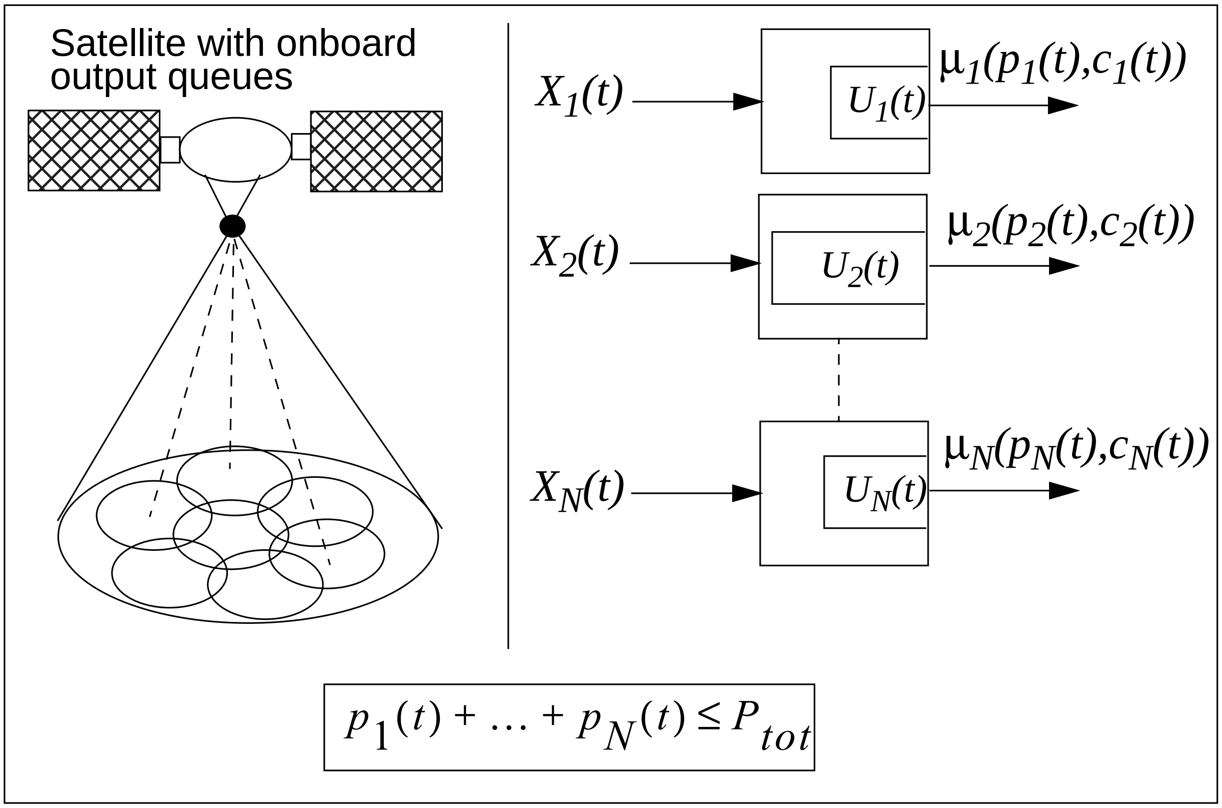
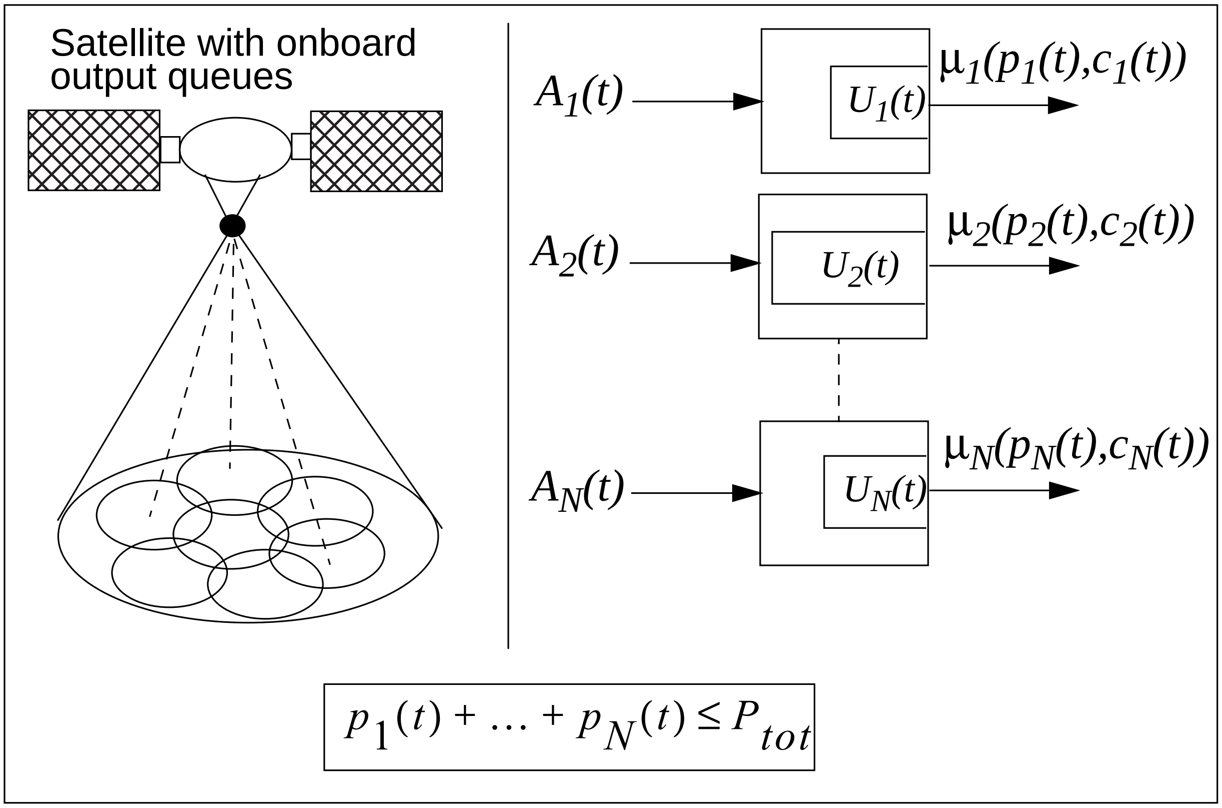
Power and Server Allocation in a Multi-Beam Satellite with Time Varying Channels



Michael Neely
MIT -- LIDS
mjneely@mit.edu

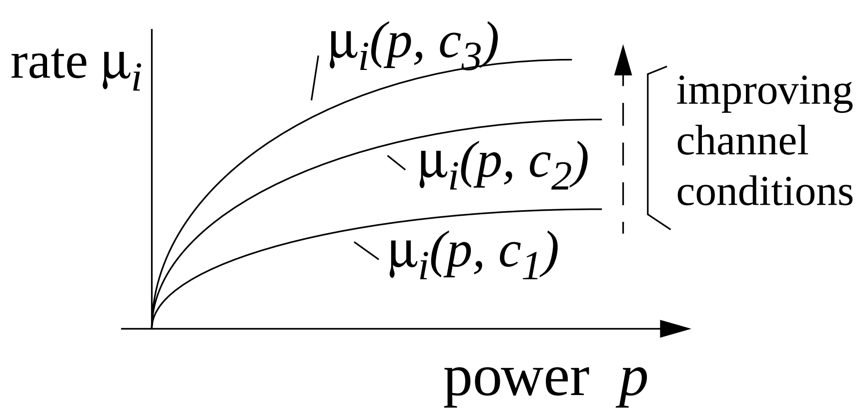
Eytan Modiano -- LIDS
Charlie Rohrs -- LIDS

Conference: INFOCOM 2002
Journal: M.J. Neely, E. Modiano, and C. E. Rohrs, "Power Allocation and Routing in Multi-Beam Satellites with Time Varying Channels," IEEE Transactions on Networking, Feb. 2003.

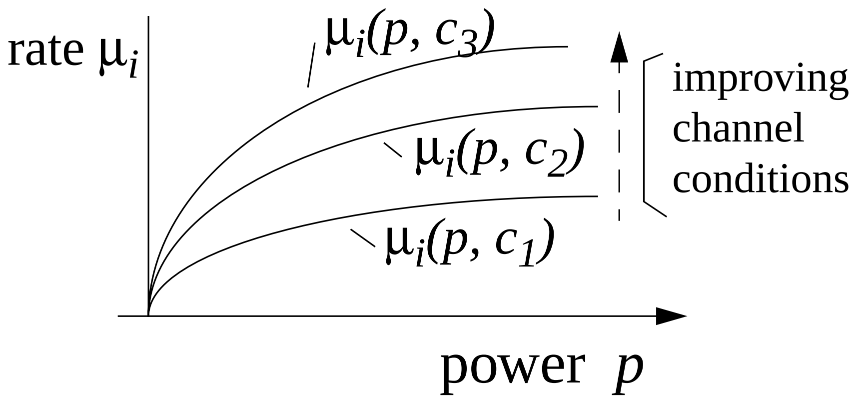


1. Data Arrives as Random Processes $\{A_i(t)\}$ (rates $\{\lambda_i\}$)
2. Time Varying Channels $\vec{C}(t) = (c_1(t), c_2(t), \ldots, c_N(t))$
3. Rate-Power curves $\mu_i(p_i,c_i)$

$$U_i(t+T) = max[U_i(t) - \mu_i(p_i(t), c_i(t)), 0] + A_i(t)$$



Link Budget Curves: (Concave, Continuous)

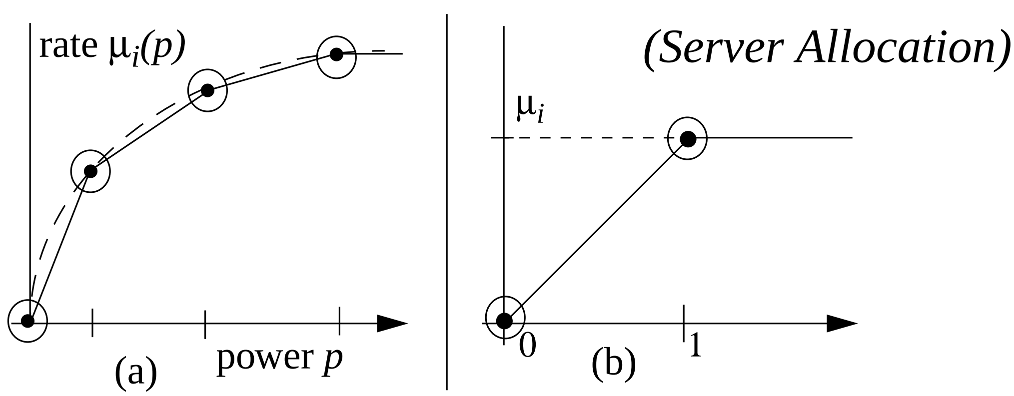


Curve Examples:

> $log(1 + p_i c_i)$ (c_i ~ attenuation-to-noise coefficient)

> Any curves for codes designed to acheive a specified low probability of error δ.

Curves may have a finite set of feasible rate-power points (corresponding to a finite databank of codes).



Create a *Virtual Power Curve* $\tilde{\mu}_i(p_i, c_i)$ = Piecewise Linear Interpolation of Feasible Points.

Goals:

-Develop Capacity region $\Omega = \{(\lambda_1, \lambda_2, ..., \lambda_N)$ rates that the system can stably support$\}$.

(We consider general ergodic arrival processes, and all possible power allocation strategies)

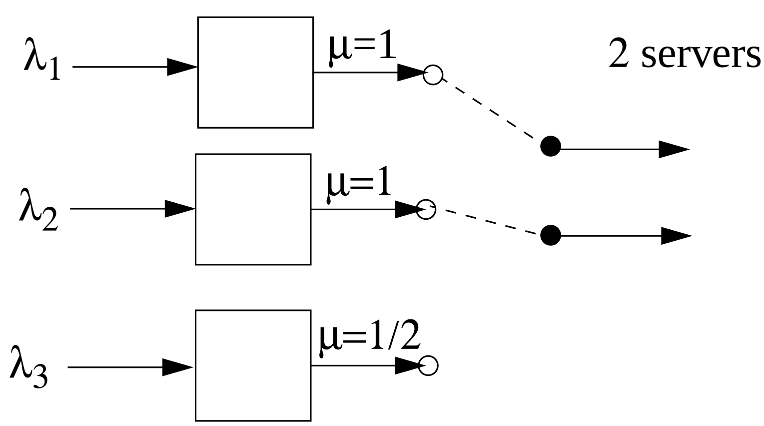
-Develop a Dynamic Power Allocation Policy to stabilize system and thereby achieve maximum throughput and maintain acceptably low levels of unfinished work backlog in all queues.

(Consider Timeslotted structure, *iid* assumptions Can be generalized to Markovian inputs/Channel states)

Example: Special Case of *Server Allocation*:

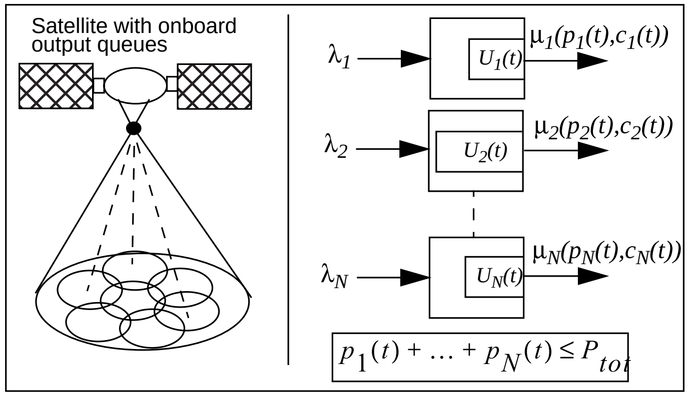
2 servers, 3 queues (static channel conditions)

Packet arrives to queue i every timeslot with probability λ_i



Serving the 2 fastest, non-empty queues does not stabilize the system in this case...

$$(\lambda_1 = \lambda_2 = p, \quad \lambda_3 = \frac{1}{2}(1 - p^2) + \varepsilon)$$



Definition of Capacity Region Ω:

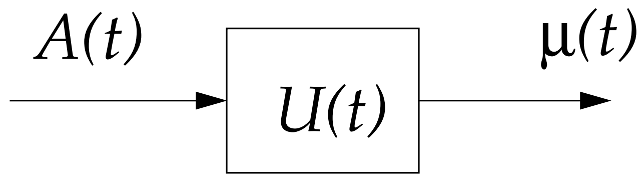
Let λ_i be the bit rate of stream $X_i(t)$

The Capacity Region Ω is the set of all rate vectors $\vec{\lambda} = (\lambda_1, \ldots, \lambda_N)$ such that:

- -The network is necessarily unstable whenever $\vec{\lambda} \notin \Omega$.

- -The network can be stabilized if $\vec{\lambda}$ is *strictly interior* to Ω.

A note on Stability:



$A(t)$ = Arrival Process (assumed ergodic of rate λ).
$\mu(t)$ = instantaneous processing rate (potentially non-ergodic).
$U(t)$ = Unfinished work (bits) in queue at time t.

Definition: The *overflow function g(M):*

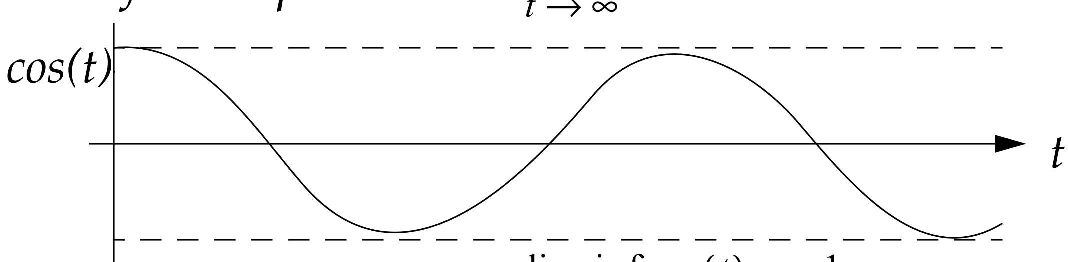
$$g(M) = \limsup_{t \to \infty} \left[\frac{1}{t} \int_0^t 1_{\{U(\tau) > M\}} d\tau \right]$$

$g(M)$ represents the average fraction of time the unfinished work is above the level M.

__*Definition*__*:* A queueing system is __*stable*__ if $g(M) \to 0$ as $M \to \infty$.

Ex. of limsup:

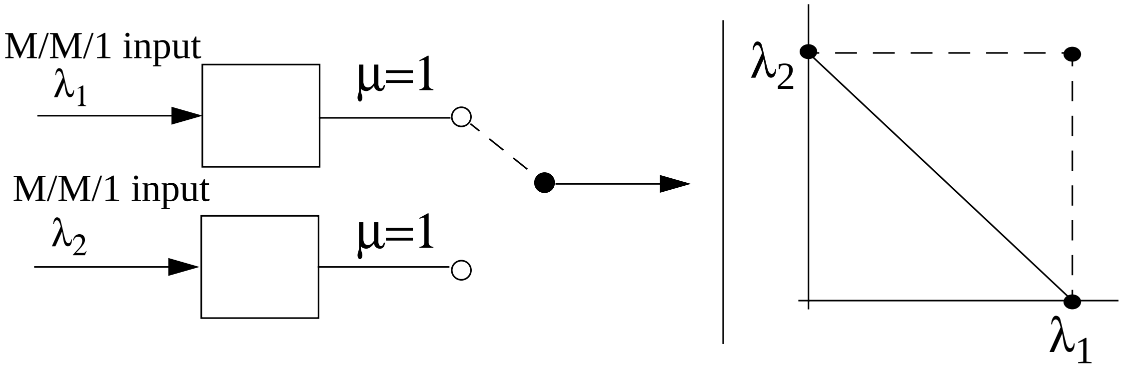
$$\limsup_{t \to \infty} \cos(t) = 1$$



$$\liminf_{t \to \infty} \cos(t) = -1$$

What can go wrong with wrong def.?

A network is stable if all queues are stable

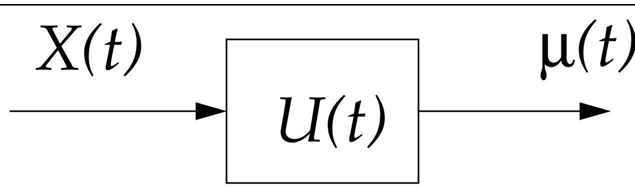


The *lim sup* definition is essential to obtain the correct notion of stability.

The above system is stable whenever $\lambda_1+\lambda_2<1$.

If *lim inf* is used, it is stable for $\lambda_1+\lambda_2<2$.

<u>Necessary Condition</u> (Single Queue):



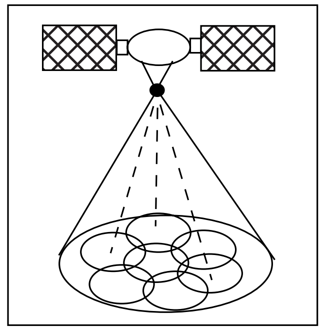
Define: $$\underline{\mu} = \lim_{t \to \infty} inf \, \frac{1}{t}\int_0^t \mu(\tau)\, d\tau$$

<u>Lemma</u>--Necessary Cond. for Stability:

$$\lambda \leq \underline{\mu}$$ ❑

Derivation of Capacity Region Ω of Satellite Downlink: The Necessary Condition

(consider constant channel case)



Suppose the inputs have rates $(\lambda_1,...,\lambda_N)$, and some power allocations $\{p_i(t)\}$ stabilize the system (perhaps designed with full knowledge of future).

$\{p_i(t)\}$ satisfy instantaneous power constraint:

$$\sum_{i=1}^{N} p_i(t) \leq P_{tot} \quad \forall t$$

Define:

$$\underline{\mu}_i = \lim_{t \to \infty} \sup \frac{1}{t}\int_0^t \mu_i(p_i(\tau))d\tau$$

By stability:

$$\lambda_i \leq \underline{\mu}_i \leq \frac{1}{t^*}\int_0^{t^*} \mu_i(p_i(\tau))d\tau + \varepsilon$$

$$\leq \mu_i\left(\frac{1}{t^*}\int_0^{t^*} p_i(\tau)d\tau\right) \quad \text{(by concavity)}$$

$$= \mu(p_i^*)$$ (and p_i^* satisfies power constraints) ❑

Similarly, for time varying channels:
We can restrict ourselves to stationary power allocation policies (allocate fixed power vector $(P_1^{\vec{C}}(t), \ldots, P_N^{\vec{C}}(t))$ whenever in state $\vec{C}(t)$).

Let: $\pi_{\vec{C}}$ = Steady state probability of being in channel state $\vec{C}$.

The capacity region Ω of the satellite downlink is the set of all rates $(\lambda_1,\ldots,\lambda_N)$ such that there exist power values $p_i^{\vec{C}}$:

$$\sum_{i=1}^{N} p_i^{\vec{C}}(t) \le P_{tot} \qquad \text{(Power Constraint)}$$

$$\lambda_i \le \sum_{\vec{C}} \pi_{\vec{C}} \mu_i\left(p_i^{\vec{C}}, c_i\right) \qquad \text{(Rate inequality)}$$

Would like to stabilize system without knowing channel statistics, input processes, or input rates $(\lambda_1,\ldots,\lambda_N)$.

A Stabilizing Policy that considers $\vec{C}(t)$ and $\vec{U}(t)$:

(consider timeslotted system under independence assumptions)

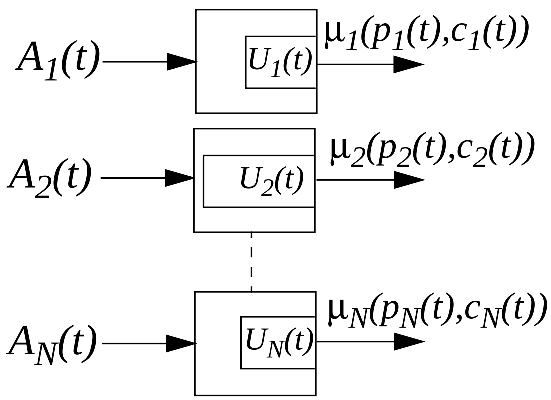
-Packets arrive to queue i with bit rate λ_i

(Bit arrivals $a_i \sim f_i(a_i)$ *iid* every timeslot, $E[a_i] = \lambda_i$).

-Channel states $\vec{C}(t)$ change every timeslot

(*iid* with probabilities $\pi_{\vec{C}}$)

-$U_i(t)$ = Unfinished work in node i at time t.



Strategy: Every timeslot, observe $\vec{U}(t)$ and $\vec{C}(t)$:

Allocate $\{p_i\}$ to Maximize: $$\sum_{i=1}^{N} \theta_i U_i(t)\mu_i(p_i, c_i(t))$$

Subject to: $$\sum_{i=1}^{N} p_i \le P_{tot}$$

(the θ_i's are arbitrary postive weights for priority service)

The policy is stabilizing whenever the input rate vector $\vec{\lambda}$ is strictly interior to the capacity region Ω.

Analysis technique uses Lyapunov Drift (Lyapunov techniques well known in switching/scheduling literature [McKeown, Tassiulas, Leonardi]).

Dynamic Equation:

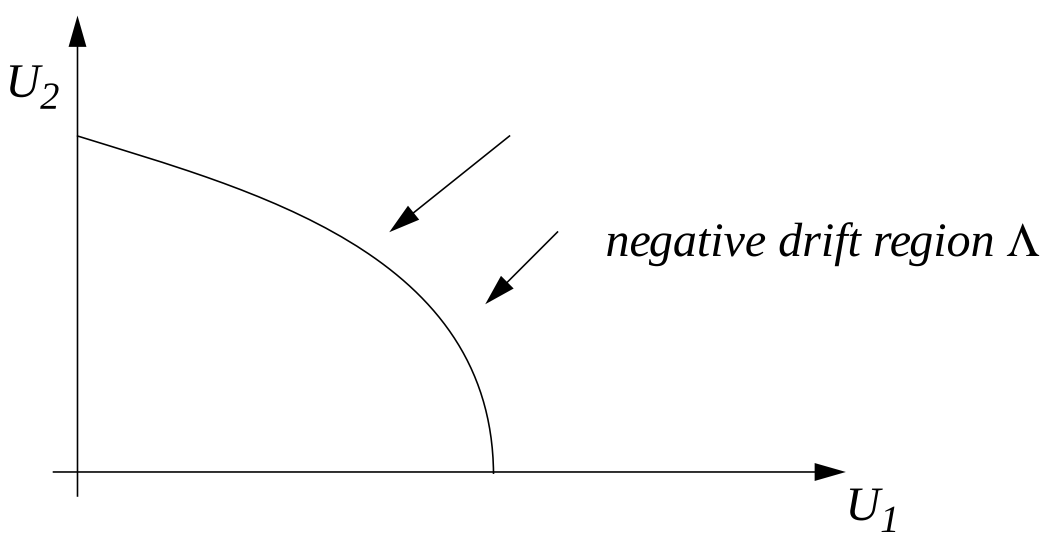
$$U_i(t+T) = max[U_i(t) - \mu_i(p_i(t), c_i(t)), 0] + A_i(t)$$

Define Lyapunov Function:

$$L(\vec{U}) = \sum_{i=1}^{N} U_i^2$$

Can show drift in Lyapunov function is negative whenever $\vec{U}(t)$ is outside some bounded region of the unfinished work state space:

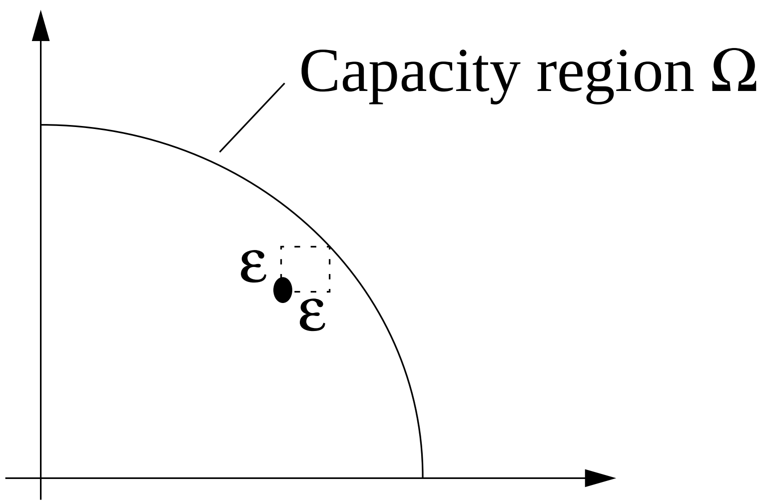
$$E[L(\vec{U}(t+T)) - L(\vec{U}(t)) \mid \vec{U}(t) \in \Lambda] \leq -\delta$$



Delay Bound:

Suppose the rate vector $\vec{\lambda}$ is strictly interior to Ω so that a positive value ε can be added to each entry such that:

$$(\lambda_1 + \varepsilon, \ldots, \lambda_N + \varepsilon) \in \Omega$$



ε can be viewed as the "distance" to the boundary of the capacity region Ω.

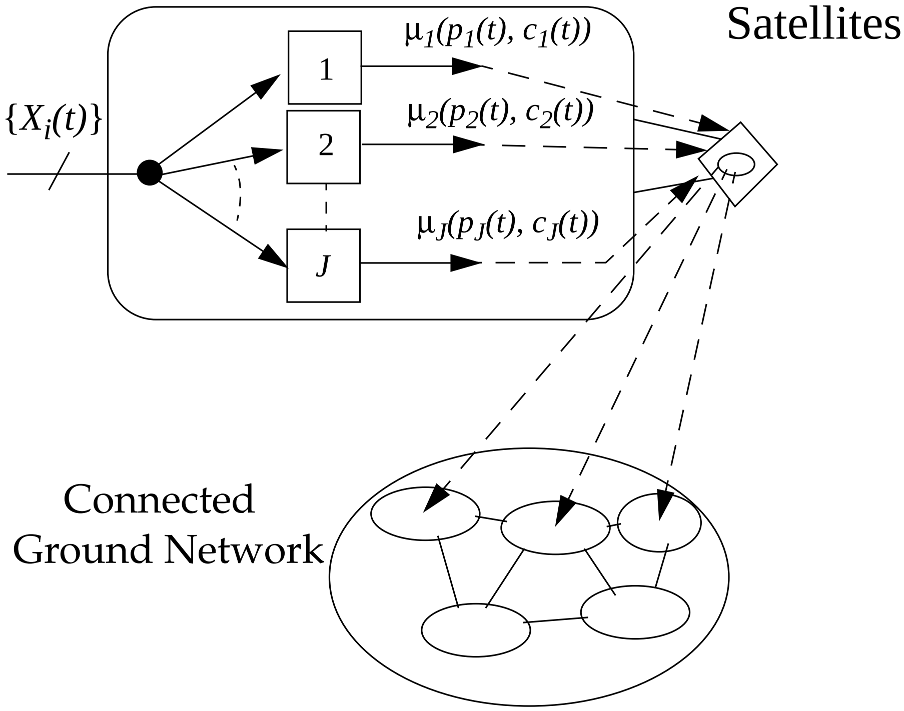
$$\text{Average Delay} \leq \frac{T\sum_i \left(E[A_i^2] + E[\mu_i^2]\right)/\lambda_{tot}}{2\varepsilon}$$

Note Fundamental Similarity to M/G/1 queue:

$$\text{Average Delay}_{(M/G/1)} = \frac{\lambda E(L^2)/\mu}{2(\mu - \lambda)}$$

2^{nd} moment of packet length

Joint Routing and Power Allocation:



Capacity Region:

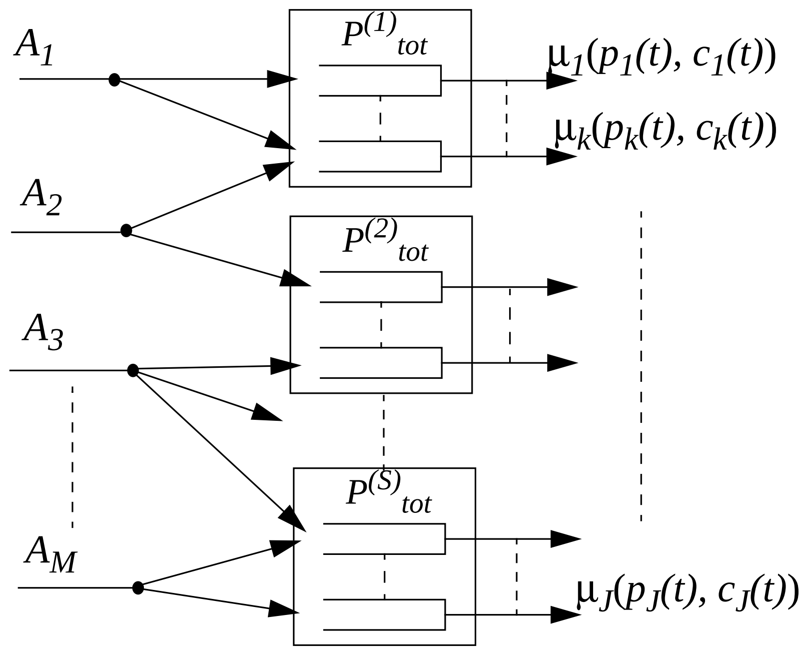
$$\lambda_1 + \ldots + \lambda_J \le \bar{\mu}_{out}$$

$$\bar{\mu}_{out} = \sum_{\vec{C}} \pi_{\vec{C}} \sum_{s=1}^{S} \max_{\left\{ \sum_{j \in Sat(s)} p_j \le P_{tot}^{(s)} \right\}} \left[\sum_{j \in Sat(s)} \mu_j(p_j, c_j) \right]$$

Decoupled Policy:

Power Alloc.: Allocate to maximize $\sum_{j \in Sat(s)} \mu_j(p_j, c_j(t))$

Routing: Route every packet arriving in a timeslot to the queue *i* with the least unfinished work.

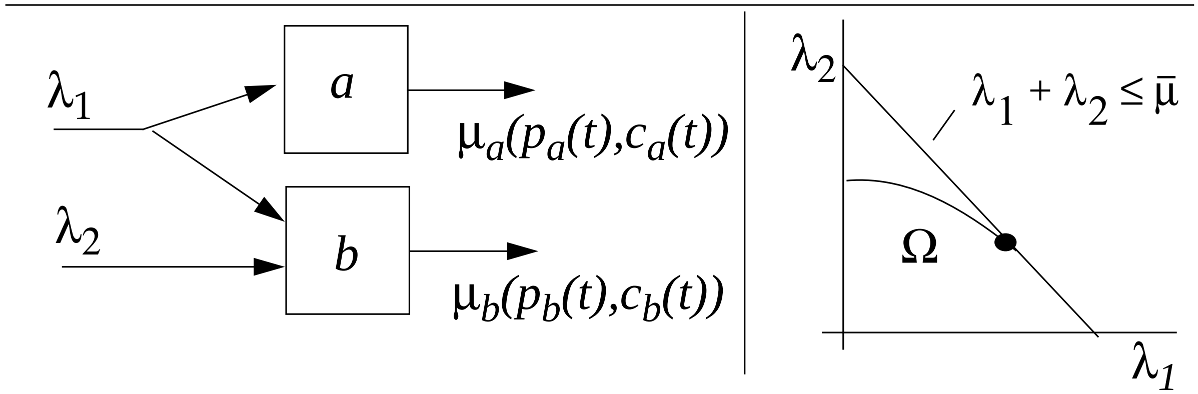


Variation of the problem: Many input types

Routing: Route packets from stream A_i to the shortest queue in its class Q_i.

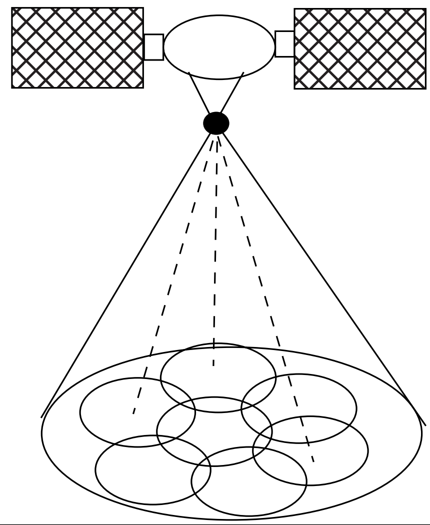
Power Allocation: Each Satellite Carries out the Dynamic Power Allocation Policy: Maximize:

$$\sum_{j \in Sat(s)} U_i(t)\mu(p_j, c_j(t))$$



Connectivity Constraints:

To limit channel interference, define *connectivity sets* to ensure power does not affect other channels:



Power $\vec{P}(t) \in \{\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_R\}$

Example -- Limited to 3 Servers:

$$\mathcal{P}_r = \left\{ (p_1, p_2, p_3, 0, \ldots, 0) \in \Re^N \middle| p_j \geq 0, \sum_{j=1}^{3} p_j \leq P_{tot} \right\}$$

Capacity Region:

$$\vec{\lambda} \in \Omega \triangleq \sum_{\vec{C}} \pi_{\vec{C}} \, Convex\ Hull\left(\{ \vec{\mu}(\vec{P}, \vec{C}) \,|\, (\vec{P} \in \mathcal{P}_r) \} \Big|_{r=1}^{R} \right)$$

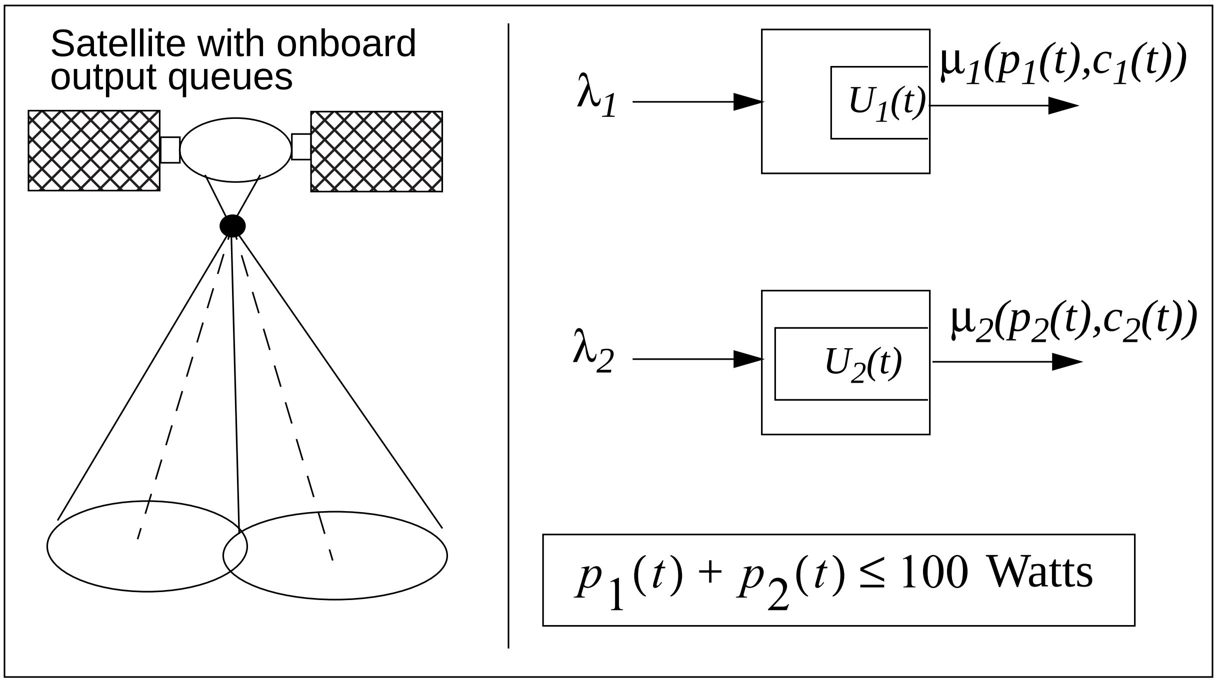
Power Allocation Policy:

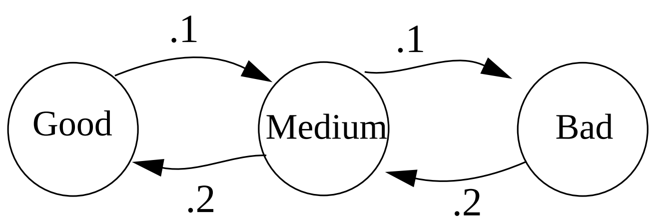
$$\text{Maximize:} \quad \sum_{j=1}^{N} U_j(t) \mu_j(p_j, c_j(t))$$

$$\text{Subject to:} \quad \vec{P} \in \mathcal{P} = \{\mathcal{P}_1, \ldots, \mathcal{P}_R\}$$

Numerical and Simulation Results:

Markovian Channel Dynamics / Poisson inputs





Log-normal distribution of α_i for each of the three channel conditions:

	mean	variance
Good	15 db	.264 db-squared
Medium	10 db	.868 db-squared
Bad	0 db	.145 db-squared

Concluding Slide: