introduced above, a distributed spectrum access Policy π in the feasible policy space Π is described by a triplet: (i) the common feedback $\mathbb{F}^\pi$; (ii) the activation strategy $\mathbf{X}^\pi$; and (iii) the encoding/decoding strategy governing the service process S^π. Next, we clarify these components.

(i) Common Feedback $\mathbb{F}^\pi$: At the end of each frame t, the base station broadcasts a common feedback $\mathbb{F}^\pi[t+1]$ to all users in the system to guide their decision in frame $t+1$. Although the exact nature of this common feedback depends on the Policy π, all common feedback design must fulfill two goals: (a) users that made transmissions in frame t can infer their success/failure status from the feedback and leave the system if successful; and (b) the remaining users (including the newcomers) can use the feedback to decide their actions in the next frame, $t+1$.

(ii) User Activation Strategy $\mathbf{X}^\pi = (X_i^\pi)_i$: At the beginning of frame $t+1$, each user i in the system uses the common feedback $\mathbb{F}^\pi[t+1]$ to decide its *activation state*, $X_i^\pi[t+1] \in \{0, 1\}$, as determined by the activation policy we will design. In particular, $X_i^\pi[t+1] = 1$ indicates that the user will be *active* and make a transmission attempt in frame $t+1$ over one or multiple of the C channels, while $X_i^\pi[t+1] = 0$ indicates that it will be silent during the frame.

Due to the distributed and homogeneous dynamics of the users, we focus on the set of activation strategies whereby each user i makes its activation decision independent of other users, based on: the last common feedback it has received; and possibly a random number it generates locally. Thus, at the beginning of frame $t+1$, user i independently decides its activation such that $P(X_i^\pi[t] = 1)$ is a function of $\mathbb{F}^\pi[t]$.

(iii) Multi-Channel Success Model governing S^π: Given the activation vector $\mathbf{X}^\pi$, the next step in the description of the Policy π is the success model for the $C = MK$ channels, over which the active users will attempt transmissions. Clearly, diverse MAC and coding schemes may be employed for such distributed uplink transmission. These can range from the simplest collision-based Aloha-like success/failure dynamics to more sophisticated space-time encoding/decoding methods.

While the performance of these schemes may vary, we aim to consider a common success model that can capture their

[1] Occasionally, we will drop this superscript to simplify the notation when the policy we refer to is clear from the context.

basic characteristics while also providing a tractable model. Accordingly, many multi-channel uplink MAC schemes will exhibit an activation-success performance that on average grows linearly with increasing number of active users until the number becomes too high for the strategy to support them. After this critical load level, which we denote as $\eta(C)$ for C channels, the channels are overloaded and the success rate drops sharply. Capturing these basic characteristics, we consider a threshold-based success model, whereby the number of successfully decoded users in frame t is given by

$$S^{\pi}[t] = \begin{cases} X^{\pi}[t], & X^{\pi}[t] \leqslant \eta(C) \\ 0, & X^{\pi}[t] > \eta(C) \end{cases},$$

whereby $\eta(C) \in \{0, 1, \ldots, C\}$ is a MAC-strategy-dependent parameter that measures the efficiency of the strategy. Accordingly, the success status of user $i \in \mathbb{N}^{\pi}[t]$ is given by

$$S_i^{\pi}[t] = \mathbb{1}(X^{\pi}[t] \leq \eta(C))\, X_i^{\pi}[t].$$

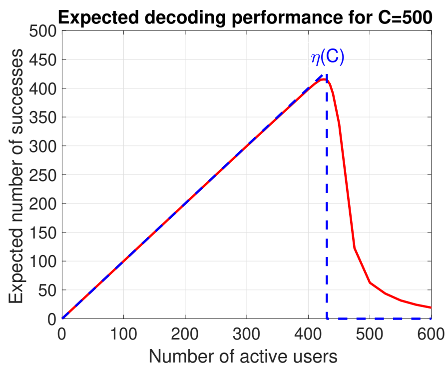

Figure 3. Average decoding performance of the SIC scheme (solid) for $C = 500$ and the threshold-based approximation model (dashed) with increasing number of active users implementing soliton-based encoding.

Figure 3 demonstrates the validity of this threshold-based success model for the case of the *Successive Interference Cancellation (SIC)* strategy (see [16]). Leaving the details of the encoding/decoding process of the SIC strategy to Section V, the figure shows how the threshold-based success model (dashed) closely follows the simulated success performance of SIC (solid) as the number of active users increase. We note that SIC is of particular interest due to its distributed and low-complexity encoding/decoding procedures. Moreover, SIC is known to be asymptotically efficient in that the fraction of successful transmissions per channel, $\eta(C)/C$, converges[2] to its maximum possible level of 1 as $C \to \infty$ (see [15], [16]). After developing our estimation and activation policies under the threshold-based success model in Sections III and IV, we will return the specific case of SIC strategy in Section V to show how they can be utilized under that strategy.

III. Distributed MAC with Perfect Network State

In this section, we design and analyze a provably good distributed-MAC strategy within the space Π (defined in Section II-B) for the threshold-based success model under the assumption of *perfect network state information* $N[t]$ under dynamic arrivals. While such perfect knowledge is impossible to achieve in the distributed system, this design is an important step in developing a provably efficient policy without perfect knowledge in Section IV. Moreover, the insights and mechanisms of these designs will also facilitate the design of similar efficient schemes that can be employed in the SIC model.

A. Policy Design: $\tilde{\pi}$

Algorithm $\tilde{\pi}$ specifies our distributed MAC strategy in the space Π (so $\tilde{\pi} \in \Pi$). The algorithm describes how the base station and each distributed user must act in each frame.[3]

Algorithm $\tilde{\pi}$: Spectrum-Access Policy with Perfect $N[t]$

Input: Network state $N[t]$ at each frame $t \in \{1, 2, \ldots\}$
Initialize: $\mathbb{F}[1] = (Z[0] = 0, N[1])$
repeat For $t = 1, 2, \ldots$
  Each user i**:** At the *beginning* of frame t **do**:
  if $i \in \mathbb{N}[t-1], X_i[t-1] = 1$, and $Z[t-1] = 1$ **then**
    $S_i[t-1] = 1$ and user i leaves the system
  end if
  if $i \in \mathbb{N}[t]$ **then**
    Set

$$X_i[t] = \begin{cases} 1, & \text{w.p. } \min\{\frac{r^*}{N[t]}, 1\} \\ 0, & \text{otherwise,} \end{cases}$$

    where

$$r^* \triangleq \arg\max_{r \geq 0} \mathbb{E}\left[Y(r)\mathbb{1}\left(Y \leq \eta(C)\right)\right], \qquad (2)$$

    and $Y(r)$ is a Poisson random variable with mean r
  end if
  if $X_i[t] = 1$ **then**
    User i performs SIC-based encoding in frame t
  end if
  Base station: At the *end* of frame t **do**:
  Set $Z[t] = \mathbb{1}(X[t] \leq \eta(C))$, where $X[t] = \sum_{i \in \mathbb{N}[t]} X_i[t]$
  Set $\mathbb{F}[t+1] = (Z[t], N[t+1])$
end repeat

Note that feedback $\mathbb{F}^{\tilde{\pi}}[t+1]$ of Policy $\tilde{\pi}$ for frame $t+1$ is composed of two components: the binary variable $Z^{\tilde{\pi}}[t] \in \{1, 0\}$, indicating whether the decoding operation at the end of frame t was successful or not; and the number of users in the system $N^{\tilde{\pi}}[t+1]$ at the beginning of frame $t+1$. Under the threshold-based success model, the binary feedback simply becomes $Z^{\tilde{\pi}}[t] = \mathbb{1}(X^{\tilde{\pi}}[t] \leq \eta(C))$. And, under the perfect network state information assumption, $N^{\tilde{\pi}}[t+1]$ is known perfectly by the base station.

At beginning of frame t, users who have made a transmission in the previous frame leave or stay (depending on the binary feedback). Those $N^{\tilde{\pi}}[t]$ users that are still in the system independently decide to become active with probability $\max\{r^*/N^{\tilde{\pi}}[t], 1\}$. Here, r^* is optimally selected according to

[2]As a side note, we find via numerical simulations that a close functional approximation to $\eta(C)$ for SIC under a Soliton-based transmission strategy can be given by $\eta(C) \approx C(1 - \frac{1}{e^{-0.29753 C^{0.33515}} + 1})$.

[3]The superscript $.^{\tilde{\pi}}$ is omitted to avoid clutter in the discussion.

an easily solvable maximization (2). This optimization is based on a Poisson approximation of the random user activations and aims to optimize spectral efficiency. Those users that decide to transmit then utilize the SIC-based encoding/decoding procedure described in Section II-B(iii).

B. Performance Analysis of Policy $\tilde{\pi}$

For each positive integer n, define $S^{\tilde{\pi}}(n)$ as the number of users successfully served in a frame under $\tilde{\pi}$ when $N[t] = n$. Define $r_n^* = \min\{r^*, n\}$ for all $n \in \{1, 2, 3, \ldots\}$, where r^* is defined in (2). Observe

$$P(S^{\tilde{\pi}}(n) = i) = \binom{n}{i} \left(\frac{r_n^*}{n}\right)^i \left(1 - \frac{r_n^*}{n}\right)^{n-i},$$

for all $i \in \{1, \ldots, \min\{n, \eta(C)\}\}$. This is a truncated $Binomial(n, r_n^*/n)$ distribution. Consequently, its limit is a truncated Poisson distribution from the Poisson approximation result of binomials. In particular, as $n \to \infty$, $S^{\tilde{\pi}}(n)$ converges in distribution to $Y(r^*)\mathbb{1}(Y(r^*) \leq \eta(C))$, where $Y(r^*)$ is a Poisson random variable with mean r^*. Accordingly, we define $\rho^*(C)$ as

$$\rho^*(C) = \lim_{n\to\infty} \mathbb{E}\left[S^{\tilde{\pi}}(n)\right] \quad (3)$$

$$= \mathbb{E}\left[Y(r^*)\mathbb{1}(Y(r^*) \leq \eta(C))\right] = \sum_{i=1}^{\eta(C)} i \frac{(r^*)^i}{i!} e^{-r^*}, \quad (4)$$

where (4) follows from the boundedness of the range $\{0, \ldots, \eta(C)\}$ of $S^{\tilde{\pi}}(n)$ and its convergence in distribution.

Theorem 1: Fix frame size K. Assume arrivals $\{R(\tau)\}_{\tau=0}^{\infty}$ are i.i.d. with mean λ and variance σ^2. Suppose $\lambda < \rho^*(C)/K$, where $C = MK$ and $\rho^*(C)$ is defined in (3). Then, Policy $\tilde{\pi}$ stabilizes the network. Moreover, the resulting average user delay satisfies (in units of slots):

$$\overline{W} \leq \frac{m(\rho^*(C) + K\lambda) + K\sigma^2 + K\lambda(\eta(C) - K\lambda)}{\lambda(\rho^*(C) - K\lambda)}, \quad (5)$$

where $m := \lceil(\rho^*(C) + K\lambda)/2\rceil \in \{0, \ldots, \lceil\rho^*(C)\rceil\}$.

Furthermore, noting that $\rho^*(C)/\eta(C) \to 1$, and $\eta(C)/C \to 1$ as $C \to \infty$ under the SIC scheme (see [15]), we get $\rho^*(C)/K \to M$ as $K \to \infty$. Since the network cannot be stabilized for any $\lambda > M$, this shows that the Policy $\tilde{\pi}$ fully utilizes the threshold-based model and is *asymptotically throughput-optimal* under the SIC scheme.

Due to limited space, the proof of this theorem is provided in [28]. In addition to providing a delay bound, Theorem 1 shows that near-perfect utilization of all M channels can be achieved (as we extend the frame duration $K \to \infty$). To appreciate this, consider a simpler scenario with exactly M users that never leave. We could assign each user to one of the M channels, each user could send one packet per slot with no contentions, and 100% throughput would be achieved. Theorem 1 shows the remarkable fact that similar throughput can also be achieved when users dynamically arrive and depart and make randomized transmission decisions. Since each user can only send one packet, there is limited time for learning or coordination. This is the strength of the SIC coding combined with the simple randomized MAC scheme (Policy $\tilde{\pi}$). Recall that Policy $\tilde{\pi}$ requires perfect state information $N[t]$, which is difficult to realize in practice. The next section shows that similar near-perfect throughput can be achieved without knowledge of $N[t]$.

IV. Distributed MAC with Estimated Network State

Our distributed MAC Policy $\tilde{\pi}$ in the previous section has desirable asymptotic optimality characteristics as well as large performance improvements in the non-asymptotic regime. However, it relies on a perfect network state $N[t]$ to be known at the base station. This section develops a new distributed-MAC policy that does not require knowledge of $N[t]$ but achieves the same throughput. The technique is inspired by an estimation method developed for basic slotted Aloha in [13].

A. Distributed-MAC Policy Design: $\hat{\pi}$

In Algorithm $\hat{\pi}$, we propose a new Policy $\hat{\pi} \in \Pi$, that uses an estimate $\hat{N}[t]$ of the (unknown) $N[t]$ value to guide the distributed spectrum access strategy.

Algorithm $\hat{\pi}$: Spectrum-Access Policy with Estimated $\hat{N}[t]$

Input: An estimate $\hat{\lambda}$ of $\lambda = \mathbb{E}[R(\tau)]$
Initialize: $\mathbb{F}[0] = (Z[0] = 0, \hat{N}[0] = r^*)$
repeat For $t = 1, 2, \ldots$
  Each user i: At the *beginning* of frame t **do**:
  if $i \in \mathcal{N}[t-1], X_i[t-1] = 1$, and $Z[t-1] = 1$ **then**
    $S_i[t-1] = 1$ and user i leaves the system
  end if
  if $i \in \mathcal{N}[t]$ **then**
    Set
$$X_i[t] = \begin{cases} 1, & \text{w.p. } \frac{r^*}{\hat{N}[t]} \\ 0, & \text{otherwise,} \end{cases}$$
    where r^* is defined in (2)
  end if
  if $X_i[t] = 1$ **then**
    User i performs SIC-based encoding in frame t
  end if
  Base station: At the *end* of frame t **do**:
  Set $Z[t] = \mathbb{1}(X[t] \leq \eta(C))$,
  where $X[t] = \sum_{i\in\mathcal{N}[t]} X_i[t]$
  if $Z[t] = 1$ **then**
    Set $\hat{N}[t] = \max\{\hat{N}[t-1] + K\hat{\lambda} - r^*, r^*\}$
  else if $Z[t] = 0$ **then**
    Set $\hat{N}[t] = \hat{N}[t-1] + K\hat{\lambda} + r^+$,
    where r^+ is the solution to
$$r^+ = \frac{\mathbb{E}\left[(r^* - Y(r^*))\mathbb{1}\left(Y(r^*) \leq \eta(C)\right)\right]}{\Pr\left(Y(r^*) > \eta(C)\right)} \quad (6)$$
  end if
  Set $\mathbb{F}[t] = (Z[t], \hat{N}[t])$
end repeat

Note that the feedback structure $\mathbb{F}^{\hat{\pi}}$ and the user activations $\mathbf{X}^{\hat{\pi}}$ of Policy $\hat{\pi}$ have the same form as that of Policy $\tilde{\pi}$, except for the difference that the exact network state $N^{\hat{\pi}}[t]$ is replaced with the estimated $\hat{N}^{\hat{\pi}}[t]$ in both the feedback content and the activation decision.

The key modification in the design is the tracking operations of the network state. The algorithm does not assume perfect knowledge of the arrival rate and therefore takes as input an estimated arrival rate $\hat{\lambda}$. We show later that this estimate can take a range of values, and can also be fixed to an upper-limit that will be given in Theorem 2. At the end of each frame, the network state estimate $\hat{N}^{\hat{\pi}}$ is incremented by the estimated arrival rate of $K\hat{\lambda}$, regardless of the decoding operation outcome. However, depending on the success/failure indicator $Z^{\hat{\pi}}[t]$ of the decoding operation, the estimate of $N^{\hat{\pi}}[t]$ is: decremented by r^* defined in (2) if $Z[t] = 1$, and incremented by r^+ defined in (6) if $Z[t] = 0$. Both of these parameters can be easily calculated for the given number of channels C. Their values are carefully selected to guarantee that the estimate $\hat{N}^{\hat{\pi}}[t]$ can track the true value $N^{\hat{\pi}}[t]$ under the system dynamics. An interesting feature of Policy $\hat{\pi}$ is that, despite its lack of knowledge of the system state $N[t]$, its feedback has the same low-overhead structure as Policy $\tilde{\pi}$.

B. Performance Analysis

In this section, we analyze the stability characteristics and prove asymptotic efficiency of our proposed Policy, $\hat{\pi}$.

Theorem 2: Assume arrivals $\{R(\tau)\}_{\tau=0}^{\infty}$ are i.i.d. with mean λ and variance σ^2. For any $\lambda \in [0, \rho^*(C)/K)$, the Policy $\hat{\pi}$ stabilizes the system for any choice of $\hat{\lambda} \in [\lambda, \rho^*(C)/K]$, where $\rho^*(C)$ is defined in (3), K is the number of slots in a frame, and $C = MK$ is the number of frequency-time channels in a frame. Consequently, as with Policy $\tilde{\pi}$, Policy $\hat{\pi}$ is *asymptotically throughput-optimal*.[4]

Due to limited space, we provide the proof in [28]. This theorem establishes that the stability region of the practical $\hat{\pi}$ Policy that works with an estimated arrival rate $\hat{\lambda}$ and estimated network state $\hat{N}[t]$ is the same as the stability region of Policy $\tilde{\pi}$ that requires knowledge of $N[t]$. It is also worth noting that the arrival rate estimate can be fixed to its upper limit $\rho^*(C)/K$ if there is no good prior knowledge of the true arrival rate. Of course, more accurate levels of $\hat{\lambda}$ can affect the delay performance, but as long as it is greater than λ, the Policy is guaranteed to stabilize the network.

V. Distributed MAC under the SIC Model

Until now we have studied the distributed MAC policies $\tilde{\pi}$ and $\hat{\pi}$ which we have specifically designed for the threshold-based success model. In this section, we build on the mechanisms of these designs to develop a new Policy, $\overline{\pi}$, that is adapted to operate efficiently under the SIC model (cf. Section V-C). We show via simulation (cf. Section VI) that our Policy $\overline{\pi}$ closely matches and can even outperform the

[4]We can also establish a delay bound on Policy $\hat{\pi}$, with asymptotic $\overline{W} \leq O(K/(\rho^*(C) - K\lambda))$ similar to (5), but the expression is more involved and we omit that due to space limitation (see final comment in proof).

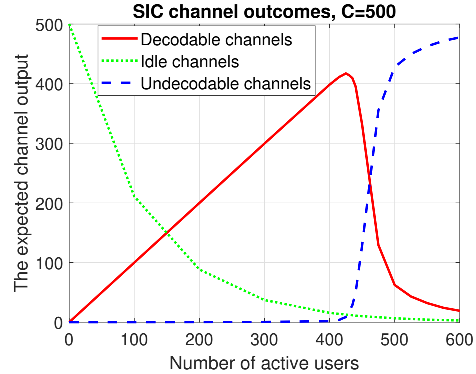

Figure 4. Success-Failure-Idleness characteristics of SIC with varying active users

policies we have developed based on the threshold-based success model.

A. Encoding/Decoding Process of the SIC Scheme

Under the SIC scheme, each *active* user i (with $X_i^{\overline{\pi}}[t] = 1$) distributedly selects any subset of the C channels to transmit the *same copy* of its packet in frame t. Then, at the receiver-end (base station), the SIC decoding algorithm works recursively as follows: at each stage all channels with a *single* packet are decoded, and their copies transmitted over the other channels[5] are cancelled. This process is repeated until no channels with a single packet are left in the frame. [15] shows that by letting each active user i choose the number of copies $d_i \in \{1, \ldots, C\}$ it will transmit according to the truncated soliton distribution, and then selecting d_i channels uniformly out of C, the number of successfully decoded packets can reach arbitrarily close to 1 packet per channel as C increases.

B. Success/Failure/Idle Characteristics of SIC

In the SIC model, we no longer observe the sharp drop in decodability when the number of active users $X[t]$ increases from $\eta(C)$ to $\eta(C) + 1$. This behaviour makes base station feedback more challenging since a simple collective success or failure paradigm no longer holds. Therefore, the base station needs to make a different decision based on other decoding metrics inherent to the SIC model. We depict several of these metrics available to the base station in Fig. 4: the number of decodable channels, undecodable channels, and idle channels. In particular, we observe that the optimal utilization of the C channels (the point corresponding to $\eta(C)$ in the threshold-based success model) occurs almost precisely at the point where the number of idle and undecodable channels meet. This is sensible, since many idle channels indicate under-utilization while many undecodable channels indicate over-congestion.

C. Distributed-MAC Policy Design: $\overline{\pi}$

Our proposed new Policy, $\overline{\pi}$, for the SIC model inherits the structure of our previous design $\hat{\pi}$ with two crucial differences: (i) the update condition at the base station, and (ii) the first

[5]Users add a small header to their packets declaring all the channels over which they are transmitting, which is negligible compared to packet size.

component of the feedback. Here, we point to these differences and omit the full definition, which remains the same.

(i) Regarding the update condition, instead of using the indicator $Z[t] = \mathbb{1}(X \leq \eta(C))$ that Policy $\hat{\pi}$ uses, Policy $\bar{\pi}$ uses $Z'[t] = \mathbb{1}(\#$ idle channels in frame $t >$ #undecodable channels in frame $t)$ to make the exact same updates to the estimate of the network state $\hat{N}[t]$. This is based on the observations from the previous subsection showing how the number of idle and decodable channels reveal the effectiveness of the SIC operation.

(ii) Regarding the feedback, Policy $\bar{\pi}$ uses $\mathbb{F}^{\bar{\pi}}[t] = (\mathbf{D}[t], \hat{N}[t])$, where $\mathbf{D}[t] \in \{0,1\}^C$ is a C-bit signal with the i^{th} dimension indicating whether the content in the corresponding i^{th} virtual channel was successfully decoded at the end of frame $t-1$. This fixed and low overhead signal allows active users in frame $t-1$ to know whether their transmission has been successful or not. This is different from the one-bit feedback $Z[t]$ under Policy $\hat{\pi}$ since users do not succeed or fail all together under the SIC model.

VI. Simulation Results

We have performed extensive numerical simulations under the three policies $\tilde{\pi}$, $\hat{\pi}$, and $\bar{\pi}$. In this section, we present some of these results, both to validate the theoretical claims, and to investigate their throughput and delay characteristics. In all these simulations, we have taken the per-slot arrival process $R(\tau)$ to be Poisson distributed with mean λ.

We start with investigating the maximum stabilizable throughput of the three policies in Figures 5 and 6.

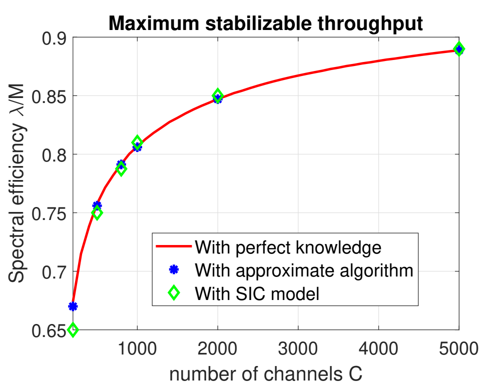

Figure 5. Maximum stabilizable throughput performances with increasing C.

In Fig. 5, we plot the theoretical limit $\rho^*(C)$ of Policy $\tilde{\pi}$ in a solid line, and samples of the maximum throughputs of Policies $\hat{\pi}$ and $\bar{\pi}$ for different values of C. The maximum stability levels of these policies are visually indistinguishable, supporting the efficiency of all these designs. We also see that all the throughputs increase towards the maximum limit of 1 packet/channel, as predicted by our Theorem 1.

Fig. 6 digs deeper in the previous figure to plot the maximum throughput ratios of the two policies, $\hat{\pi}$ and $\bar{\pi}$, without state knowledge to the benchmark Policy $\tilde{\pi}$ with perfect state knowledge. First, the plot confirms that the throughput performance of $\hat{\pi}$ matches that of $\tilde{\pi}$, as predicted by Theorem 2. We see that the performance of Policy $\bar{\pi}$

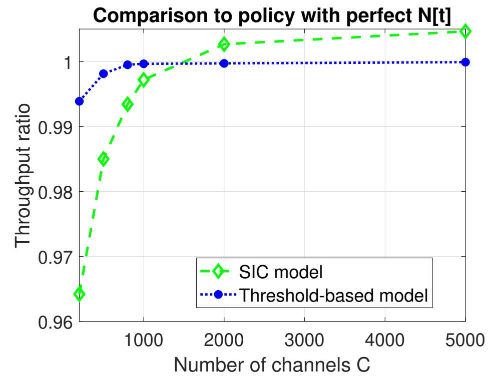

Figure 6. Ratio of maximum throughputs of $\bar{\pi}, \hat{\pi}$ to $\tilde{\pi}$ with increasing C.

under the SIC model performs closely to $\tilde{\pi}$ over all C. More interestingly, we also see that as the number of channels C increase, Policy $\bar{\pi}$ can even outperform the benchmark, despite its lack of perfect state information. This is due to the fact that the approximation to the SIC performance (see Fig. 3) is typically an underestimate of the capacity of SIC encoding/decoding process. Accordingly, this extra capacity in the SIC implementation can allow Policy $\bar{\pi}$ to outperform the design $\tilde{\pi}$ under the threshold-based success model with perfect state knowledge.

In Figures 7 and 8, we move on from throughput to average delay performances of the proposed policies. In Fig. 7, we take

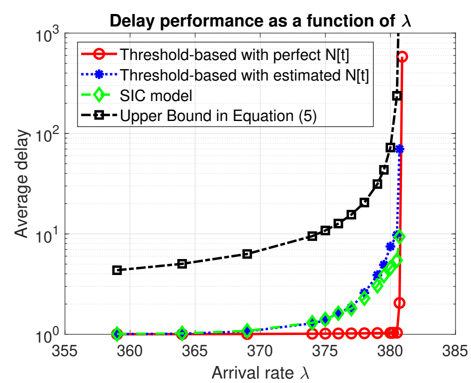

Figure 7. Average delay performance of policies for increasing $\lambda \leq \rho^*(C) \approx 380.7$ with $C = 500$ (with $M = 500, K = 1$).

the case of $C = KM = 500$ channels, where $\rho^*(C) \approx 380.7$, and plot the average delays under all policies, as well as the delay upper bound in (5), for increasing $\lambda < \rho^*(C)$. The plot confirms that all three policies are stabilizing for the same range of arrival rates, as predicted before. However, it also shows the additional delay cost that Policy $\hat{\pi}$ must endure with respect to Policy $\tilde{\pi}$ to pay for its lack of perfect state information. We also see that Policy $\bar{\pi}$ yields very close delay performance to $\hat{\pi}$ that may go above or below its counterpart depending on its efficiency level (discussed in Fig. 6).

In Fig. 8, we turn to another interesting aspect of delay performances, where the focus is on selecting the frame duration K parameter to minimize the delay performance for a given arrival rate λ. We fix $M = 10$ and $\lambda = 7.58$ per slot and plot the average delay performance as K varies. Several interesting observations emerge from this: (i) if K is not large

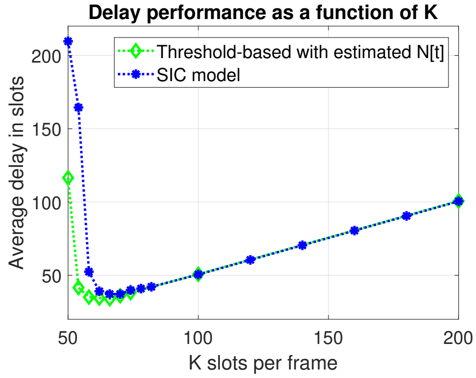

Figure 8. Average delays for varying frame durations K, and fixed $\lambda = 7.58, M = 10$.

enough, below 50 in the plot, the throughput level $\rho^*(C)$, with $C = MK$, of the SIC channel is not high enough to stabilize the arrival rate λ; (ii) after K exceeds this stabilizing level of 50, we see that the delay performance *decreases* for a while for both Policies $\hat{\pi}$ and $\bar{\pi}$, then starts to increase. This happens because initially increasing K increases $\rho^*(C)$ with small additional cost in waiting for the frame to complete. However, as K increases the delay from the increased duration of the frame starts to dominate the marginal gains received from the increase in $\rho^*(C)$. This reveals that finding the best choice of frame duration as a function of the arrival rate λ can yield non-negligible delay reductions while maintaining stability guarantees.

VII. Conclusions

In this paper, we addressed the efficient design of distributed MAC strategies for future wireless networks that will serve increasingly dynamic users with minimal coordination and low-complexity. Recognizing the instability of MAC strategies designed for static user populations, we developed distributed MAC strategies for dynamic users with a low-overhead estimation and feedback mechanism. Specifically, we considered a threshold-based success model that can capture essential characteristics of many spectrally-efficient multi-channel uplink MAC schemes. Under that model, we characterized the maximum supportable throughput of our design in terms of the system parameters and arrival rates, which fully utilizes the threshold-based success model as the frame length grows. Considering the spectrally-efficiency Successive-Interference-Cancellation (SIC) mechanism as a particular protocol, we also extended our design to incorporate the characteristics of the SIC operation. We also provided a delay analysis and various simulation results that confirm their efficiency and further reveal interesting delay characteristics.

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