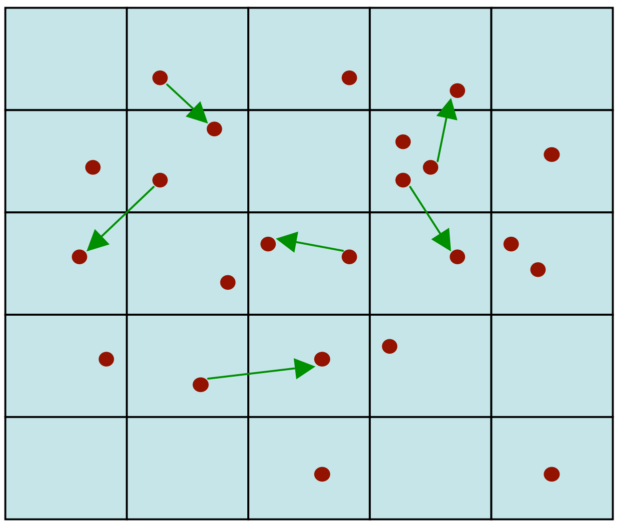
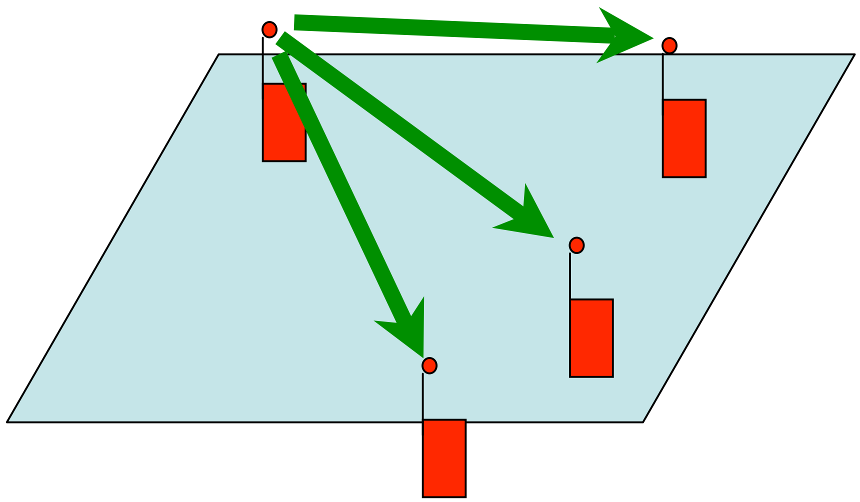
Energy Optimal Control for

Time Varying Wireless Networks





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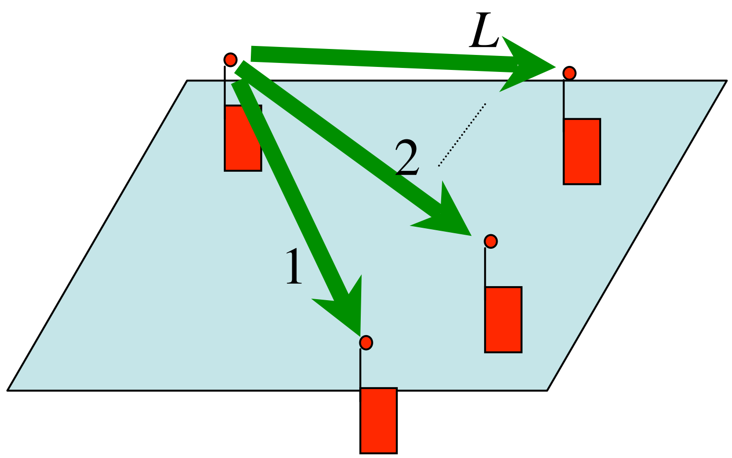
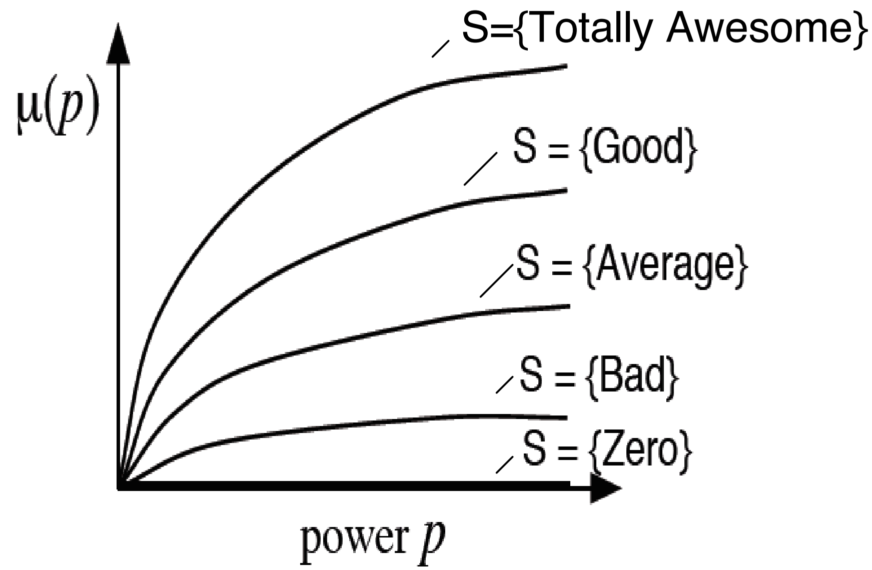
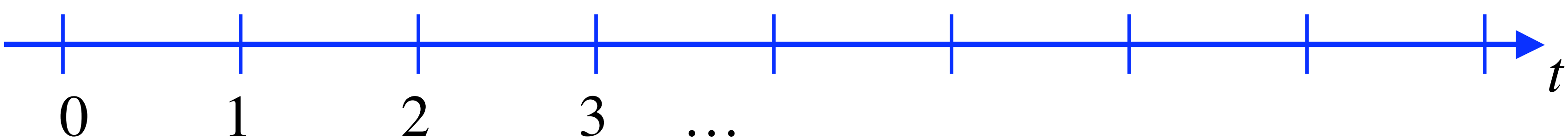
Part 1: A single wireless downlink (*L* links)





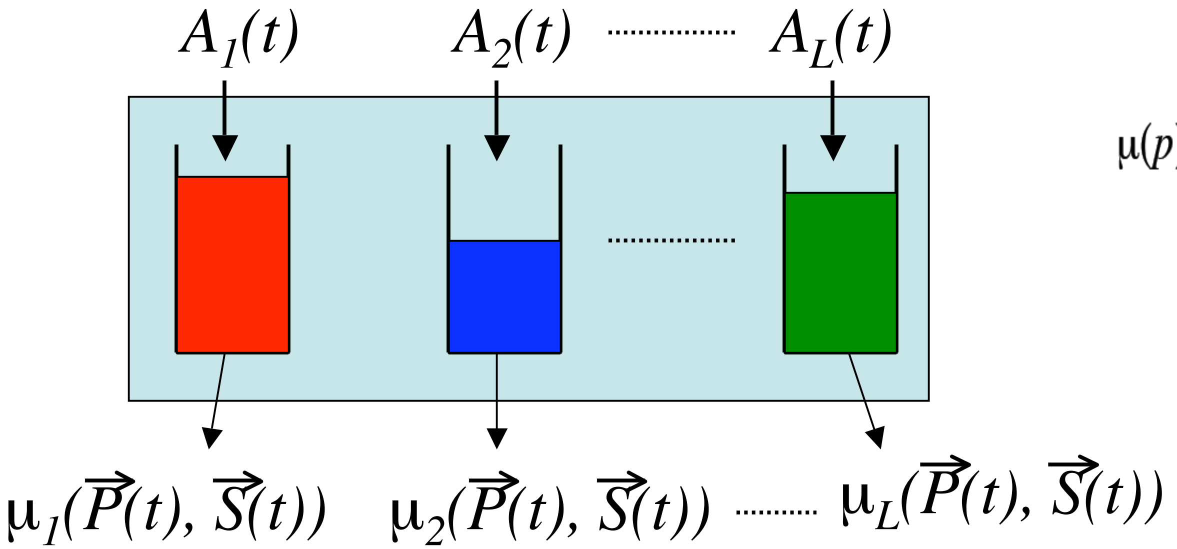
Slotted time $t = 0, 1, 2, \ldots$

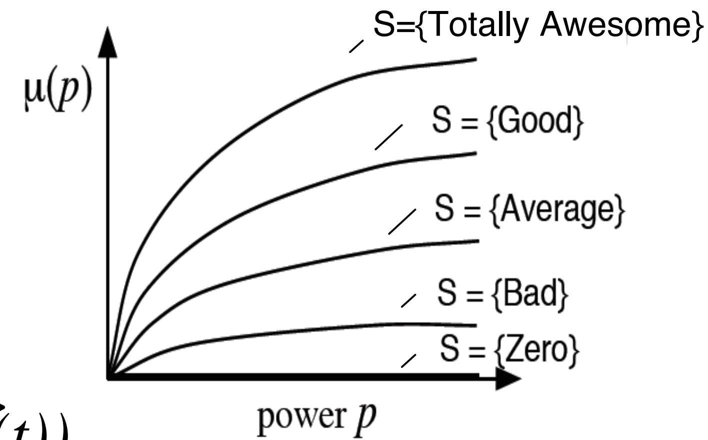


Power Vector: $\vec{P}(t) = (P_1(t), P_2(t), \ldots, P_L(t))$

Channel States: $\vec{S}(t) = (S_1(t), S_2(t), \ldots, S_L(t))$ (i.i.d. over slots)

Rate-Power Function: $\vec{\mu}(\vec{P}(t), \vec{S}(t))$ (where $\vec{P}(t) \in \Pi$ for all t)





Random arrivals : $A_i(t)$ = arrivals to queue i on slot t *(bits)*

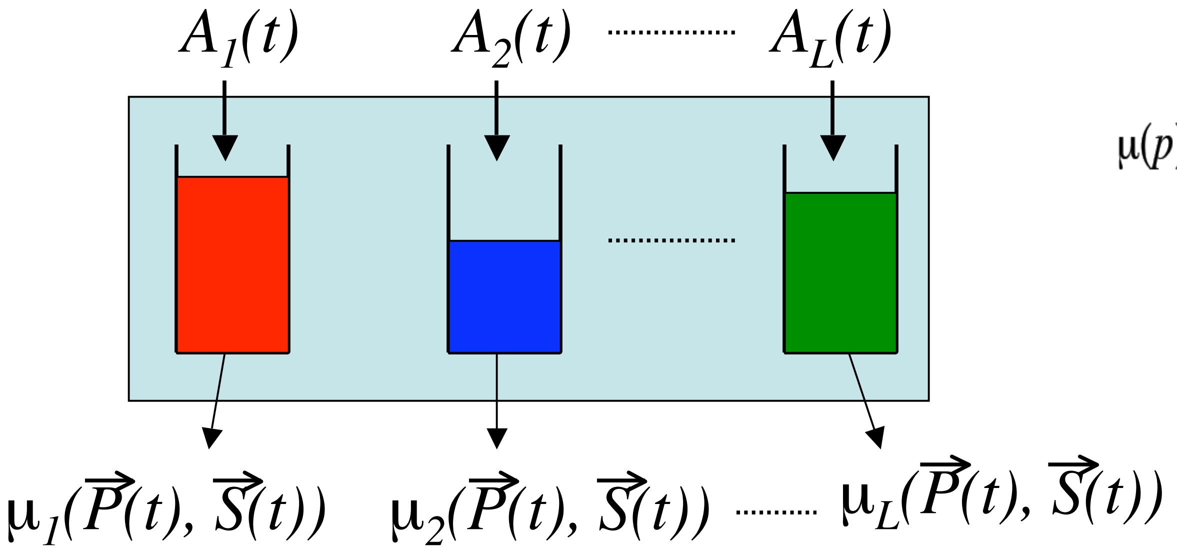
Queue backlog : $U_i(t)$ = backlog in queue i at slot t *(bits)*

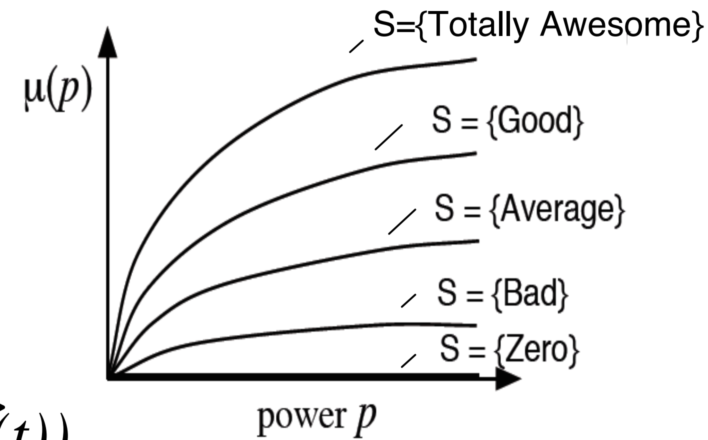
Arrivals and channel states i.i.d. over slots (unknown statistics)

Arrival rate: $E[A_i(t)] = \lambda_i$ (bits/slot), i.i.d. over slots

Rate vector: $\vec{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_L)$ (potentially unknown)

Allocate power in reaction to queue backlog + current channel state…





Random arrivals : $A_i(t)$ = arrivals to queue i on slot t *(bits)*

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Two formulations: (both have peak power constraint: $\vec{P}(t) \in \Pi$)

1. Maximize thruput w/ avg. power constraint: $\mathbb{E}\left\{\sum_{i=1}^{L} P_i\right\} \leq P_{av}$

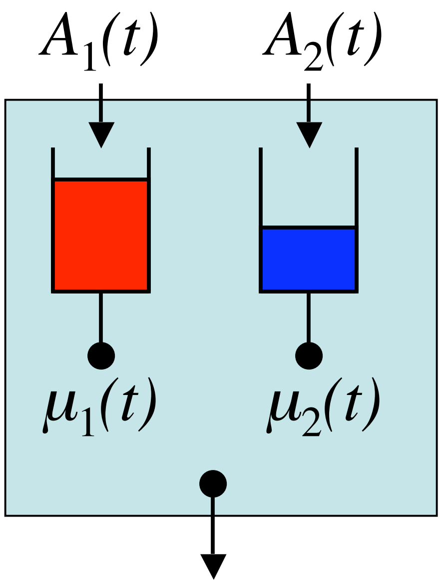
2. Stabilize with minimum average power (will do this for multihop)

Some precedents:

Energy optimal scheduling with known statistics:
- -Li, Goldsmith, IT 2001 [no queueing]
- -Fu, Modiano, Infocom 2003 [single queue]
- -Yeh, Cohen, ISIT 2003 [downlink]
- -Liu, Chong, Shroff, Comp. Nets. 2003 [no queueing, known stats or unknown stats approx]

Stable queueing w/ Lyapunov Drift: $\boxed{\text{MWM -- max } \mu_i U_i \text{ policy}}$
- -Tassiulas, Ephremides, Aut. Contr. 1992 [multi-hop network]
- -Tassiulas, Ephremedes, IT 1993 [random connectivity]
- -Andrews et. Al. , Comm. Mag. 2001 [server selection]
- -Neely, Modiano, TON 2003, JSAC 2005 [power alloc. + routing]

(these consider stability but not avg. energy optimality…)



	t	0	1	2	3	4	5	6	7	8
Arrivals	$A_1(t)$	3	0	3	0	0	1	0	1	0
	$A_2(t)$	2	0	1	0	1	1	0	0	0
Channels	$S_1(t)$	G	G	M	M	G	G	M	M	G
	$S_2(t)$	M	M	B	M	B	M	B	G	B
Max $U_i \mu_i$ Policy	$U_1(t)$	0	$\boxed{3}$	0	$\boxed{3}$	$\boxed{1}$	0	1	1	$\boxed{2}$
	$U_2(t)$	0	2	$\boxed{2}$	2	2	$\boxed{3}$	$\boxed{2}$	$\boxed{1}$	0
Better Choices	$U_1(t)$	0	3	3	6	$\boxed{6}$	$\boxed{3}$	1	1	$\boxed{2}$
	$U_2(t)$	0	2	2	$\boxed{3}$	1	2	3	$\boxed{3}$	0

<u>Example</u>: Can either be idle, or allocate 1 Watt to a single queue.

$$\vec{P}(t) = (P_1(t), P_2(t)) \in \Pi = \{(0,0), (1,0), (0,1)\}$$

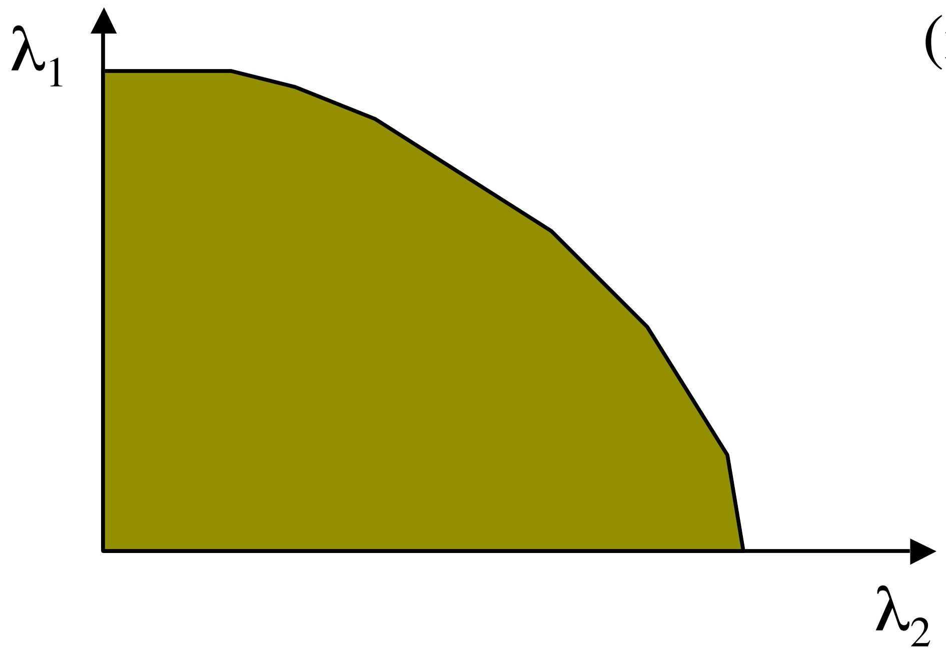
$$S_1(t), S_2(t) \in \{Good, Medium, Bad\}$$

Assume identical rate functions for $i = 1, 2$, given by:

$$\mu_i(0, S_i) = 0 \text{ units/slot} \quad \text{for all } S_i \in \{G, M, B\}$$

$$\mu_i(1, G) = 3, \mu_i(1, M) = 2, \mu_i(1, B) = 1 \text{ (units/slot)}$$

Capacity region Λ of the wireless downlink:



(i) Peak power constraint:

$\vec{P}(t) \in \Pi$

$(\sum_{i=1}^{L} P_i(t) \leq P_{peak})$

Λ = Region of all supportable input rate vectors $\vec{\lambda}$

Capacity region Λ assumes:

-Infinite buffer storage

-Full knowledge of future arrivals and channel states

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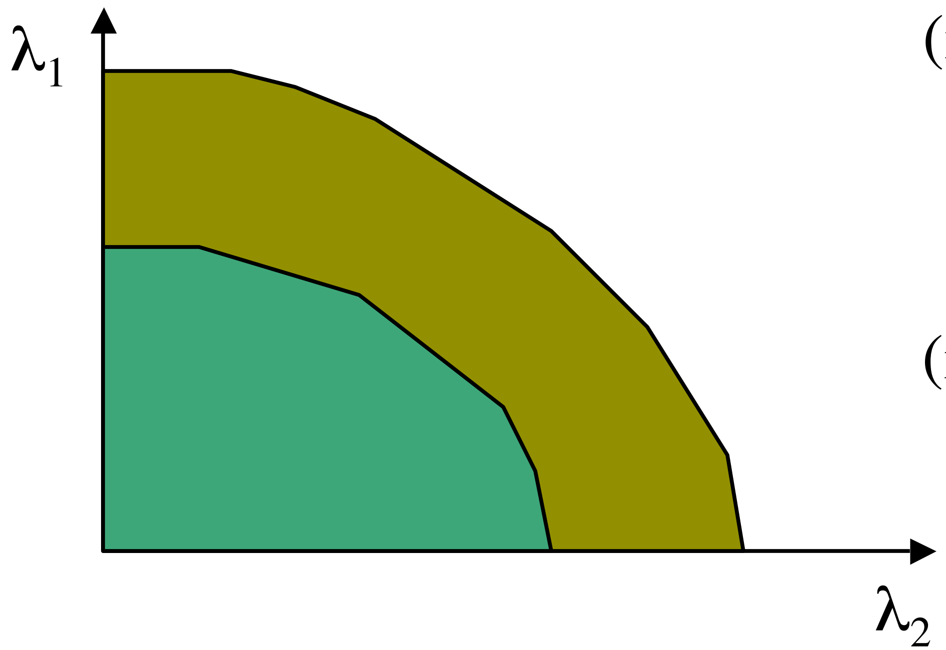

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(ii) Avg. power constraint:

$$\boxed{\mathbb{E}\left\{\sum_{i=1}^{L} P_i\right\} \leq P_{av}}$$

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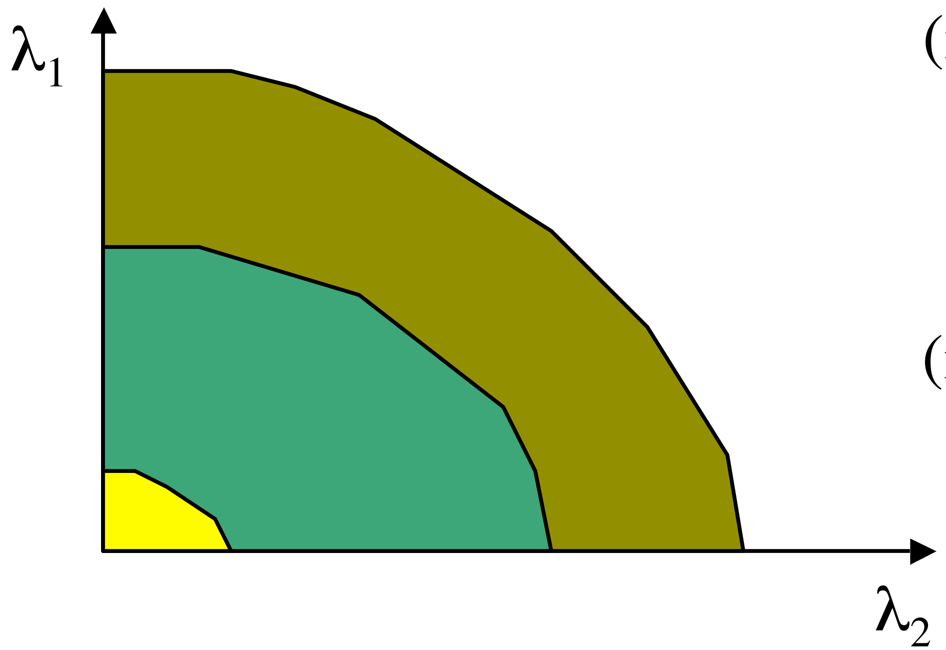

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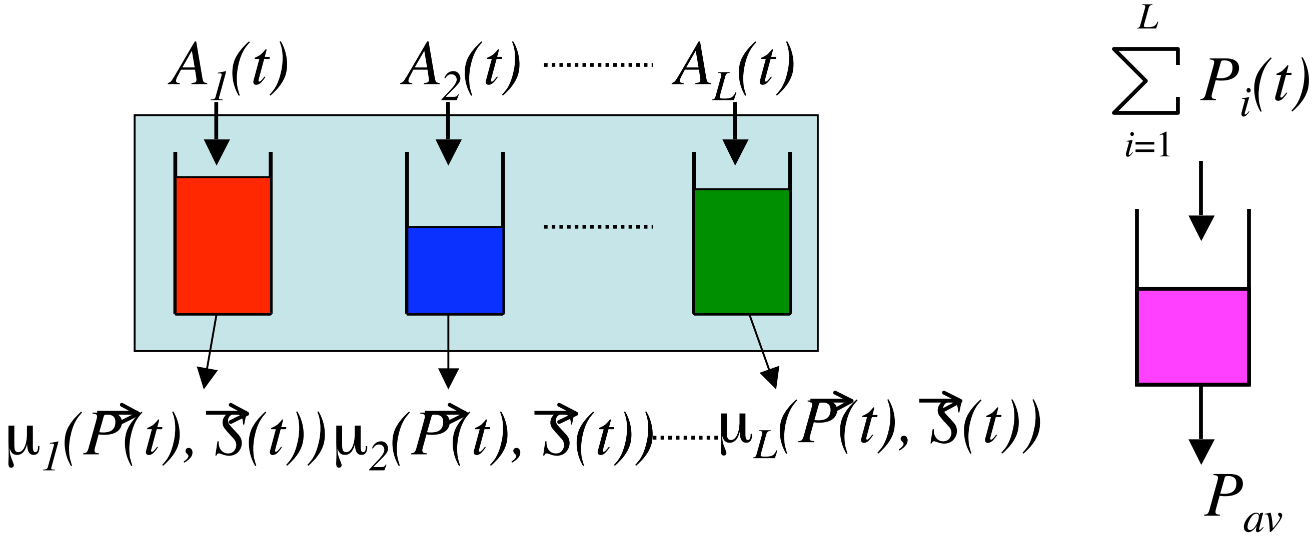
To remove the average power constraint $\mathbb{E}\left\{\sum_{i=1}^{L} P_i\right\} \leq P_{av}$,

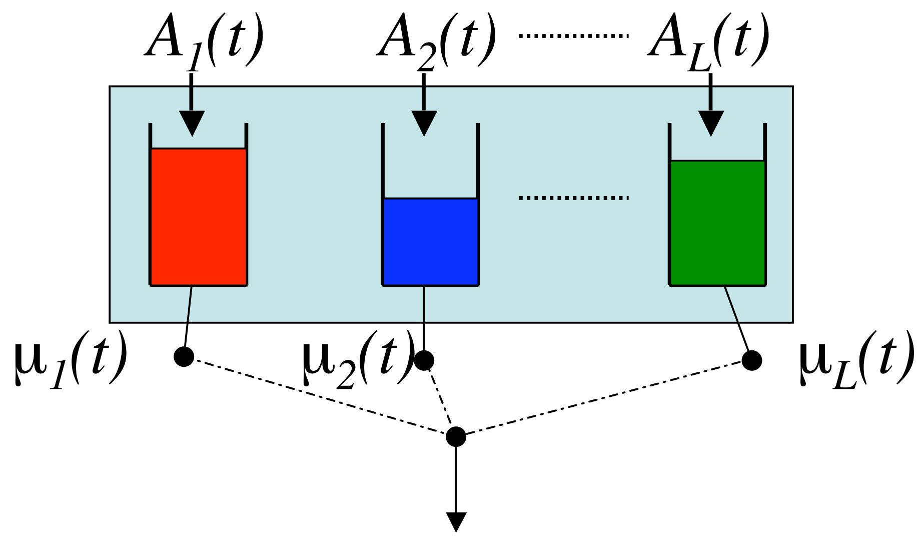
we create a *virtual power queue* with backlog $X(t)$.

Dynamics: $$\boxed{X(t+1) = \max[X(t) - P_{av}, 0] + \sum_{i=1}^{L} P_i(t)}$$



Observation: If we stabilize all original queues *and* the virtual power queue subject to only the peak power constraint $\vec{P}(t) \in \Pi$, then the average power constraint will automatically be satisfied.

Control policy: In this slide we show <u>special case</u> when Π restricts power options to full power to one queue, or idle (general case in paper).



Choose queue i that maximizes: $\boxed{U_i(t)\mu_i(t) - X(t)P_{tot}}$

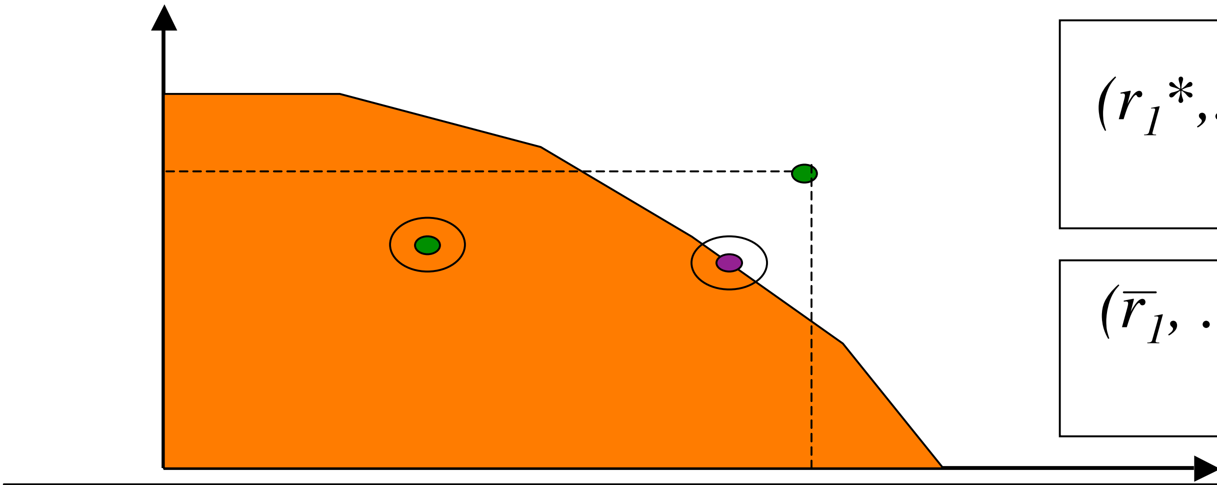
Whenever this maximum is positive. Else, allocate no power at all.

Then iterate the $X(t)$ virtual power queue equation:

$$X(t+1) = \max[X(t) - P_{av}, 0] + \sum_{i=1}^{L} P_i(t)$$

Performance of Energy Constrained Control Alg. (ECCA):

Theorem: Finite buffer size B, input rate $\vec{\lambda} \in \Lambda$ or $\vec{\lambda} \notin \Lambda$



$(r_1^*, \ldots, r_L^*)$ = optimal vector

$(\overline{r}_1, \ldots, \overline{r}_L)$ = achieved thruput vec.

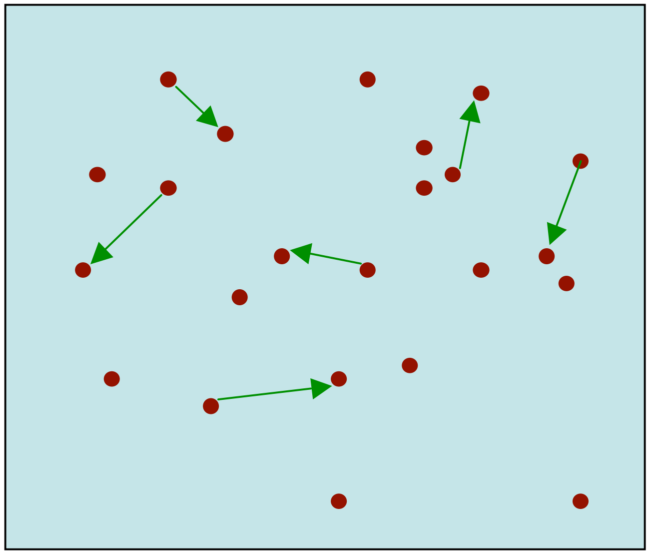
(a) Thruput: $\sum_{i=1}^{L} \overline{r}_i \geq \sum_{i=1}^{L} r_i^* - C/(B - A_{max})$

(b) Total power expended over *any interval* $(t_1, t_2) \leq P_{av}(t_2 - t_1) + X_{max}$

where C, X_{max} are constants independent of rate vector and channel statistics.

$C = (A_{max}^2 + P_{peak}^2 + P_{av}^2)/2$

Part 2: Minimizing Energy in Multi-hop Networks



N node ad-hoc network

(λ_{ic}) = input rate matrix

= (rate from source i

to destination node j)

(Assume $(\lambda_{ic}) \in \Lambda$)

$S_{ij}(t)$ = Current channel state between nodes i,j

Goal: Develop *joint* *routing*, *scheduling*, *power allocation* to minimize

$$\boxed{\sum_{n=1}^{N} E[g_i(\sum_{j} P_{ij})]}$$

(where $g_i(\bullet)$ are arbitrary convex functions)

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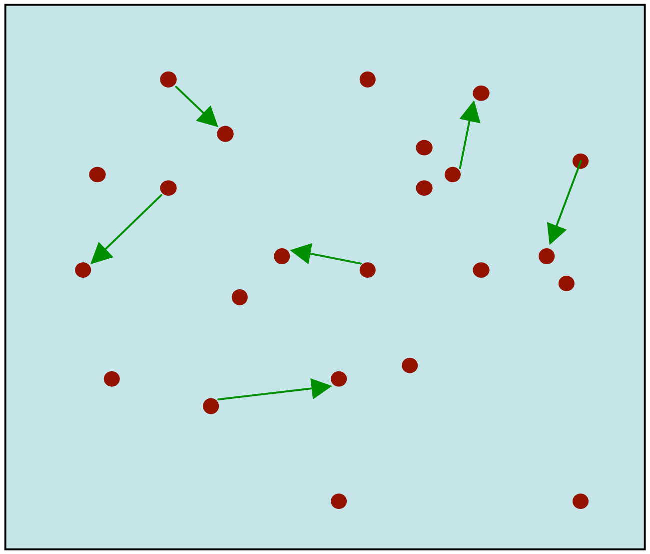

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To facilitate distributed implementation, use a *cell-partitioned model*…

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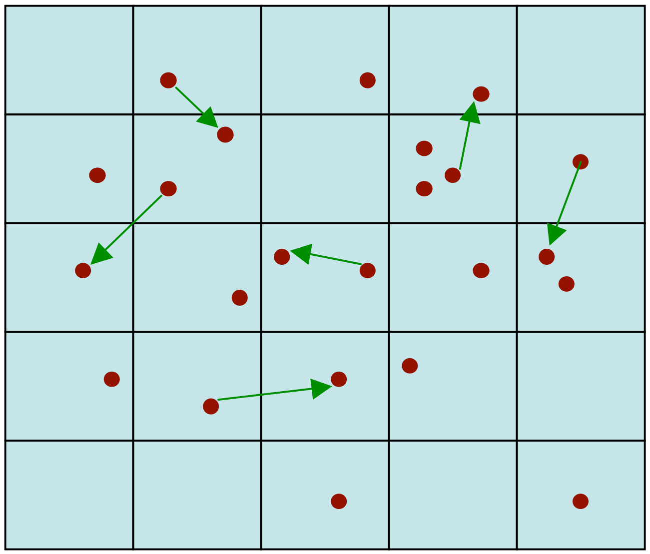

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To facilitate distributed implementation, use a *cell-partitioned model*…

Analytical technique: ***Lyapunov Drift***

Lyapunov function: $L(\vec{U}(t)) = \sum_n U_n^2(t)$

Lyapunov drift: $\Delta(t) = E[L(\vec{U}(t+1) - L(\vec{U}(t)) \mid \vec{U}(t)]$

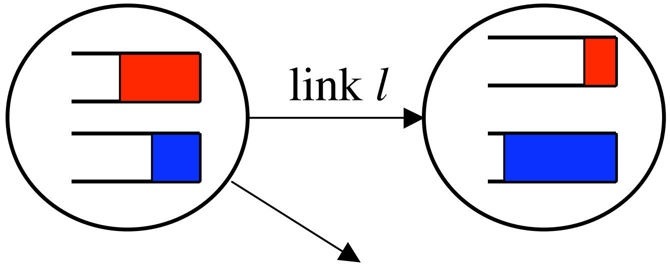
<u>Theorem</u>: (Lyapunov drift with Cost Minimization)

If for all t: $\Delta(t) \leq C - \varepsilon \sum_n U_n(t) + Vg(\vec{P}(t)) - Vg(\vec{P}^*)$

Then: (a) $\sum_n E[U_n] \leq \frac{C + VGmax}{\varepsilon}$ (stability and bounded delay)

(b) $E[g(\vec{P})] \leq g(\vec{P}^*) + C/V$ (resulting cost)

Joint routing, scheduling, power allocation:



$c_l*(t)$ =

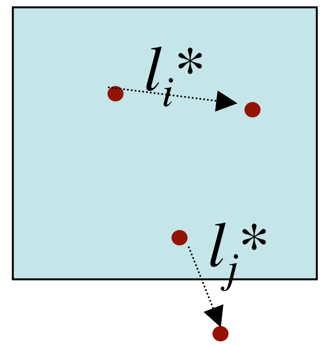
(1) For all links l, find the commodity $c_l^*(t)$ such that:

$$c_l^*(t) = \arg\max_c \left\{U^c_{tran(l)}(t) - U^c_{rec(l)}(t)\right\}$$

and define:

$$W_l^*(t) = \max[U^{c_l^*}_{tran(l)}(t) - U^{c_l^*}_{rec(l)}(t), 0]$$

(similar to the original Tassiulas <u>*differential backlog*</u> routing policy [92])



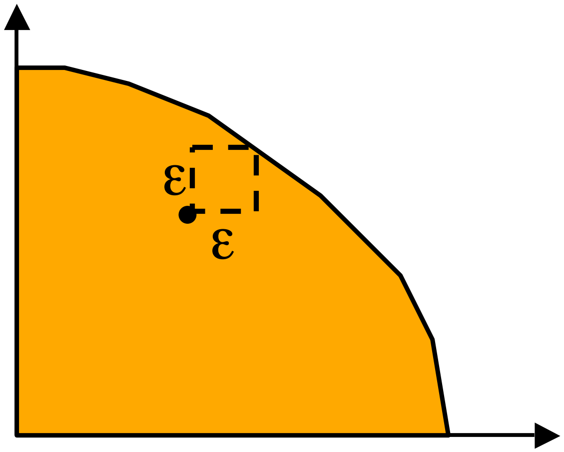
(2) Each node computes its optimal power level P_i* for link l from (1):

$$P_i^* \text{ maximizes: } \quad \underbrace{\mu_l(P, S_l(t))W_l^* - Vg_i(P)}_{Q_i^*} \qquad (\text{over } 0 < P < P_{peak})$$

(3) Each node broadcasts Q_i* to all other nodes in cell.

Node with largest Q_i* transmits:
Transmit commodity c_l* over link l*, power level P_i*

Performance:



ε = "distance" to capacity region boundary.

Theorem: If ε>0, we have…

a time average congestion bound of:

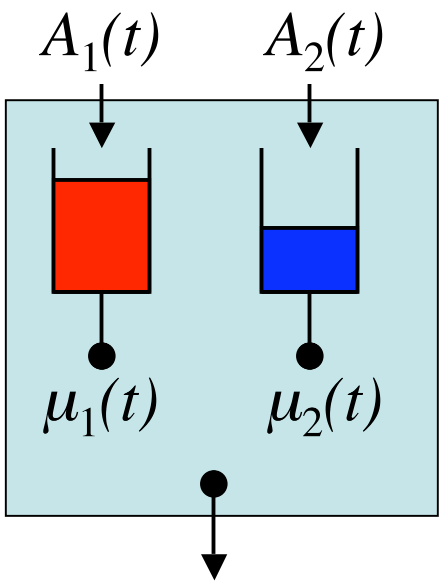
$$\sum_{nc} \overline{U}_n^c \leq \frac{DN + V\sum_n g_n(P_{peak})}{2\epsilon_{max}}$$

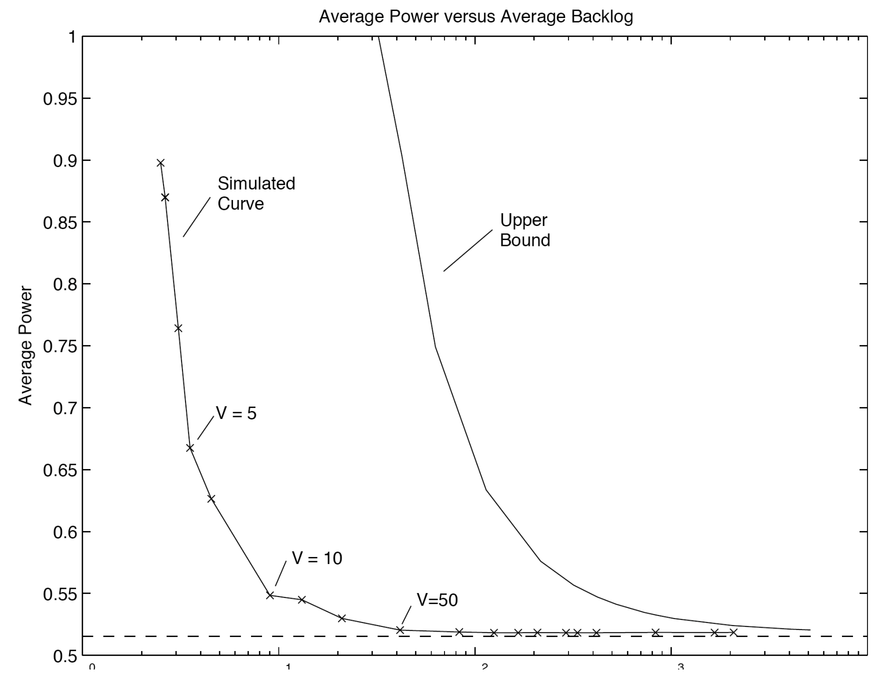
(where ϵ_{max} is the largest ϵ such that $(\lambda_{nc} + \epsilon) \in \Lambda$).

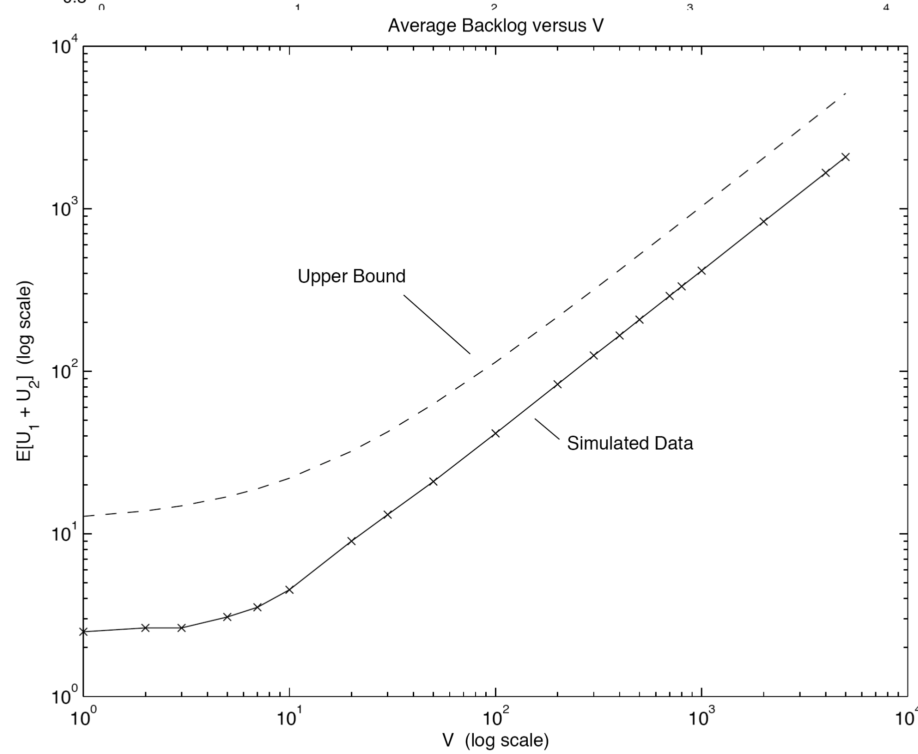
Further, the time average cost satisfies:

$$\sum_n \overline{g}_n \triangleq \lim_{t\rightarrow\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \left[\sum_n g_n\Big(\sum_{l \in \Omega_n} P_l(\tau)\Big) \right] \leq g^* + \frac{DN}{V} \qquad \square$$

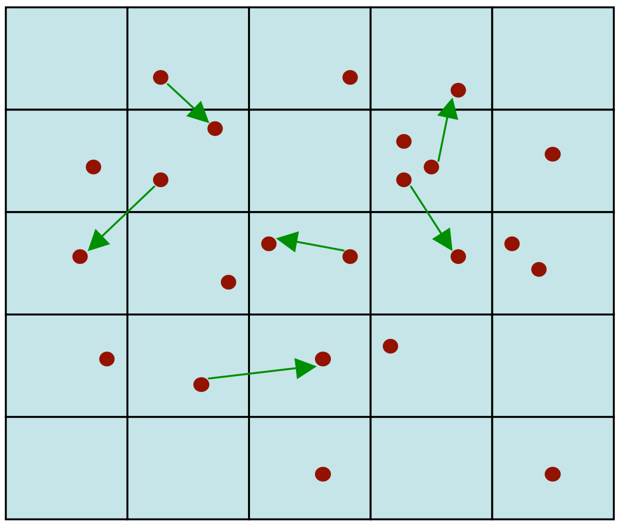
Example Simulation:
Two-queue downlink
with $\{G, M, B\}$ channels

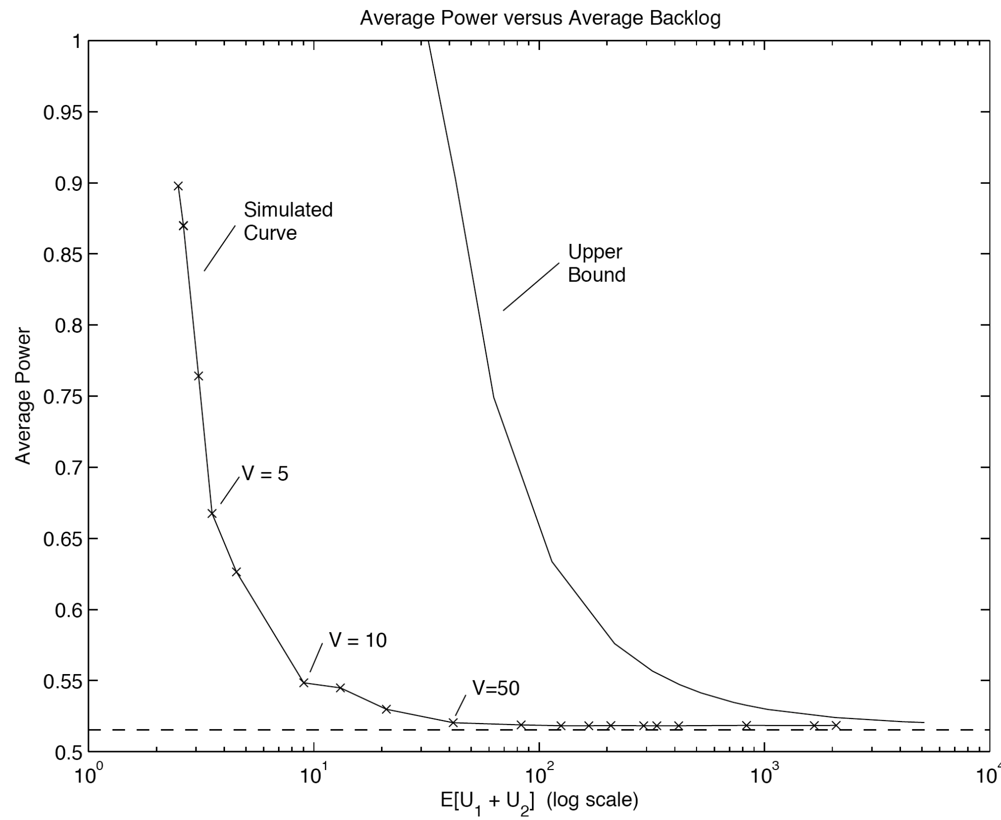






Conclusions:





1. Virtual power queue to ensure average power constraints.
2. Channel independent algorithms (adapts to any channel).
3. Minimize average power over multihop networks over all joint power allocation, routing, scheduling strategies.
4. Stochastic network optimization theory

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the end