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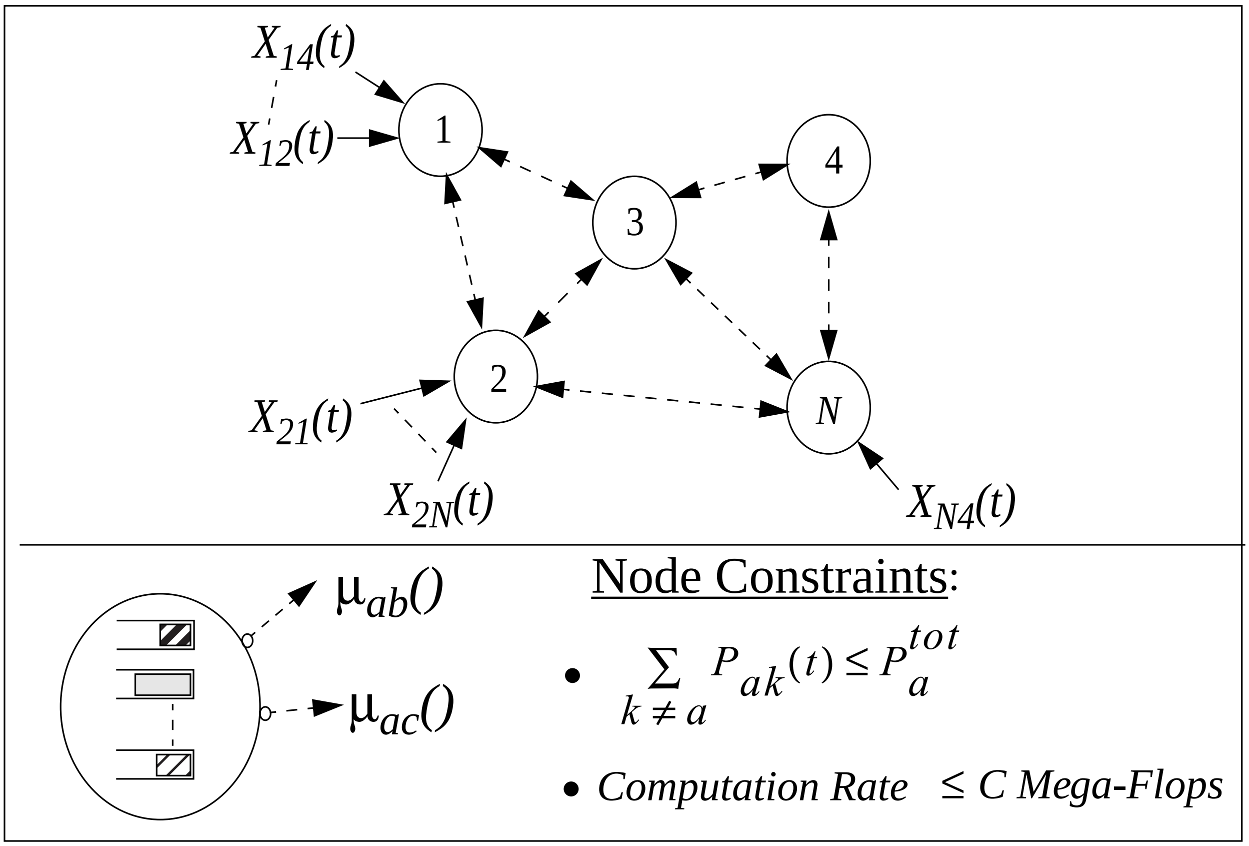
Delay and Complexity Tradeoffs for Dynamic Routing and Power Allocation in a Wireless Network

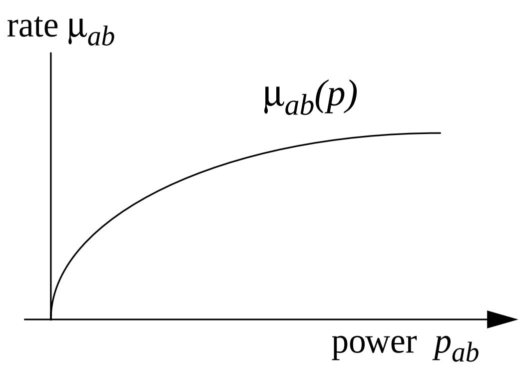


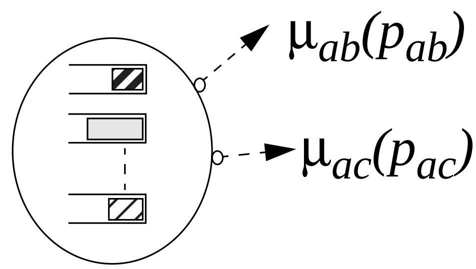
MIT -- Laboratory for Information and Decision Systems (LIDS)

Michael J. Neely
Jun Sun
Eytan Modiano

{mjneely@mit.edu, junsun@mit.edu, modiano@mit.edu}

Network Model:



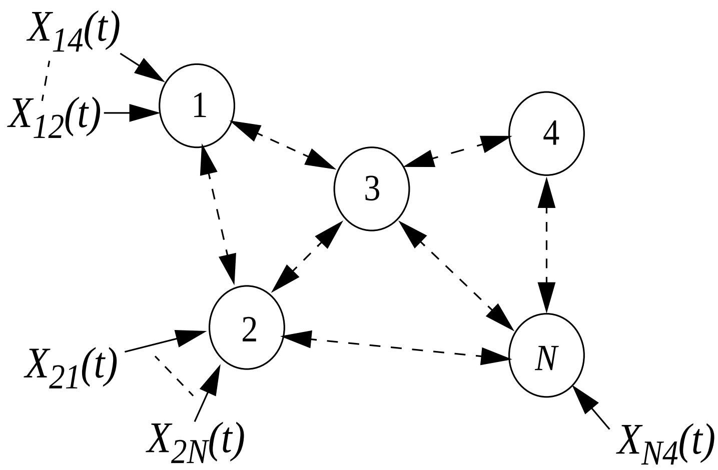


Problems with the model:

- No Interference Effects: $\mu_{ab} = \mu_{ab}(p_{ab})$
- No Time Variation
- Fluid Model of data flow

What does the model capture?

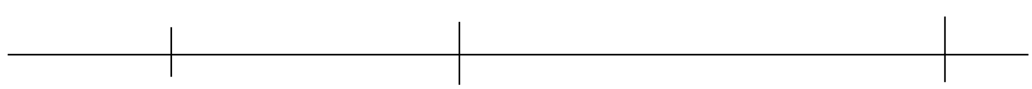
- Nonlinear Power Allocation Problem
- Complexity of scheduling optimal strategy

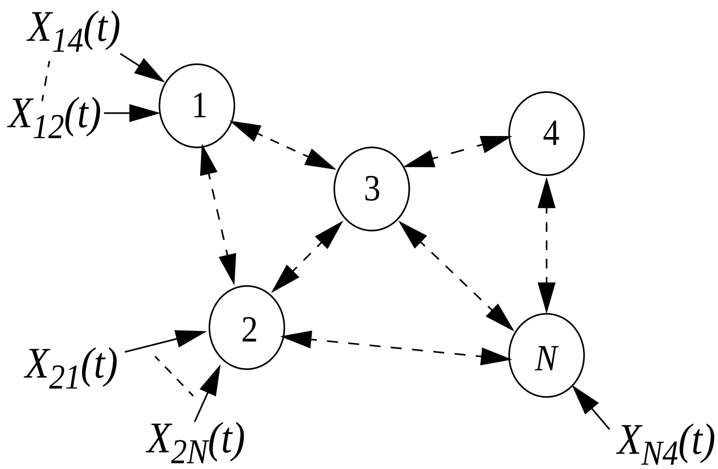


2 ideas of this paper:

1. Capacity (100% thru-put) strategy obtained by iteratively solving a min-clearance time problem.

2. Complexity/Delay tradeoff by solving the min clearance problem over longer time intervals.





Min Clearance Problem:
No arrivals. Have backlog at time 0.

U_{ij} = Unfinished bits in node i (to be delivered to node j).

Find routing and power controls $p_{ij}(t)$ to clear in min time.

<u>Observation</u>: Optimal control can be restricted to constant power allocation strategies.
<u>Proof sketch</u>:

Given optimal $p_{ij}(t)$ (clears in minimum time T).
Let $\bar{p}_{ij}$ represent the empirical avg. during [0, T].

$$\frac{1}{T}\int_0^T \mu_{ij}(p_{ij}(\tau))d\tau \leq \mu_{ij}(\bar{p}_{ij}) \qquad \square$$

(by concavity of $\mu()$ and Jensen's inequality)

From this, it is straightforward to form the min clearance time solution as a convex optimization problem:

Problem π_{min}

Maximize γ

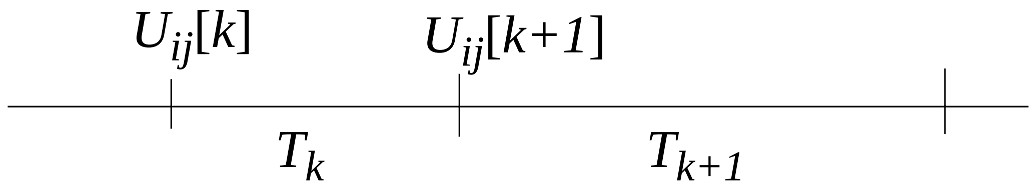
Subject to: $f_{ij}^{(c)} \geq 0$

$$\sum_{a=1}^{N} f_{ai}^{(c)} - \sum_{b=1}^{N} f_{ib}^{(c)} = -\gamma U_{ic} + \delta_{i-c} \sum_{j=1}^{N} \gamma U_{jc}$$

$$\sum_{c=1}^{N} f_{ij}^{(c)} \leq \mu_{ij}(\bar{p}_{ij})$$

$$\sum_{j=1}^{N} \bar{p}_{ij} \leq P_i^{tot}$$

Dynamic Scheduling Using Iterative solution of π_{min}:



Iterative Minimum Emptying Time algorithm (IMET):

1. If the system is empty, wait for new data to enter.

2. Start iteration k by observing the current backlog $U_{ij}[k]$, and solve π_{min} for this backlog, clearing it in time T_k. Hold routing and scheduling fixed for duration T_k.

3. Repeat for iteration k+1.

Let:
Λ = set of data rates (λ_{ij}) the network can stably support.

Can be shown that Λ is the set of all rates λ_{ij} such that there exists a constant power allocation p_{ij}^* for which a multi-commodity flow can be set up over the network (with link capacities $\mu_{ij}(p_{ij}^*)$) that satisfies the λ_{ij} rates.

<u>Traffic Assumptions</u> -- Time varying leaky bucket:

$X_{ij}(t)$ = Bits arrived to node i destined for j during $[0, t]$.

$$X_{ij}(t+T) - X_{ij}(t) \leq \sigma + \int_t^{t+T} \lambda_{ij}(\tau)\, d\tau$$

where $(\lambda_{ij}(t) + \varepsilon) \in \Lambda$ for all t

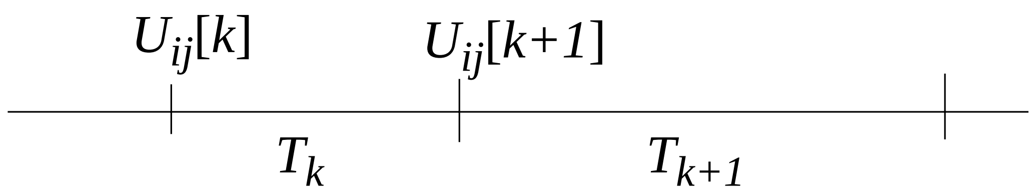
$\lambda_{ij}(t)$ = instantaneous data rate of $X_{ij}(t)$ stream
σ = traffic burst parameter
ε = distance the instantaneous data rate is from the boundary of the capacity region

These above parameters are unknown to the network controller.

Theorem: The IMET Algorithm guarantees:

$$T_{worst-case} \leq 2\sigma/\varepsilon$$

Proof:



Let λ_{ij} represent the rate of traffic during interval T_k. By assumption, there is a $\lambda_{ij}^* \in \Lambda$ such that $\lambda_{ij} + \varepsilon \leq \lambda_{ij}^*$.

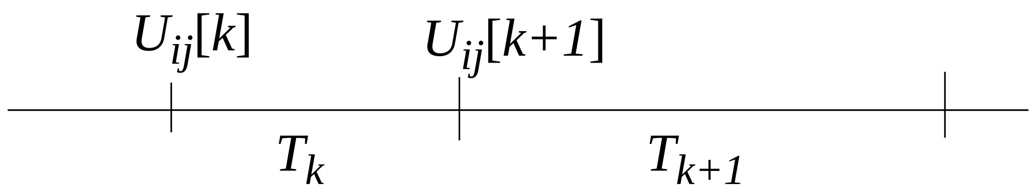
$$T_{k+1} = \text{min time to clear}$$

$$\leq \max_{(i,j)} \frac{U_{ij}}{\lambda_{ij}^*}$$

$$\leq \max_{(i,j)} \frac{\sigma + \lambda_{ij} T_k}{\lambda_{ij}^*}$$

$$\leq \max_{(i,j)} \frac{\sigma + (\lambda_{ij}^* - \varepsilon) T_k}{\lambda_{ij}^*}$$

$$\leq \frac{\sigma}{\varepsilon} \qquad \square$$



Complexity Constraint:

The IMET algorithm requires the solution to a convex optimization to be computed instantaneously at the beginning of a slot.

Idea: Compute solution of $U_{ij}[k]$ problem during T_{k+1}.

Computational Processing Speed Constraint:
C = Processing Rate (floating point ops / second)

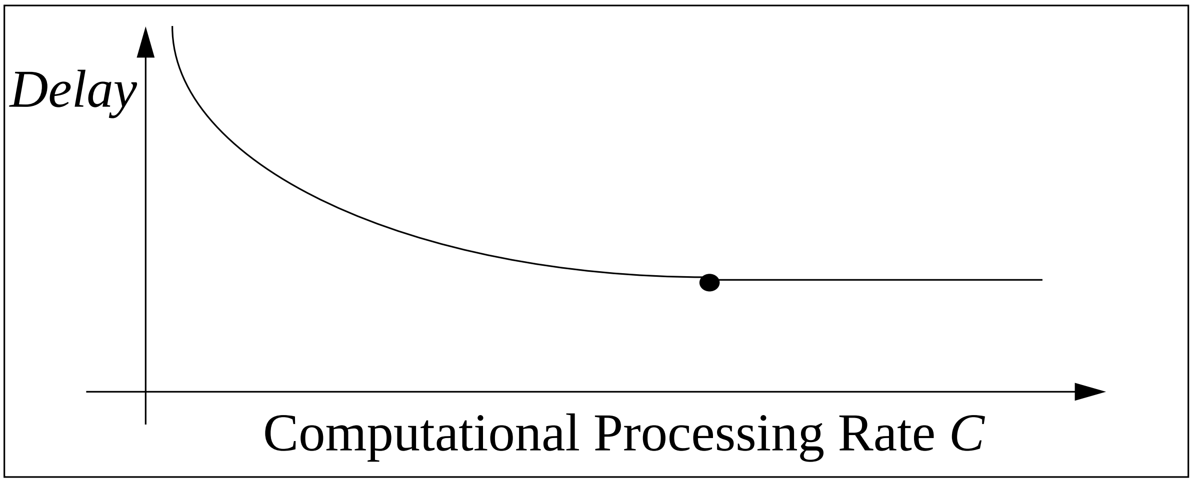
Let a_N = # operations required to compute the solution of the convex optimization for a net. of size N.

Modified IMET:
-Shift computations by one interval T_k.
-Hold solutions fixed for $max\{emptying\ time,\ a_N/C\}$

Theorem (for modified IMET):

$$T_{worst-case} \le 3max\left[\frac{\sigma}{\varepsilon}, \frac{a_N}{C}\right]$$

(compared to original IMET bound of $2\sigma/\varepsilon$).



<u>Conclusions</u>:

-Iterative Min Emptying Time algorithm IMET

-Acts without knowledge of rate or burst parameters ($\lambda_{ij}(t)$), ε

-100 % thruput, Worst Case Delay Bound

Future Work... Time varying systems
Fairness issues