Full length article

Meniscus process optimization for smooth surface fabrication in Stereolithography

Yayue Pan a, *, Yong Chen b

a Department of Mechanical and Industrial Engineering, University of Illinois at Chicago, 842 W. Taylor St., ERF 2039, Chicago, IL 60607, USA
b Epstein Department of Industrial and Systems Engineering, University of Southern California, 3715 McClintock Ave., GER 240, Los Angeles, CA 90089, USA

A R T I C L E   I N F O

Article history:
Received 1 January 2016
Received in revised form 19 February 2016
Accepted 8 May 2016
Available online 21 June 2016

Keywords:
Stereolithography
Surface finish
Meniscus
Accuracy
Process optimization

A B S T R A C T

In the layer-based additive manufacturing processes, it is well known that the surface finish is usually poor due to the stair-stepping effect. In our previous study, it was shown that the meniscus approach can be used in the Stereolithography process to achieve much better surface finish. A related challenge is how to optimize parameters such that the formed meniscuses can lead to high surface finish and accurate shape. In this paper, a systematic process planning method is presented for the Stereolithography process that is integrated with the meniscus approach. Process planning, parameters characterization and meniscus parameters optimization are presented with experimental verifications. To demonstrate the systematic process, surface profile simulation, meniscus database construction, meniscus forming parameter optimization, and shape accuracy prediction have been performed using a test case based on convex surfaces. A comparison between the experimental results with and without the process planning method is presented, illustrating the effectiveness of the developed meniscus optimization method in simultaneously controlling the shape and surface finish of fabricated objects. Subsequently the simulation results are also verified with experimental results.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

Stereolithography (SL) Apparatus is the first commercialized Additive Manufacturing (AM) technology and also one of the most commonly used AM technologies [1–3]. In SL process, liquid photosensitive resin is cured by light irradiation, usually through the use of a UV laser beam or a Digital Light Projection (DLP) system that uses a Digital Micromirror Device (DMD). Controlled light irradiation induces a curing reaction, forming a highly cross-linked polymer. Compared to other polymer-based AM technologies such as the extrusion or jetting based processes, the SL process can produce parts with fine features, good accuracy, and using various polymers [4–9]. SL also surpasses the processing speed and yields of subtractive-type mask-based processing [10] as well as other micro-patterning processes [11]. Its potential applications in three dimensional (3D) structure fabrications include prototyping, tooling and manufacturing, medical devices, artwork, and architectures, to name a few. A schematic diagram of a Mask Image Projection based Stereolithography (MIP-SL) system is shown in Fig. 1a.

Similar to other additive manufacturing processes, the objects fabricated by the SL process also have stair-stepping effect due to the use of two-dimensional (2D) layers [11–13]. As shown in Fig. 1b–d, a given 3D model is first sliced into a set of 2D layers. By stacking the sliced 2D layers together, a physical object can be fabricated to approximate the original Computer-aided Design (CAD) model. Due to the stacking of 2D layers, the fabricated curved surfaces especially the ones whose normals are close to the building direction (Z axis) may have large approximation errors. This stair-stepping effect greatly limits the use of AM in the applications that require smooth surfaces, for example, the optical lens, the rotating shaft and socket in mechanism design, etc.

Approaches such as controlled curing depth [14,15] methods have been developed for the fabrication of smooth down-facing surfaces. As to the up-facing surfaces fabrication, a meniscus approach for achieving improved surface finish in the SL process was presented in our previous work [16]. The developed meniscus approach for the Mask Image Projection based Stereolithography (MIP-SL) system is shown in Fig. 2. Supposing a CAD model that is to be built has a curved surface, as illustrated in Fig. 2a, the meniscus approach could be used. The first step is to solidify liquid resin layer by layer using the conventional Stereolithography process, as shown in Step-1 in Fig. 2b. The cured layers form intersecting planes in corners. Due to surface tension, meniscuses will be formed after
raising the built layers out of the liquid resin by moving the platform up using the Z-stage with a proper velocity. The proper velocity setting for different resins has been identified by experiments in our previous work [16]. Accordingly, another mask image can be used to solidify the resin in the meniscus areas as Step-2 in Fig. 2b. In brief, using the meniscus approach, a set of 2D layers is solidified and the meniscuses are cured afterwards. By adjusting the slicing strategy and when to raise the built layers out of liquid resin, we can change the height and width of corners, thus manipulating the meniscuses to better suit the given shape. To conclude, in SL process with meniscus approach, the 3D design is still approximated by stacking 2D slices as the conventional SL process. However, the staircase effect problem caused by the 2D layer stacking mechanism is eliminated by coating the 2D layers and filling the corners between layers using meniscus principle. Hence, this approach can be used to fabricate smooth curved surfaces no matter convex or concave.

Preliminary studies were presented in [16] and the use of the meniscus approach in fabricating smooth up-facing surfaces has been verified. The primary challenge in planning meniscus process is also identified, that is, how to achieve the required accuracy and build time when using the meniscus approach to improve surface finish. Such manufacturing process optimization is a multi-objective problem with nonlinear functions. How to solve such an optimization problem in the meniscus process planning is still an open question. In this paper, we present a systematic process planning method to optimize the meniscus forming parameters based on a simulation of predicted meniscus shape.

The remainder of the paper is organized as follows. Section 2 discusses the models of formed meniscus for various cases, and challenges in planning meniscus forming process. Based on them, a meniscus process planning approach is presented in Section 3 to consider the meniscus of multiple cured layers. Important process parameters related to forming the meniscus are also identified and characterized in the section. Section 4 presents the formulation of the optimization problem and a general process to solve the formulated optimization problem. The experimental setup for performing physical experiments and test results are discussed in Section 5. Finally, conclusions with future work are drawn in Section 6.
2. Challenges in meniscus process planning

As shown in Fig. 3, meniscus is the liquid residue that exists in the corner of two intersecting planes. As presented in [16,17], the developed meniscus shape equation relates liquid interface with gravitational influence and interfacial tension that is represented by contact angle $\theta$ and capillary height $h_c$, as shown in Eq. (1).

$$ \rho g - \frac{1}{2} h^2 + \frac{\rho \gamma}{(1 - \sin\theta)(1 + y^2)^{2/3}} = 0; $$

where $\rho$ is the density of the liquid, $g$ is gravitational acceleration, and $y$ is the height of the meniscus above the horizontal plane surface. The contact angle $\theta$ in Eq. (1) is the angle at which the liquid resin interface meets the solidified resin surface. Specific to the given liquid and solid system, the contact angle is determined by the interactions between the liquid resin, solidified resin and air interfaces. The capillary height $h_c$ is the maximum height that the fluid can reach on an infinite vertical wall. $h_c$ is a characteristic length for the fluid subject to gravity and surface tension.

The value of $h$ and $b$ of a corner determines the meniscus profile, accordingly, determining the geometrical accuracy of the fabricated curved surface. When and where to form and cure meniscuses will not only affect surface accuracy, but also influence other performance metrics such as build time. Therefore the prediction of meniscus profiles and the optimization of process parameters are critical for the meniscus method. To model the meniscus shape, different boundary conditions have been used in the following five cases [16,17]. For completeness, the boundary conditions are listed as follows:

**Case 1.** both $|h|$ and $b$ can be considered infinite. The related boundary conditions are:

$$ y(x = 0) = -\tan^{-1}\theta; y(x = \infty) = 0 $$

**Case 2.** $|h|$ is smaller than $h_c$, and $b$ is bigger than $b_0$. Hence the curvature of the meniscus is decided by $h$. The boundary conditions are as follows.

$$ y(x = 0) = -\tan^{-1}\theta; y(x = 0) = -h; $$

$$ y(0) = -h; \quad y(b) = 0; $$

**Case 3.** $|h|$ is bigger than $h_c$, and $b$ is smaller than $b_0$. Thus the curvature of the meniscus is decided by $b$. The boundary conditions are as follows.

$$ y(x = 0) = -\tan^{-1}\theta; y(x = b) = 0 $$

$$ y(0) = -h; \quad y(b) = 0; $$

**Case 4.** $h$ is smaller than $h_c$, and the length of the horizontal plane $b$ is smaller than $b_0$. Hence the curvature of the meniscus is decided by both $b$ and $h$. The problem follows the boundary conditions:

$$ y(0) = -h; \quad y(b) = 0; $$

**Case 5.** $|h|$ is smaller than $h_c$, and the discontinuity is connected with a horizontal plane that the size is smaller than the liquid droplet size. The problem turns into a wetting puddle in the bottom-up projection system and the boundary conditions are:

$$ y(x = 0) = 0; \quad y(b) = 0; $$

by using the governing equation Eq. (1) and boundary conditions Eqs. (2)–(5), a meniscus profile could be modelled by solving the mathematical problem. However, to build a part, the manufacturing process could involve multiple meniscuses, with an infinite number of choices on forming meniscus segments to fabricate the final part. Hence, how to plan the meniscus-based SL process based on surface shape prediction is quite challenging. Improper meniscus forming process parameters will cause large approximation errors or even surface defects of the fabricated parts. An example is shown in Fig. 4. The CAD model shown in Fig. 4 (Left) was fabricated with the conventional layer-by-layer SL method (M1) and the meniscus-based SL method (M2). As shown in the microscopic images of Area A1 and A2 in Fig. 4, the meniscus approach improves the high surface finish, but leads to a much larger approximation error in the fabricated part in the area A1.

To address the challenge in predicting and optimizing meniscus surfaces, a systematic process planning and modeling method is presented in Section 3. First, a pipeline of the meniscus-based process is reviewed in Section 3.1. Two important properties for modeling the meniscus surface profile and establishing parameter characterization are presented in Sections 3.2 and 3.3. A database is built to store the shape information of all meniscus segments, and a discrete point-based representation method is developed to represent 3D meniscus shapes. Based on the models presented in Section 3, meniscus optimization method is presented in Section 4 in order to achieve desired building accuracy. Moreover, an example of the process optimization based on a convex lens is given in Section 4.

3. Meniscus process planning in the SL processes

3.1. Pipeline of the meniscus-based Stereolithography process

Fig. 5 shows the pipeline of the meniscus-based Stereolithography process. An input CAD model is first divided into a set of curved segments using a given meniscus thickness $D$. Notice that the used meniscus thickness is determined by the capillary height
in the meniscus models and is usually much larger than the layer thickness as used in SLA, which is related to the curing depth of the liquid resin. Hence, a meniscus segment can be sliced into multiple 2D curing layers using the curing layer thickness. After curing all the 2D layers of each meniscus segment in a layer-by-layer fashion, the stacked layers will be raised out of liquid resin to form meniscuses. For each divided meniscus segment, we could extract curved surface information from the CAD model and save the surface profile as a set of points \( P(x, y, z, n) \), where \( x, y \) and \( z \) are coordinates and \( n \) is the norm vector. The process of point-based surface profile extraction is discussed in Section 4.2. The normal information \( n \) is used to determine the surface type. The meniscus approach will only be applied to the up-facing surfaces whose normal is positive with the \( z \) axis [16]. If a meniscus segment does not contain any curved surfaces, it will be built using the layer-by-layer fabrication approach without forming meniscuses. Otherwise, if a meniscus segment has curved up-facing surfaces, the meniscus forming process will be planned and optimized. The meniscus process planning approaches and related algorithms are presented in Section 4.3. After the optimal meniscus process parameters have been identified for one meniscus segment, the next meniscus segment would be processed in the same way until the last one. Based on the planned 2D layers and meniscus forming segments, a physical object can be fabricated using the meniscus-based Stereolithography process.

3.2. Modeling of surface profile of a meniscus segment

As discussed in Section 2, two important parameters in determining the shape of the formed meniscus profile are \( h \) and \( b \). When the meniscus is formed in the corner defined by two neighbouring layers, as shown in Fig. 3, the meniscus model is straightforward and the mathematic model in Section 2 can be directly applied. This method has been presented in [18], in which the meniscuses are formed and cured in each layer. However, this approach will cause extremely slow build speed, that is, the build time will be at least doubled due to the additional meniscus curing for each layer. In addition, such an approach was found to only work for up-facing surfaces that are \( 40^\circ \) or less from the horizontal surface [18, 19].

Different from all others’ work, in this research, the meniscus is not formed for each layer. Instead, meniscuses are formed after multiple layers have been cured. By forming meniscuses among multiple 2D layers, we are able to improve the building speed as well as to achieve better geometric accuracy. The challenge is then how to model the meniscus when the corner structure is not formed by just two intersecting planes, but a set of small stairs. An illustrative example is shown in Fig. 6b and c, where the meniscuses are formed after two 2D layers are cured (1st–2nd). To use the equations developed in Section 2 to model the meniscus, the primary question that needs to be answered is:

**What is height \( h \) and width \( b \) in the equations to describe the meniscuses formed by multiple cured layers?**

To simplify the discussion of the problem, we will present our work for a 2D case. The work could be extended to 3D cases as shown in Section 4. Suppose that a meniscus segment starting from \( z_0 \) to \( z_n(z_n - z_0 = \psi) \), is further sliced into multiple 2D layers and the \( i \)th layer is at the level \( z = z_i \). In order to model the meniscus, we have the following observation with two properties to address the above question.

**Observation.** If a layer is inside the meniscus, the layer has smaller value of \( z \) than the meniscus \( M(x, z) \), that is, \( p(x') = z_i < M(x') \) where \( p \) is a boundary point of the layer. If a layer is outside the meniscus, the layer has bigger value of \( z \) than the meniscus \( M(x, z) \), that is, \( p(x') > M(x') \) where \( p \) is a boundary point of the layer.

As shown in Fig. 6b and c, the dotted orange curve denotes the meniscus which is formed among two layers (1st and 2nd layers). In Fig. 6b, the boundary point \( p \) of the 1st layer is inside the meniscus, while in Fig. 6c, \( p \) is outside the original meniscus curve.

**Property 1.** If the 2D layer \((z = z_i)\) is inside the meniscus curve \((p(x') < M(x'))\) for all \( p(x') = z_i \), its effect on the formed meniscus shape is negligible.

As shown in Fig. 6b, the 1st layer is inside the meniscus curve. The meniscus \( M \) could be modelled by choosing \( h = L_1 + L_2 = D \), and \( b = t_1 + t_2 = t \). For some resins and some geometry where the 2D layer contour inside of the meniscus is very close to the meniscus profile, the effect of the 2D layer on the meniscus curve may not be negligible. An undesired bulge may occur due to the surface tension and affect the accuracy. To keep Property 1 true in such cases, we will manipulate the mask image for that layer curing, so that the boundary of the problem layer will align with the boundary of the top layer of the meniscus and have no effect on the meniscus curve. For example, supposing the \( p(x) \) in Fig. 6b will affect the meniscus curve greatly, the geometry of this layer, \( L_1 \), will be modified by editing its mask image so that the fabricated layer \( L_1 \) will align with the top layer \( L_2 \), and the resulted geometry will be the same as Fig. 6a.

**Property 2.** If the cured layer \((z = z_i)\) is outside of the meniscus curve \((p(x') > M(x'))\) for all \( p(x') = z_i \), the formed meniscus profile \( M \) is split by this layer. Let \( p(x', z_i) \) be a boundary point on this layer, and \((x_0, z_0), (x_n, z_n)\) are the boundary points of the bottom and top 2D
layers in the meniscus segment. Then the meniscus could be modelled by: 
\[ M = f(h_1, b_1) + f(h_2, b_2) = f(z_n - z_0, x') + f(z_n - z_i, x') \]

For example, in Fig. 6c, the 1st layer boundary \( x' \) extends out of the meniscus profile. So the meniscus formed between the 0th layer and 2nd layer is constructed by two curves \( f_1(L_1, t_1) \) and \( f_2(L_2, t_2) \), instead of the curve \( f(h = D, b = t) \) in Fig. 6b.

3.3. Meniscus process parameters characterization

With the two aforementioned properties and the mathematical models in Section 2, we can model the meniscus profiles that are formed among multiple 2D slicing layers. Fig. 7 is a 2D illustration of a meniscus formed over multiple layers. In this example, after the 1st meniscus profile is formed and cured for \( D_1 \), assume \( n \) 2D layers (i.e. 1st–nth) are built using the conventional Stereolithography process for 2nd meniscus segment \( D_2 \). After the nth layer is built, all the layers will be raised out of the liquid resin to form a meniscus profile to be cured. The process parameters that affect the meniscus shape in such a multiple layers case include:

1. Starting and ending positions: \( p_0, p_n \). They are called the meniscus points, where the meniscus forming process is performed. That is, the object is fabricated layer by layer until the nth layer has been built and the fabricated object is then raised out of liquid resin to form meniscus. All the layers from the 1st layer to the nth layer are called a meniscus segment, with a thickness of \( D \). It is the thickness that is used to divide curved surfaces into segments, as described in Section 3.1.

2. Boundaries of each cured layer in the meniscus segment: \( |p_i| i = 1, 2, \ldots, n \). Fig. 7 is an illustration of a 2D case. In this 2D example, \( p_1, \ldots, p_n \) are the boundaries of the 1st, \ldots, nth layers, respectively. Based on the two aforementioned properties, the boundary positions of the sliced 2D layers will determine the meniscus profiles.

3. Stair dimensions: \( t_i \) (i = 1, \ldots, n). They can be derived from the boundary positions of each 2D layer. Based on the two properties, if all the layers \( i = 1, 2, \ldots, n - 1 \) sit inside of the meniscus profile \( f(h = D, b = \sum t_i) \), the meniscus is this simple curve \( f(h = D, b = \sum t_i) \). However, if the ith layer breaks the meniscus curve \( f \), the meniscus will be calculated separately on the two sides of the ith layer, leading to \( f = f_1 \left( h = \sum t_i, b = \sum t_i \right) \cup f_2 \left( h = \sum t_i, b = \sum t_i \right) \).

Hence, the meniscus formed among multiple cured layers \( |p_i| i = 1, 2, \ldots, n \) could be written as:

\[ M(x) = f(p_i | i = 0, 1, \ldots, n) = \sum_{i=1}^{n} f \left( \sum_{i=k_m+1}^{k_m} l_i, \sum_{i=k_m}^{i=k_m+1} t_i \right) \]

\[ k_1 = 1; k_{M+1} = n; M < n; \]

\[ t_i = x(p_{i-1}) - x(p_i); \]

\[ L_i = y(p_i) - y(p_{i-1}); i \in \{1, 2, \ldots, n\} \]

As denoted in Eq. (7), the meniscus formed among layers \( |p_i| i = 1, 2, \ldots, n \) is a combination of \( M \) segments. The mth segment covers \( (k_m+1 - k_m) \) cured layers, i.e., from the \( p_{k_m} \) layer to the \( p_{k_m+1} \) layer.

3.4. Meniscus database construction

In a SLA system, the achievable resolution is limited by the resolution of mask image and material deposition. Ideally, it is desired to achieve infinitely small resolution for material deposition using the meniscus approach. However, regardless of whether the input energy is based on mask image projection or laser beam scanning, the light spot size is limited. Therefore, there is no need to calculate the continuous mathematical model of the formed meniscus. Instead, the performance will not be affected if the meniscus profile is sampled to a set of points with a controlled sampling density. By sampling the continuous meniscus curve into discrete points, the NP-hard optimization problem that is to be presented in the following sections can be solved approximately. Also the computation time can be greatly reduced based on sampling data.

A meniscus database is constructed to store the sampled points of functions \( f(h,b) \). As shown in Eq. (7), any meniscus formed among multiple cured layers is a combination of segments which are represented by function \( f(h,b) \) with different values of \( h \) and \( b \). With the constructed database of the meniscus sampling points, the process planning will not need to repeatedly solve and sample a meniscus segment. Instead, it can just search the corresponding point set in the database with a given setting \( (h,b) \). Fig. 8 shows the meniscus database structure used in our study. The material properties and model parameters of the two materials that are used in the experimental tests (E-shell and SI 500 from EnvisionTEC Inc.) are characterized in Table 1.

In our tests, the two materials have residue modeling parameters “Maximum wetting \( b_0 \)” and “Capillary height” as calibrated in Table 1. Also the Stereolithography system used in our tests has 5 \( \mu \)m XY plane resolution and 10 \( \mu \)m minimum curing layer...
Fig. 8. Database of the meniscus profiles M(x) with different h and b.

Fig. 9. An illustration of the curved surface information extraction process.

Fig. 10. Approximation error check process in optimized meniscus approach.
thickness in the Z axis. So in the database, the meniscus segment thickness is considered to be 1500 μm of $b$ and 1700 μm of $h$. It is further sampled into a set of segments $M(x) = f(h, b)$ by an increment of 5 μm in $b$ direction and 10 μm in the building direction $h$. Then each segment is sampled into a list of points $(x, M(x))$ by an increment of 5 μm in the X direction.

Fig. 8 shows the table of all meniscus segments $f(h, b)$. Each table element contains a list of sampling points. Therefore, the nonlinear equation solving problem is substituted with a simple database searching problem. Due to the limited system resolution and the material properties, the meniscus sampling point database is relatively small, and the data searching time can be rather fast.

### 3.5. Sampling CAD model into point-based meniscus segments

Similar to representing formed meniscus profiles, there is no need to use a continuous representation model for storing the curved surface information. A discrete point-based model representation can be used to meet the accuracy requirement of the meniscus-based SLA processes. Moreover, a point-based representation could be processed faster and more easily.

Since both boundary and normal information are required for planning the meniscus forming process, the Layered Depth-normal Images (LDNI) representation [20,21] is used to slice the original CAD input and generate the point set of a segment of curved surfaces with normal information. LDNI is an implicit representation of solid models that sparsely encodes the shape boundary in three orthogonal directions [20,21].

According to the definition of LDNI, an orthogonal projection is used in extracting LDNI points of a certain $Z$ segment from input polygonal models. That is, the coordinate of a query point used in the LDNI projection procedure is calculated as follows:

\[
x' = \text{minExt}(0) + I_x \times \Delta x
\]

\[
y' = \text{minExt}(1) + I_y \times \Delta y
\]

\[
z' = \text{minExt}(2) + I_z \times \Delta z
\]

\[
\Delta z = \text{ceil}(\text{maxExt}(2) - \text{minExt}(2)) / (s - 1)
\]

where $\Delta x$, $\Delta y$ denote the resolutions in the $X$ and $Y$ directions, respectively, and $\{\text{minExt}(k), k = 1, 2, 3\}$ and $\{\text{maxExt}(k), k = 1, 2, 3\}$ are vectors that represent the minimum and maximum values of the $X$, $Y$, and $Z$ coordinates of the calculated boundary box. $I_x$ and $I_y$ are the index of the $X$ and $Y$ axis, respectively. $s$ is the number of points.
of points per ray, and \( i \) is an integer between 0 and \( S \). Because only the current layer is of interest, \( z \) follows the boundary condition:

\[
\min \text{Ext}(2) = n \times D; \quad \max \text{Ext}(2) = (n + 1) \times D
\]

(14)

where \( n \) is the current segment number, and \( D \) is the meniscus segment thickness. Notice that \( D \) does not have to be the normal slicing thickness in AM processes. As mentioned in Section 4.1, the thickness of a meniscus segment is defined by the capillary height of the liquid, which is usually larger than the curing thickness in the SLA process.

By adjusting the values of \( \Delta x \), \( \Delta y \) and \( \Delta z \), the accuracy of the extracted LDNi sampling points can be controlled accordingly. It is reported that even for very complex geometries and high accuracy requirements, the construction process is fast with the aid of graphics hardware [22]. Fig. 9 is an illustration of the point-based meniscus segment of a given CAD model using the LDNI algorithms. Similar research in efficient slicing using LDNI has been reported before [23,24].

Positive normal denotes that the meniscus segment contains up-facing surfaces \((N \cdot Z > 0)\). Let \( \varepsilon \) be the threshold that indicates the curves that need the meniscus method. If \((N \cdot Z > \varepsilon)\), meniscus will be formed in order to better approximate the up-facing surfaces. The meniscus planning and process optimization will be presented in the following section.
4. Meniscus planning and process optimization

4.1. Optimization objectives

As discussed in Section 3, the optimization problem is a multi-objective problem:

1. Smoothness. This is achieved by the choice of the curve indicator $\varepsilon$ that was discussed in Section 3.5. The smaller $\varepsilon$ is, the bigger portion of the up-facing surfaces will be taken care of by using the meniscus approach.

2. Approximation errors: $(-e_{\text{min}}, e_{\text{max}})$. The dimensional accuracy is not guaranteed, and in some cases, may be worse if the formed meniscus is not planned well. Therefore, the dimensional accuracy is an important objective when we plan the meniscus-based SLA process. The required accuracy should be met together with the smoothness by applying the meniscus approach. In this research, $-e_{\text{min}}$ and $e_{\text{max}}$ is used to denote the bounds of the allowed approximation error.

3. Minimum build time. After the planned 2D layers are cured, the platform is raised up above the liquid surface to form meniscuses. After a certain waiting time, light will be projected to cure the formed meniscuses. This additional meniscus forming and curing process will elongate the build time. Hence using a minimum number of meniscus segments is desired for reducing the required build time.

4.2. Optimization strategies

Fig. 10 shows the meniscus process planning framework. Based on the $i$th meniscus segment information that is extracted from CAD model as discussed in Section 3, the $Z$ height map of curved surfaces is generated as $C(x,y)$. Accordingly, the $Z$ height map of the planned meniscus is shown as $M(x,y)$. By deducting the $C(x,y)$ from $M(x,y)$, an error $Z$ height map $E(x,y)$ could be obtained.

The initial meniscus plan is a simple meniscus curve with $h = D$ and $b = f$, as the example shown in Fig. 6a. If the error $Z$ height map is bounded between $[-e_{\text{min}}, e_{\text{max}}]$, it will be saved; otherwise, the meniscus process parameters as discussed in Section 3.3 will be adjusted. The following two strategies are used in modifying the meniscus shape:

1. If the left bound $-e_{\text{min}}$ is exceeded, new cured layer slices will be inserted to adjust the process parameters $\{p_i | i = 1, 2, \ldots, n\}$ to push the meniscus closer to the CAD profile.

2. If the right bound $e_{\text{max}}$ is exceeded, new meniscus starting point and ending point will be inserted to adjust the process parameters $p_0, p_n$ to pull the meniscus back to the CAD profile.

Fig. 11 shows the meniscus segment curing plan optimization algorithm.

Fig. 12 is a graphic illustration based on a simple 2D example. The orange curve represents formed meniscus profiles; the dotted orange curves represent sliced layers; and the black curve is the given surface profile defined in the CAD model. The meniscus shape in the area of $y \in (0, y_0)$ is first estimated and then compared with the input geometry. If the error map $E(x)$ is within the
acceptable range, points \( p_0(b_0, 0), \ldots, p_n(b_n, y_n) \) are selected as the meniscus points \( M_0 \) and \( M_1 \). If the meniscus curve is outside of the CAD profile and the error is represented by a positive \( \epsilon_g \), a new point will be selected as an additional meniscus point, as \( M_{0-1} \) in Fig. 12c. On the contrary, if the meniscus curve is inside the CAD profile, for example, the orange curve between \( M_{0-1} \) and \( M_0 \) in Fig. 12c, new slices would be added to push the meniscus curve outward to better approximate the CAD profile. Curves 50–1 is an example. Obviously, by selecting different positions for the inserted meniscus point M0–1, the modified meniscus would have different accuracy. With the same meniscus curing times and smoothness, it is desired to have the minimum approximation error.

Therefore, when the current meniscus plan cannot meet the accuracy requirements, how to identify the optimal position to insert new slices or new meniscus points, is the key to the meniscus process planning and optimization problem. Methods like brutal searching, trial and error, or guessing are not efficient and may not work for more complicated geometries. Consequently an optimization algorithm is required for the meniscus process planning problem.

4.3. Meniscus process optimization algorithms

To find the optimal position for inserting a new slice, the minimization problem can be formulated as follows.

**Policy for determining sub-meniscus point position:**

**Problem 1.1.** Input: \( C(x, y), \{ P_0, P_1, \ldots, P_n \} \)

\[
\min \left( C(x, y) - M'(x, y) \right) \mid y \in (y_0, y_n)
\]

\[
M'(x, y) = \sum_{i=1}^{n} k_i \cdot \left( f(P_y, b) = 0, 1, \ldots, n \right) + f(P_i | t = i, i+1, \ldots, n)
\]

\[
\sum_{i=1}^{n} k_i = 1, 0 \leq k_i \leq 1, k_i : \text{integer}
\]

where \( M'(x, y) \) is the modified meniscus among the 0th layer \( P_0 \) and the nth layer \( P_n \) as shown in Fig. 7, so the portion of the meniscus segment where \( y \in (y_0, y_n) \) is considered here. \( k_i \) is the indicator that whether the point \( P_i \) on the ith layer is selected as an inserted meniscus point \( M_{0-1} \). If \( k_i = 1, P_i \) is selected as a meniscus point. \( L_i \) and \( t_i \) are the height and width of the step created by the ith layer and the (i−1)th layer, as shown in Fig. 7.

Similarly, the position of the inserted additional sub-slice could be optimized by solving the following minimization problem:

**Problem 1.2.** Input: \( C(x, y), L, \{ P_0(x_0, y_0), P_1(x_1, y_1) \} \)

\[
\min \left( C(x, y) - M'(x, y) \right) \mid y \in (y_0, y_1)
\]

\[
M'(x, y) = \sum_{i=1}^{u} s_i \cdot \left( f(i = i \cdot l, |x - x_0|) + f((u - i) \cdot l, |x - x_1|) \right)
\]

\[
\sum_{i=1}^{u} s_i = 1, 0 \leq s_i \leq 1, s_i : \text{integer}
\]

where \( M'(x, y) \) is the modified meniscus among the 0th layer and 1st layer after having inserted a new slice, as the orange dotted curve shown in Fig. 12d. There are \( u \) candidate sub-slices generated by further slicing the 1st layer (from \( P_0 \) to \( P_1 \)) with a layer thickness \( l \). If \( s_i \) equals to 1, ith sub-slice is selected to be inserted between layer \( P_0 \) and layer \( P_1 \), at \( y = y_1 = y_0 + i \cdot l \). With the above two policies, the optimized meniscus segment fabrication plan could be generated for one meniscus segment and then the next one can be repeated until the whole CAD model is processed. The algorithms are described as follows.

**Algorithm 1.1. Insert meniscus point.**

Input: \( C(x, y), P_0, P_1, \ldots, P_n \), \( \epsilon_g \), \( \epsilon_{signal} \), meniscus point set \( M_0 \).

Output: an updated meniscus point set \( M_0 \).

1. Solve the minimization problem 1.1:
   \[
   \text{min}(\min(C(x, y) - M'(x, y)) \mid y \in (y_0, y_n))
   \]
   \[
   M'(x, y) = \sum_{i=1}^{n} k_i \cdot f(P_y, b) = 0, 1, \ldots, n + f(P_i | t = i, i+1, \ldots, n);
   \]
   \[
   \sum_{i=1}^{n} k_i = 1, 0 \leq k_i \leq 1, k_i : \text{integer}
   \]
   \[
   \text{modified}=\text{false};
   \]
   
   Then:
   \[
   \text{Update } M(x, y): (M(x, y) = M'(x, y) \mid y \in (y_0, y_n));
   \]
   \[
   E(x, y) = C(x, y) - M'(x, y) \mid y \in (y_0, y_n));
   \]
   \[
   \text{modified}=\text{true}.
   \]

2. If \( \text{modified}=\text{true} \):
   
   Add \( P_i \) to \( M_0 \).

3. If \( \text{modified}=\text{false} \): \( \text{add } P_i \) to \( M_0 \).

4.4. Meniscus process optimization example

To better demonstrate the presented optimization algorithms, an example of the meniscus process planning and optimizing process of Figs. 13–16. Suppose the maximum allowed approximation error is \( \pm 56 \mu m \), \( 30 \mu m \). Fig. 13a is the extracted CAD model of a sliced meniscus segment with \( D = 250 \mu m \) and Fig. 13b is its height map of the CAD model. We first set the curing layer thickness \( L \) the same as \( D \), which is the case as illustrated in Fig. 19, with \( D = 250 \mu m \) and \( b = 90 \mu m \). The meniscus curve information \( f(250, 90) \) relates to the element \( i = 24, j = 17 \) can be found in the table shown in Fig. 8. The meniscus model and the related meniscus height map are generated by using the sampling points in the corresponding list, as shown in Fig. 13c and d.

By deducing Fig. 13b from d, the error model and the related error height map can be generated as shown in Fig. 14a and b. It is found that the approximation error is in the range of \( \pm 56 \mu m \) after applying the planned meniscus. To demonstrate the benefit of the optimization algorithms, the meniscus process parameters were first modified randomly 20 times to find the meniscus model that gives best result. The optimal meniscus model generated by the
brutal search and its relevant error models are shown in Fig. 15a–d. It is shown that only a slight improvement was made with a random search with a pool size of 20. The approximation error is in the range (−41 μm, 26 μm) with the modified meniscus.

Instead of the brutal search, the optimization algorithm as described in Section 4.3 was used to find the optimal meniscus process parameters. The original meniscus result gives a perfect error upper bound, but the lower bound of the approximation error exceeds the given error bound, which leads to the second “if” condition in “Algorithm 1” being executed. Hence, Algorithm 1.2 is used to insert a new sliced 2D layer to modify the meniscus. The optimal meniscus process plan was identified after iterating once. The optimized meniscus models and its resulting error models are shown in Fig. 16. It is shown that the result is significantly improved by reducing the approximation error to (−20 μm, 0 μm), which has met the given accuracy requirement. Compared to the brutal search optimization, it returns much better results with less computation time.

5. Experimental verification of the optimized meniscus method

A Mask Image Projection based Stereolithography (MIP-SL) system has been built to verify the meniscus-based SLA approach presented in the paper. A photo of the prototype system is shown in Fig. 17. In addition, a Matlab program has been built to simulate the meniscus profiles for different variables. Accordingly, a meniscus database can be pre-computed and stored.

The model shown in Fig. 4 was fabricated with an optimized meniscus process method. In the optimized meniscus process, to reduce the positive approximation error, a new meniscus point was added into the previous meniscus point set \( \{M_i\} \), which was used for fabrication of Fig. 4b. The new meniscus point pulls the meniscus profile back to get closer to the CAD model profile. A graphical and quantitative comparison of the fabrication results using conventional method M1, previous meniscus method M2-1, and the newly optimized meniscus method M2-2, are shown in Fig. 18. It is shown that the part fabricated with optimized meniscus method M2-2 is
The developed MIP-SL testbed for verifying optimized meniscus method.

not only smoother than part fabricated with conventional method, but also closely approximates the given CAD model. The approximation error and roughness measurements of the parts are listed in Table 2. Because of the large approximation error, the part fabricated by previous meniscus method M2-1 is considered a failed part and its surface finish has not been measured quantitatively, thus the corresponding cell in Table 2 is denoted as NA.

In addition to the concave test case, a convex case was also tested to verify the optimization algorithm. Fig. 19 shows the fabricated results of a micro-lens with a convex surface. The lens size is 2.54 x 2.54 x 0.5 mm. Fig. 19a shows the CAD model and Fig. 19f shows a piece of paper with blue and red lines for testing. A layer thickness of 100 μm was used to fabricate the part. Fig. 19b and c are the microscopic images of the lens that is fabricated by the conventional layer-based MIP-SL process (lens A), and Fig. 19g shows the optical performance of the fabricated micro-lens. The paper image is distorted by the micro-lens due to its rough surface. In comparison, Fig. 19d and e are the microscopic images of the lens that is fabricated by the optimized meniscus method (lens B). The optical performance of lens B on the same piece of paper is shown in Fig. 19h, which demonstrates a significant improvement over that of lens A. Profiles of the two surfaces as showed in Fig. 19c and e were sampled. Quantitative measurements of the accuracy and surface finish were performed.

Table 2 shows statistics of the approximation errors and surface finish of the fabricated parts using conventional method, previous meniscus method, and the new meniscus method that is optimized. The experimental results agreed with simulation results, and verified the effectiveness of the optimized meniscus method in improving the surface finish of curved surfaces and meeting accuracy requirements.

6. Conclusion

A meniscus-based Stereolithography process and the related process planning and optimization methods have been developed.

Fig. 17. The developed MIP-SL testbed for verifying optimized meniscus method.

Fig. 18. Geometry profile of the models fabricated with the CAD model given in Fig. 4: using conventional layer-by-layer method M1, previous meniscus method M2-1, and the newly optimized meniscus method M2-2.

Fig. 19. A test case of a micro-lens using E-shell: (a) CAD model (c–d) microscopic image of the part fabricated using conventional method (e–f) microscopic image of the part fabricated using the optimized meniscus method, (g) surface measurements of the two parts.
for the fabrication of smooth curved surfaces. Theoretical models, process parameters characterization and optimization algorithms have been presented in the paper.

An example based on a convex lens has been presented to illustrate the meniscus optimization process. Based on the given CAD model, the meniscus models in a database, and the computed approximation error models, optimized meniscus process parameters have been identified using the developed algorithms. The result has been compared with the optimization results generated by random search. The example shows the improved efficiency and effectiveness of the optimization algorithm.

In addition, both hardware and software systems have been developed for physical experimental verifications. Test examples on up-facing surfaces have been performed. The comparison between experimental results with and without the process planning method illustrated the effectiveness of the developed method in simultaneously controlling the accuracy and surface finish of the fabricated objects. It also showed that the optimized meniscus approach can reduce the build time by minimizing the number of meniscus segments.

Although the presented micro-lens test case does not have a complex shape, the developed frame could be applied to meniscus process planning of any arbitrary complicated up-facing surfaces. Potential challenges include computation cost and meniscus structure construction. Moreover, by combining this approach with approaches for fabricating smooth down-facing surfaces, such as controlled curing depth approach, complicated geometries with curved surfaces could be fabricated with high smoothness by using Stereolithography equipment. Therefore, future work will include:

1. to improve meniscus database structure,
2. to improve prediction accuracy and test more complicated free-form surfaces, and
3. to integrate meniscus approaches for fabrication of both up-facing and down-facing curved surfaces.

References